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JEAN DES MURS AND THE RETURN TO BOETHIUS ON MUSIC

The Musica speculativa *of Jean des Murs played a key role in renewing interest in the teaching of Boethius in the fourteenth century. We argue that this treatise is much more than a summary of the Boethian* De institutione musica *in presenting its core teachings as fully consistent within an Aristotelian theory of knowledge. Two versions of its prologue (1323 and 1325 respectively) are examined together with their relationship to Jean's* Notitia artis musicae *(1321) and the innovative significance of its mathematical-style presentation of the teaching of Boethius about proportions with its appeal to clear diagrams. We aim to guide the modern reader through the thought patterns and diagrams of Jean des Murs, demonstrating why the* Musica speculativa *was so widely studied in the later Middle Ages. The two different prologues are presented in English translation for the first time.*

While much has been written about the absorption of Aristotelian texts into the curriculum of the University of Paris, much less attention has been given to the role of Jean des Murs (*c.* 1290–5 to after 1344), whose interest in the teaching of Boethius on music went much further than was normally the case in the University of Paris in the early fourteenth century.[1] In recent years, there have been several editions of his *Musica speculativa secundum Boetium*.[2] There have been relatively few efforts, however, to

The following abbreviations are used:
BnF Bibliothèque nationale de France
SM Jacobus, *Speculum musicae*

[1] There have been many variants of the vernacular version of the name of Johannes de Muris, including Jean de Murs, used in New Grove and John of Murs in *Dictionary of Scientific Biography*, ed. C. C. Gillispie (New York, 1973), pp. 128–33, article by E. Poulle. Biographical details are given by L. Gushee in 'New Sources for the Biography of Johannes de Muris', *Journal of the American Musicological Society*, 22 (1969), pp. 3–26, and G. L'Huillier, 'Aspects nouveaux de la biographie de Jehan des Murs', *Archives d'histoire doctrinale et littéraire du moyen âge*, 54 (1979), pp. 272–6.

[2] The *Musica speculativa* has been edited by C. Falkenroth, *Die* Musica speculativa *des Johannes de Muris: Kommentar zur Überlieferung und kritische Edition*, Beihefte zum Archiv für Musikwissenschaft, 34 (Stuttgart, 1992); E. Witkowska-Zaremba: Musica Muris *i nurt*

appreciate the originality of a treatise which, we argue, is much more than a derivative summary of all five books of the Boethian *De institutione musica*.[3] We hold that Jean draws on a mathematical frame of mind to present the key arguments of Boethius as eighteen conclusions, but framed within epistemological principles provided by Aristotle.

THE STUDY OF THE *DE INSTITUTIONE MUSICA* OF BOETHIUS IN THE THIRTEENTH CENTURY

The interest of des Murs in the Boethian *De institutione musica*[4] went much deeper than the limited focus on its first two books mandated by the University of Paris.[5] Far fewer copies of the Boethian treatise survive from the thirteenth century than from the twelfth.[6] A vivid example of how

spekulatywny w muzykografii średniowiecznej/Musica Muris *and Speculative Trend in the Medieval Musicography*, Studia Copernicana, 32 (Warsaw, 1992); S. Fast, *Johannis de Muris Musica <speculativa>*, Institute of Mediaeval Music, Musicological Studies, 61 (Ottawa, 1994), online at http://www.chmtl.indiana.edu/tml/14th/MURMUSI_TEXT.html, based on her PhD dissertation, submitted under the name of S. Scea, 'A Critical Edition of Johannes de Muris's "Musica (speculativa)"' (University of Iowa, 1990); her choice for the principal text is the manuscript Milan, Biblioteca Ambrosiana, H. 165 inf. The edition and translation of C. Meyer, Jean de Murs, *Écrits sur la musique: Traduction et commentaire* (Paris, 2000), pp. 134–95, is based on that of Witkowska-Zaremba. In his edition Meyer says: 'nous a semblé offrir le texte le plus sûr' (p. 55). The text as edited by Falkenroth is also available in TML (Thesaurus Musicarum Latinarum) for Version A at http://www.chmtl.indiana.edu/tml/14th/MURMSP, and for Version B at http://www.chmtl.indiana.edu/tml/14th/MURMSPE. In this study, we refer to the editions of the *Musica speculativa* simply by the editor's name and within the work by the numbers in the text provided by Witkowska-Zaremba (which are also used by Meyer) and the page numbers in Witkowska-Zaremba and Meyer.

