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Editorial

Special Issue: The many facets of the Vlasov equation

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The Vlasov equation is a pillar of plasma physics. Proposed more than 80 years ago by Vlasov (and even earlier by Jeans in the context of gravitational dynamics), it encompasses a great variety of phenomena ranging from wave propagation, instabilities, nonlinear effects and turbulence. Coupled to the electromagnetic Maxwell equations, it constitutes the paradigm of a mean-field theory in plasma physics.

The usefulness of the mean-field approach can be hardly overstated. For instance, solving the full dynamics of a typical fusion plasma, containing roughly $N \approx 10^{20}$ charged particles per cubic metre, is a totally unrealistic task today, and very likely to remain so for a very long time, quantum computers notwithstanding. The mean-field approach simplifies this extremely complex scenario by neglecting all particle–particle correlations (the 'collisions') and retaining only the global electromagnetic fields generated by the charge and current distributions as if they were a continuous medium (the 'mean field').

Further, the accuracy of the Vlasov approximation can be quantified by a single adimensional coupling parameter,

$$g = \frac{1}{n\lambda_D^3} \propto \frac{n^{1/2}}{T^{3/2}},$$

where λ_D (cm), the Debye length, is the characteristic length of influence of a test charge beyond which the system behaves as a collective system, namely a plasma. Furthermore, n (cm^{-3}) is the number density, and T ($^\circ$K) is the temperature. The above coupling constant is proportional to the average ratio of the interaction (Coulomb) energy between two particles and their thermal energy. When this ratio is small, meaning that there is a very large number of particles in a Debye sphere, the two-body and higher-order correlations can be neglected and the mean-field theory is an accurate approximation, although a rigorous mathematical justification of the limit $g \to 0$ is a tricky, and not fully solved, problem.

The mean-field approximation has momentous implications on the complexity of the mathematical and numerical description of the plasma. Indeed, while the original N-body problem is characterized by a $6N$ dimensional phase space (considering three positions

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and three velocities per particle), the corresponding mean-field problem only needs a six-dimensional phase space. Thus, the mean-field plasma dynamics is fully described by a single-particle probability distribution $f(\boldsymbol{r}, \boldsymbol{v}, t)$ which evolves according to the Vlasov equation in the $(\boldsymbol{r}, \boldsymbol{v})$ phase space

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} + \frac{q}{m} (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial f}{\partial \boldsymbol{v}} = 0,$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ are given by the sum of external and self-consistent electric and magnetic fields. The latter are solutions of the Maxwell equations with sources given by the self-consistent charge density $\rho = q \int f \,\mathrm{d}\boldsymbol{v}$ and current density $\boldsymbol{j} = q \int f \boldsymbol{v} \,\mathrm{d}\boldsymbol{v}$.

In spite of its apparent simplicity, the Vlasov–Maxwell system underpins a huge variety of phenomena in all domains of plasma physics where two-body collisions can be neglected. In particular, both space and nuclear fusion plasmas are characterized by small values of g, thus justifying the collisionless mean-field approximation. In addition, the Vlasov equation is also currently used in other research areas, most notably the dynamics of self-gravitating systems such as galaxies and galaxy clusters, where it is usually referred to as the collisionless Boltzmann equation. Many of the theoretical and numerical methods developed in the context of either plasma physics or gravitational dynamics can be profitably transposed to the other research area.

The present Special Issue 'The many facets of the Vlasov equation' originates from a recent conference on the Vlasov equation held in Strasbourg (France) in July 2019.[1] This is part of the series of the 'Vlasovia' conferences, organized every three years alternatively in Italy and France since 2003. Previous editions were held in Nancy (2003), Florence (2006), Marseilles (2009), Nancy (2013) and Copanello (2016). Each of these conferences attracted between 60 and 100 participants, testifying to the existence of a vibrant interdisciplinary community of scientists interested in these topics.

Most papers appearing here originate from contributions and discussions presented at the 2019 Vlasovia conference. Nevertheless, this Special Issue was open to contributions from all scientists working on the physical, mathematical and computational aspects of the Vlasov equation. These papers reflect the diversity of the topics addressed at the Vlasovia conference and will hopefully constitute a valuable reference for future work on the Vlasov equation.

[1]Vlasovia 2019 – Sixth International Workshop on the Theory and Applications of the Vlasov Equation: https://vlasovia2019.sciencesconf.org/.

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