[3] Joseph Dyer refers to the renewal of interest in *musica speculativa* in the early fourteenth century, but claims 'Muris's *Musica speculativa* was intended to rectify the neglect of Boethius by providing a summary of the first two books of *De institutione musica* for the benefit of those lacking advanced mathematical skills' in his 'Speculative "Musica" and the Medieval University of Paris', *Music & Letters*, 90 (2009), pp. 177–204, at pp. 179–80. Gilles Rico offers a detailed exposition of interest in Boethius in a university context in his DPhil thesis, 'Music in the Arts Faculty of Paris in the Thirteenth and Early Fourteenth Centuries' (Oxford University, 2005), especially pp. 76–148 on glosses on Boethius. Christian Meyer describes the treatise as 'un résumé axiomatisé' of the teaching of Boethius in 'Un abrégé universitaire des deux premiers livres du "de institutione musica" de Boèce: Édition et commentaire', *Archives d'histoire littéraire et doctrinale du moyen âge*, 65 (1998), pp. 91–121, at p. 96.

[4] From now on all references in the form 'Boethius II.1' will refer to *De institutione musica* unless otherwise indicated.

[5] Dyer lays out the evidence of these manuals in 'Speculative "Musica"', pp. 189–92. See also Rico, 'Music in the Arts Faculty of Paris', p. 30.

[6] See the introduction to *Codices Boethiani: A Conspectus of Manuscripts of the Works of Boethius*, i, ed. M. T. Gibson and L. Smith, Warburg Institute Surveys and Texts, 25 (London, 1995), pp. 26–7; C. Bower, 'Boethius' *De institutione musica*: A Handlist of Manuscripts', *Scriptorium*, 42 (1988), pp. 205–51; and M. Bernhard, 'Glosses on Boethius' *De institutione musica*', in A. Barbera (ed.), *Music Theory and its Sources* (Notre Dame, Ind., 1990), pp. 136–49. An abridgement of the first two books produced no later than 1273/4 has been edited by Meyer, *Jean de Murs*.

Aristotle's criticism of Pythagorean tradition in the *De caelo* affected the standing of Boethius in the 1270s is provided by Johannes de Grocheio's criticism of the notion of an audible music of the spheres, upheld by followers of John of Garland.[7] Grocheio himself only refers to the first two books of the *De musica*.[8]

That only the first two books of Boethius were studied in the late thirteenth century is also evident from comments made by Jacobus (*c.* 1260–after *c.* 1330) in his vast *Speculum musicae*, written sometime after 1325, that in his youth he had heard lectures on the first two books of the *De institutione musica*.[9] Afterwards, having acquired a complete copy of this work of Boethius 'from a certain worthy man (*quodam valente*)', he started to read much more widely to deepen his understanding.[10] Jacobus reports that while there are some people who extract a few things from that treatise, 'they say things that hardly accord at all with the intention of Boethius'.[11] The possibility must be considered that Jacobus was directing his polemic against those, like Jean des Murs (whose *Musica speculativa* Jacobus quotes almost verbatim),[12] who were interpreting Boethius in a very different way from traditional abridgements of the *De institutione musica*.

[7] Johannes de Garlandia, *De mensurabili musica: Kritische Edition mit Kommentar und Interpretation der Notationslehre*, ed. E. Reimer (Beihefte zum Archiv für Musikwissenschaft, 10–11; Wiesbaden, 1972), online at http://chtml.indiana.edu/tml/13th/GARDMM. Johannes de Grocheio, *Ars musice* 5.5, ed. C. J. Mews, J. N. Crossley, C. Jeffreys, L. McKinnon, and C. J. Williams, TEAMS (Kalamazoo, 2011), pp. 56–8.

[8] Grocheio, *Ars musice* 1.2–4.18, *ibid.*, pp. 43–56, largely taken from Book 1 of Boethius, *De institutione musica*. On the date of Grocheio's treatise, see the editors' discussion in the Introduction, pp. 10–12.

[9] Margaret Bent has discovered an early fifteenth-century inventory recording that Jacques, traditionally identified as 'of Liège', was referred to as Magister Jacobus de Hispania, leading her to propose that he could possibly be identified with an Oxford-educated master James of Spain, related to the royal family of Spain; M. Bent, *Magister Jacobus de Ispania, Author of the* Speculum musicae, Royal Musical Association Monographs, 28 (Farnham, 2015), pp. 6–7 and 82–91. Rob Wegman suggested that *Hispania* instead could refer to Hesbaye, an archdeaconry of Liège (R. C. Wegman, 'Jacobus de Ispania and Liège', *Journal of the Alamire Foundation*, 8 (2016), pp. 253–74, at p. 253). See Jacques de Liège, *Speculum musicae*, ed. Roger Bragard, Corpus Scriptorum de Musica, 3 (Rome, 1961), Book II, ch. 56: 'Timens autem ne tacta Boethii Musica mihi concessa tolleretur a me, ut de ea memoriale <aliquid> mihi retinerem, ut amplius in ea proficerem, ut confidentius illa uti possem, qui de duobus primis libris, quos Parisius audieram, aliqua extraxeram, plura coepi et de illis et de aliis excerpere, in aliquibus locis textum Boethii quem habebam nudum, sine scriptis, sine glossis abbreviare, in aliquibus locis qui mihi difficiliores videbantur, ut occurrebat, exponere in textu et figuris.' References to this edition will be in the form '*SM* II.56'.

[10] *SM* II.56.

[11] *SM* III.1, 3, p. 11: 'Hic venerabilis Boethii Musica, quae nunc magna ex parte derelicta videbatur, ad memoriam revocatur. Nam, etsi aliqui in suis musicae tractatibus de illa sumant aliqua, illa sunt pauca et de assumptis quaedam dicunt quae minime ad intentionem vadunt Boethii, ut patebit infra.'

[12] *SM* VII.6.

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If Jacobus was born around 1260, his interest in exploring all five books of Boethius may have been influenced by the Dominican author Jerome de Moravia (of Moray), who seems to be one of the earliest theorists to initiate renewed interest in all five books of the Boethian treatises.[13] Jerome incorporates almost all of Books I–IV into his *Tractatus de musica*, with a few paragraphs from Book V.[14] He was particularly interested in combining what Boethius had to say with more recent treatises on mensural music by John of Garland, Franco, and Petrus Picardus, all masters of what came to be known as the *ars antiqua*. Exactly when Jerome completed his *Tractatus* is not certain, but it must have been after 1272 (as he incorporated part of the commentary on the *De caelo* of Thomas Aquinas, composed between late 1271 and his death on 7 March 1274) and before 1306, when the manuscript was bequeathed by Peter of Limoges (*c.* 1240–1306) to the library of the Sorbonne. Jerome's importance in broadening attention to beyond the first two books of the *De institutione musica* deserves attention in appreciating the achievement of Jean des Murs in creating an epitome of the entire treatise, not just of its first two books.

THE *NOTITIA ARTIS MUSICAE* AND THE TWO VERSIONS OF THE *MUSICA SPECULATIVA*

Jean des Murs does not explicitly acknowledge Boethius by name in his first exposition of the theory and practice of music, the *Notitia artis musicae*, a work dated to 1319 according to a false *explicit*, but in fact 1321.[15] In this

[13] There had been a Dominican convent at Moray (Moravia) since 1233/34. The vitality of Moray is evident from the database People of Medieval Scotland (although Jerome of Moray is not mentioned there), at https://www.poms.ac.uk. It is quite possible that Jerome's interest in all five books of Boethius may have been stimulated by comments of John of Garland in his *Musica plana* as preserved in Paris, BnF lat. 18514, fols. 85r–94r, ed. C. Meyer, *Musica Plana Johannis de Garlandia* (Baden-Baden, 1998), pp. 3–21, quoted by Jerome at the outset of his *Tractatus*. Speaking about chromatic, enharmonic and diatonic scales, John comments (ed. Meyer, p. 4): 'De hiis tribus supra Boecius 4o que 5. Libro diffusius est prosequtus.' John seems to refer to the glosses on all five books on Boethius that precede the *Musica plana* in this manuscript (fols. 1r–85r); the authorship of these glosses deserves further investigation.

[14] See the Index fontium in Hieronymus de Moravia, *Tractatus de musica*, ed. G. Lobrichon and C. Meyer, Corpus Christianorum Continuatio Mediaeualis, 250 (Turnhout, 2012), pp. 272–5. Jerome refers to Book V. 2–3, 8, 9, 11 at the opening and close of his *Tractatus*. On his reading of Boethius, see C. Meyer, 'Lecture(s) de Jérôme de Moravie, lecteur de Boèce', in Michel Huglo and Marcel Peres (eds), *Jérôme de Moravie. Un théoricien de la musique dans le milieu intellectuel parisien du XIIIe siècle* (Paris, 1992), pp. 55–74.

[15] MS Paris, BnF lat. 7378a, fol. 60vb. The *Notitia artis musicae* is edited by U. Michels in Johannes de Muris, *Notitia artis musicae et Compendium musicae practicae*. Petrus de Sancto Dionysio: *Tractatus de musica*, Corpus scriptorum de musica, 17 (Rome, 1972), and edited and translated by Meyer in Jean de Murs, *Écrits sur la musique*, pp. 58–111. It is available in TML at http://chmtl.indiana.edu/tml/14th/MURNOT_TEXT.

treatise, he sets out his understanding of the theory and practice of music, introduced by a prologue in which he draws on the teaching of Aristotle in the *Metaphysica* to emphasise that while theoretical understanding is superior to experience, all art must be based on experience.[16] The first of its two parts begins with discussion of the generation of sound and its relationship to number (chapters 1–2), followed by discussion, drawn from Boethius, of what Pythagoras had discovered about *musica*, namely the relationship between consonances and numerical proportions (chapter 3). Chapters 4–5 are similarly based on core teachings of Boethius (taken mostly from Books I–II) about the impossibility of dividing the tone into two equal parts and the overlapping of the hexachordal system on an octave-based system. The second part of the *Notitia* is not related to Boethius, being concerned with the measure of time, and is organised into nine 'conclusions'. Jean's way of identifying time according to both imperfect and perfect measures would be fundamental to the stylistic revolution of the *ars nova*. He was taking care to explain how he could respect the teaching of the ancients while insisting that no one should ever say that they had ever fully defined the discipline: 'For opinions and revolutions of knowledge run as if coming back in a circle.'[17] Jacobus referred to this very passage of the *Notitia* in his *Speculum musicae*, but added a reference to Jean's source, 'from Aristotle in the book of *Meteorology*', in order to mock his argument, since he thought it absurd to compare new revolutions in musical knowledge to the passage of the seasons.[18] Jacobus was suspicious of how Jean des Murs seemed to disparage tradition in speaking of revolutions in this way.

The decision of Jean des Murs to write the *Musica speculativa* as a synthetic overview of the core teachings of Boethius can be seen as a response

Karen Desmond argues that the date 1319 given in MS Paris, BnF lat. 7378A, fol. 60vb implies only that the *Notitia* was written between 1319 and 1321; K. Desmond, *Music and the Moderni, 1300–1350* (Cambridge, 2019), p. 28, but the date 1321 comes from Murs himself; see *Notitia*, ed. Michels, 9.

16 *Notitia*, Prologue, ed. Meyer, p. 58. The innovative character of the *Notitia* is emphasised by Dorit Tanay, 'The Transition from the "Ars Antiqua" to the "Ars Nova": Evolution or Revolution?', *Musica Disciplina*, 46 (1992), 79–104.

17 *SM* VII.1: 'Currunt enim, ut aiunt, et ab Aristotele in libro metheorum sumunt opiniones et scientiae revolutiones nam et, ubi nunc est arida, prius fuit aqua.' *Notitia* II.15, ed. Meyer, p. 110: 'Currunt enim opiniones et scientiae revolutiones ad circulum revertentes, quamdiu summae placuerit voluntati eius, qui non necessitatus omnia condidit in hoc mundo et omnia voluntarie segregabit.'

18 *SM* VII.1. Aristoteles Latinus, *Liber Meteorum* X.2.II, ed G. Vuillemin-Diem (Brussels, 2008), p. 85: 'Accidit enim sepe in talibus repulsa prima parte fluentis corporis propter non subcedere, aut propter artitudinem aut propter repercutere, circulum et reuolutionem fieri spiritus. Hoc quidem enim in anterius prohibet procedere, hoc autem posterius impellit, quare compellitur in latus, qua non prohibetur, ferri, et sic semper habitum, donec utique unum fiat, hoc autem est circulus: cuius enim una latio figure, hoc necesse circulum esse.'

to criticism that he had not drawn sufficiently from Boethius in his *Notitia*. According to a rubric recorded in two manuscripts of the initial version of the *Musica speculativa*, des Murs composed it in June 1323, while teaching at the Sorbonne.[19] This version, edited by Witkowska-Zaremba from mostly fifteenth-century manuscripts conserved in Central Europe, but taken there by masters and students from Paris during the Great Schism (1378–1417), begins with a prologue (*Etsi bestialium voluptatum*), which argues from the authority of Aristotle in the *Ethics* that the senses, when used in moderation, are not bad in themselves, but can lead, in the case of music, to the greatest delight. The primary example is that of Ulysses, who, as Aristotle reports in the eighth book of the *Politics*, delighted in song.[20] This leads Jean to rectify what he sees as the prevailing neglect of Boethius:

> Because the books of the ancient philosophers, not just about music but also [those] of the other mathematicians, are not read and because of this it happens that they are abhorred as unintelligible or much too difficult, it seemed to me a good thing that, from the *Musica* of Boethius which, according to the power given to me by God, I have studied closely and understood to some extent with God's favour, I have been devising a short treatise in which I shall try to make manifest the more beautiful and essential conclusions pertaining to the very art of music with clarity of discussion and vividness of meaning.[21]

In this initial version (A) Jean then begins the first part of the treatise with four distinctly non-Boethian principles which will guide his argument:

[19] Witkowska-Zaremba provides an English language summary of the manuscripts of Version A on pp. 148–66. Using her sigla, the closing rubric to the *Musica speculativa* in (O) Oxford, Bodleian Library, Bodley, 77, fol. 99va (1450–1500), describes the work as an abbreviation of the *De institutione musica* of Boethius: 'Explicit musica Boecii abbreviata a magistro Johanne de Muris anno domini 1323 mense iunii parisius in sarbona [*sic*].' By contrast, (S) St Paul in Lavantthal, Benediktinerstift, 264/4 (*c.* 1400) offers the title by which it is more often known: 'Explicit musica speculativa secundum Boetium per magistrum Johannem de Muris abbreviata Parisius in Sorbona anno domini 1323.' This copy concludes with a diagram of the monochordal instrument of des Murs and explanatory notes. While questions about the dates offered by U. Michels, *Die Musiktraktate des Johannes de Muris*, Beihefte zum Archiv für Musikwissenschaft, 8 (Wiesbaden, 1970), are raised by Desmond in *Music and the Moderni*, p. 100, the fact that des Murs was very attentive to chronology in his astronomical writings suggests that these rubrics deserve respect.

[20] Aristotle, *Politics* VIII.3, ed. F. Susemihl (Leipzig, 1872), p. 339: 'propter quod quidem Homerus ita poetizavit: "sed est quidem velut ad epulas <vocare> congaudere" et ita dicens alteros quosdam "qui vocant cantitaturam naturam tamquam delectantem omnes".'

[21] *Musica speculativa*, I.10 W-Z (hereafter I.10 W-Z), p. 172; Meyer, p. 136: 'Verum quia istis diebus libri antiquorum philosophorum nedum de musica, sed et de ceteris mathematicorum pluribus non leguntur, et ob hoc accidit eos tamquam inintelligibiles au nimis difficiles abhorreri, visum est mihi bonum, ut ex Musica Boetii, quam secundum vires mihi a Deo datas perstudui eamque favente Deo aliqualiter intellexi, tractatum brevem elicerem, in quo conclusiones pulchriores et essentialiores ad ipsam artem musicae pertinentes cum sermonis claritate et evidentia sententiae manifestare conabor.'

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- All teaching and every discipline arise from pre-existing knowledge.
- No other [knowledge] is found prior to that from sensory knowledge.
- States [of any knowledge] rest on manifold experience as its endpoint.
- The experience of sensible things creates art.[22]

Jean's development in the *Musica speculativa* of these principles or axioms marks a significant evolution from the *Notitia*, in which he had only spoken in general terms about how musical art must rest on experience. They draw on Aristotle's *Posterior Analytics* and *Metaphysics* but are formulated in condensed form. The sense of the third axiom (*Experientiae multiplici ut in termino status acquiescere*) is clarified by the closing assertion of the *Notitia*, that 'no one should say we have attained the state of music and its immutable end'.[23] These principles, with their implicit argument that music must rest on experience, run directly against the opening claims of Boethius about *musica mundana* and *humana* as harmonies not based on audible sound, unlike *musica instrumentalis*. Significantly, Jean avoids all reference to the opening discussion of Boethius about different kinds of *musica*, the first two of which cannot be known by experience.

The *Musica speculativa* differs from the *Notitia* in not having any discussion of time and mensuration, but rather demonstrates profound familiarity with the argument of all five books of the *De Musica*. Instead, Jean expands on the core elements he had provided more briefly in the *Notitia*, structured around four propositions:

- Pythagoras passed on to us the art of sounds
- He attached the force of numbers for the sake of symphony
- The three harmonies are perfect consonances
- These three consonances [*melodiae*] offer clarifying numbers.[24]

What Jean here calls three harmonies or melodies are the perfect consonances of the fourth, fifth and octave (*diatessaron*, *diapente* and

[22] I.11–14 W-Z, p. 173; Meyer, p. 136:
'Omnem doctrinam et omnem disciplinam ex praeexistenti cognitione fieri.
'Ante cognitionem sensitivam non aliam inveniri.
'Experientiae multiplici ut in termino status acquiescere.
'Experientiam circa res sensibiles artem facere.'
We have followed the reading of Milan, Biblioteca Ambrosiana, H. 165 inf, fol. 1r.

[23] *Notitia* II.14, in Meyer, *Jean de Murs*, p. 110: 'Nemo tamen dicat nos statum musicae et finem eius immutabilem tetigisse.'

[24] The four propositions are in I.15–77 W-Z, pp. 173–6; Meyer, pp. 138–42: 'Pythagoram nobis artem tradidisse sonorum; . . . Propter symphoniam subiungere vim numerorum; . . . Iam tres harmonias perfectas esse sonantes; . . . Has tres melodias numeros dare clarificantes.'

diapason). His theme is that they first exist as sounds rather than as numbers, although Pythagoras was primarily concerned with defining the consonances in terms of numeric proportions.

Jean's decision to modify the *Musica speculativa* by going back to the introduction of the *Notitia* has never been fully explained. According to a coded rubric in one important manuscript of the second (B) version of the treatise (Paris, BnF lat. 7378A; P_3 in Falkenroth's edition),[25] Jean des Murs revised this version in 1325.[26] This means that he was writing soon after Pope John XXII had issued his decree (of 1324/25) condemning the new style of music, *ars nova*.[27] In this revised preface, Jean goes back to the core opening arguments of the *Notitia*, omitted in Version A, to produce a more rounded justification for his manner of proceeding. In the later version, he did not make any allusion to contemporary neglect of Boethius, perhaps because the claim was no longer relevant. Instead he went back to his earlier argument that while theory is superior to practice, all knowledge rests on experience. He now included, however, the four central principles that he had introduced without explanation in Version A. Des Murs may have produced a transitional version prior to producing Version B, as found in Milan, Biblioteca Ambrosiana H. 165 inf., fols. 1^{r}–16^{r}.[28] It begins in the same way as the *Notitia* about Aristotle's teaching on the

[25] Concerning the two versions see F. Hentschel, *Sinnlichkeit und Vernunft in der mittelalterlichen Musiktheorie: Strategien der Konsonanzwertung und der Gegenstand der Musica Sonora um 1300* (Stuttgart, 2000), pp. 314–22.

[26] Michels, *Die Musiktraktate*, p. 21, n. 21 acknowledges the assistance of Frobenius in understanding the explicit: 'Hic liber expletur, si <sic *ms.*> quid nimis est, resecetur, si minus <nimis *ms.*> addatur, et sic ars vera paratur. Summe decem cubice, duplate, terque quadrate et semis: armoniae sunt hec <hee *ms.*> sic ab<b>reviate.' ('Here ends the book, which, if it is too much, can be shortened, if too little, added to, and thus the true art is obtained. Take ten cubed, a double and three squares and a half: these *Harmonies* are thus abbreviated. *Summe decem cubice* ... may be interpreted as 'ten cubed [plus] a double [of 10] and three times the square, and a half', that is to say $10^3 + 2 \times 10 + 3 \times 10^2 + 10/2$, or 1325. Michels interprets this as saying that this 1325 version was a shortening of the 1323 version, which was no longer being read in the Sorbonne.

[27] On the decree, see Helmut Hucke, 'Das Dekret "Docta Sanctorum Patrum"', *Musica Disciplina*, 38 (1984), 119–31.

[28] This version is transcribed in TML http://chmtl.indiana.edu/tml/14th/MURMUSM_MMBAH165, accessed 27 August 2020. It was labelled as Version A/B by Falkenroth, *Die* Musica speculativa *des Johannes de Muris*, pp. 13–15. More comprehensive textual comparison is needed to identify precisely how the Version B, preserved in several copies, including Ambrosiana H. 165 inf., fols. 126^{r}–132^{v}, relates to the 'mixed' (possibly transitional) version preserved in three MSS: Milan, Bibl. Ambrosiana C. 241 inf. (copied in Paris in 1401); Berlin, Staatsbibliothek Preußischer Kulturbesitz, lat. fol. 600 and Paris, BnF lat. 7295; see Michels, *Notitia artis musicae*, p. 16, and Hentschel, *Sinnlichkeit und Vernunft*, pp. 317 and 319.

priority of theory over practice (there also included in the prologue) and then follows with chapters about the relationship of music to sound introduced with a slightly revised statement: 'Since *musica* (music theory) is adaptation (*adequatio*) of sound related to number, it is necessary to consider both, namely number and sound.'[29]

THE CONCLUSIONS AND DIAGRAMS OF THE *MUSICA SPECULATIVA*

The bulk of the first part of the *Musica speculativa* is devoted to four propositions and eighteen conclusions concerned with the proportions of various consonances as presented by Boethius, demonstrating the competence of Jean des Murs as an accomplished mathematician. The second part is, according to Jean, more about practice, although it might be more accurate to say 'the theory of practice'.

In addition to Paris, BnF lat. 7378A, a fourteenth-century manuscript (*c.* 1362), identified by Lawrence Gushee as particularly important in the manuscript tradition of Version B of *Musica speculativa*, the manuscript BnF lat. 7207 (*c.* 1430–60, of Italian origin) is remarkable for its clarity and accuracy, in particular of its diagrams.[30] Some manuscripts do not include the diagrams at all, e.g. Oxford, Bodleian Library, Bodley 77, fols. 95ra–99va; others contain mistakes in the figures in both senses of the word: numbers in the text and the diagrams themselves.[31] Des Murs frequently advises the reader to consult the diagrams, as will be seen below.

The approach of Jean des Murs in formulating propositions and conclusions was surely influenced by his study of Euclid, probably at the University of Paris. He had knowledge of not only the arithmetic but also the algebra of al-Khwārizmī,[32] and he would demonstrate his mathematical abilities more fully in subsequent writings on

[29] Milan, Biblioteca Ambrosiana, H. 165 inf., fol. 1^{r}: 'QVoniam musica est de sono relato ad numeros adequatio necesse est utrumque numerum, scilicet, et sonum simul consyderare.'

[30] Gushee, 'New Sources', p. 6. As noted in J. N. Crossley, 'The Writings of Boethius and the Cogitations of Jacobus de Ispania on Musical Proportions', *Early Music History*, 36 (2017), pp. 1–30, at p. 26, n. 127, the numbers in the diagrams seem to have been copied by someone other than the main scribe.

[31] The texts in the editions of the *Musica speculativa* of Jean des Murs are exemplary insofar as the editors have consulted a huge range of manuscripts and noted variant readings. The diagrams, however, fare much less well. Witkowska-Zaremba's, which Meyer uses, is almost error free. The editions by Fast and Falkenroth, however, have numerous errors.

[32] Al-Khwārizmī's *Arithmetic* was the basis for algorism, the methods of calculating with Arabic numerals. Jean's *Quadripartitum numerorum* contains an extended exposition of al-Khwārizmī's *Algebra*, though it must be admitted this work only dates from 1343, twenty years or so after the *Musica speculativa*; G. L'Huillier, *Le "Quadripartitum numerorum" de Jean de Murs: Introd. et éd. critique* (Geneva and Paris, 1990).

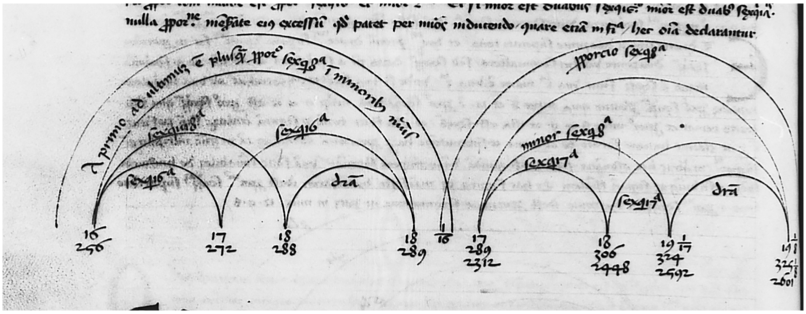

Figure 1 *Figura* D from Paris BnF lat. 7207, fol. 295ᵛ, courtesy of the Bibliothèque nationale de France

astronomy.[33] He constantly refers his readers to the diagrams.[34] The *Conclusio sexta* remark is particularly striking and he rightly says 'everything is clear in the figures', and the diagram explaining the sesquioctava proportion is perhaps the most complicated in the work (see Figure 1).[35] Since readers, medieval and contemporary, have often found these figures hard to understand, a few words of explanation may be helpful. Most of the diagrams comprise semicircles, often with their diameters. The diameters may be regarded as the string on a monochord and the semicircle is simply to give an idea of the extent. In his edition of Boethius Friedlein used brackets instead of semicircles (presumably because of typographical constraints), which may be clearer.[36] However the lengths of the semicircle diameters in Jean des Murs manuscripts bear no direct relation to the length of the string.[37] The redrawn diagrams in the present article *do* usually bear such a relation, as will be explained below.

The propositions and conclusions in Part I of the *Musica speculativa* can easily be related to corresponding elements in the *De institutione musica* of Boethius, but the same cannot be said of Part II; see Table 1

[33] See Poulle, 'John of Murs'.
[34] I.85 W-Z, p. 177; Meyer, p. 144: conclusio prima: 'patet in hoc figura'; I.154 W-Z, p. 181; Meyer, p. 152: conclusio sexta: 'In figuris haec omnia declarantur'; I.176 W-Z, p. 182; Meyer, p. 154: conclusio septima: 'Et hoc patet in figura'; I.217 W-Z, p. 185; Meyer, p. 158: conclusio duodecima : 'Quae autem nunc dicta sunt, praesens figura declarat; I.249 W-Z, p. 188; Meyer, p. 164: conclusio quarta decima: 'ut videri potest in hac figura'.
[35] The designations '*Figura* D', etc., are taken from Witkowska-Zaremba. The calculations for this figure will be discussed below; see Table 2.
[36] See, for example, Boethius III.9, in *Anicii Manlii Torquati Severini Boeti:. De institutione arithmetica libri duo. De institutione musica libri quinque*, ed. G. Friedlein (Leipzig, 1867), p. 279.
[37] See also below, n. 42 on Part II where Jean des Murs uses frequencies as well as string lengths.

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Table 1 Comparison of Jean des Murs, *Musica speculativa* and Boethius, *De institutione musica*

Jean des Murs	Boethius (except for Jean I, Prop. 4)
Part I	
Propositio 1	I.9, 10
Propositio 2	I.10 (end)
Propositio 3	I.18, II.18, 20, 32
Propositio 4	*Arithmetica* II.54[1]
Conclusio 1	II.23
Conclusio 2	II.24
Conclusio 3	II.25
Conclusio 4	I.18
Conclusio 5	I.10, II.22
Conclusio 6	III.1
Conclusio 7	I.17, II.31
Conclusio 8	II.31
Conclusio 9	II.31
Conclusio 10	I.16, III.1
Conclusio 11	I.17, II.28
Conclusio 12	II.30
Conclusio 13	II.31. III.4
Conclusio 14	III.4
Conclusio 15	II.31, III.4, 6, 7, 12
Conclusio 16	I.16, 26, II.20
Conclusio 17	I.16, II.26
Conclusio 18	II.27, V.9[2]
Part II	Approximate correlatives
Propositio 1	I.27
Propositio 2	III.9
Propositio 3	III.10
Propositio 4	
Propositio 5	IV
Propositio 6	IV.18

[1] See also the last chapter of Jean's *Arithmetica speculativa* in H. L. L. Busard, 'Die "Arithmetica speculativa" des Johannes de Muris', *Scientiarium Historia*, 13 (1971), 103–32, at p. 132.
[2] Jean supports the Pythagoreans and rejects Ptolemy, while Boethius simply states both views (see the main text).

and below. In Proposition 4 Jean starts with the diagram with which Boethius ends *De institutione arithmetica*,[38] which is mentioned again in

[38] Boethius, *De institutione arithmetica*, II.54; this is the last figure of the *De institutione arithmetica* and may be found at the very end, p. 173, in Friedlein's edition. Note that the orientation of the numbers varies; see n. 42 below.

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De institutione musica, I.10: 'Haec ultima figura est in suprema calce Arithmeticae.'[39] Jean then proceeds to extract all he can from this figure; at the same time he makes his audience (or students) so familiar with this figure that the succeeding ones should be less forbidding and more obvious.

The first *propositio* (see above) is not purely mathematical in nature; along with the third *propositio* it relates the Pythagorean tradition of associating numeric proportions with musical consonances. In the second *propositio* he lists the basic consonances and their proportions: octave (*diapason*): double; fifth (*diapente*): sesquialtera; and fourth (*diatessaron*): sesquitertia, commenting that the remaining basic interval, the tone, is not itself a consonance but is in sesquioctava proportion. The third *propositio* expounds the connections between pitches (*voces*) and consonances. In the fourth *propositio* he explains the relation between the basic consonances (plus the tone) with a Boethian-style diagram, *Figura* A (see Figure 2).[40] He uses the description from *propositio tertia* in terms of pitches with associated intervals in this diagram but inserts the numbers 6, 8, 9, 12, which he had treated in *propositio secunda*. In this way he guides the reader from the familiar area of the musical scale to the numeric proportions. The first five *Conclusiones* all use these numbers, and the accompanying figure, for their elucidation.

It should first be pointed out that there is a significant difference between proportions and ratios.[41] Medieval music theorists rarely use the word *ratio*, except in the sense of reason, favouring *proportio* for the numerical aspects. A ratio is a relation between two numbers, for example 3 : 2. On the other hand, a ratio *represents* a proportion; thus the sesquialtera proportion is represented by 3 : 2 but also by 6 : 4, 18 : 12, etc. Further, medieval writers did not necessarily specify the order of the terms in a proportion, so, unlike modern mathematical language, Jean does not need to specify the relative positions of the items he is comparing.[42]

[39] This sentence does not occur in Friedlein's edition and may be a later addition, but does in Anicius Manlius Severinus Boethius, *De institutione musica*, Patrologia cursus completus, series latina, ed. J. P. Migne, 221 vols. (Paris, 1844–1904), 63, col. 1178B.

[40] In Witkowska-Zaremba, p. 176, there are errors in the text of the proportions in the diagram, but the correct proportion names are to be found in Paris, BnF lat. 7207, fol. 294^{v}, though the numbers are reversed from left to right; on the variation in orientation see n. 42 below. Meyer's version is correct (p. 142).

[41] See Crossley, 'Cogitations', p. 10.

[42] The orientation of the figures with the numbers increasing from left to right or *vice versa* seems not to have troubled medieval readers. Cf. Boethius, *Fundamentals of Music*, trans., with Introduction and notes by C. M. Bower, ed. C. V. Palisca (New Haven and London, 1989), p. 98, n. 19, discussing the diagrams in Boethian manuscripts, specifically those for Boethius III.3: 'The diagrams ... reflect a stage of musical thought in which direction (left or right) had no implication with respect to pitch. In modern terms one can think of the numbers as representing the lengths of a string in one direction and the

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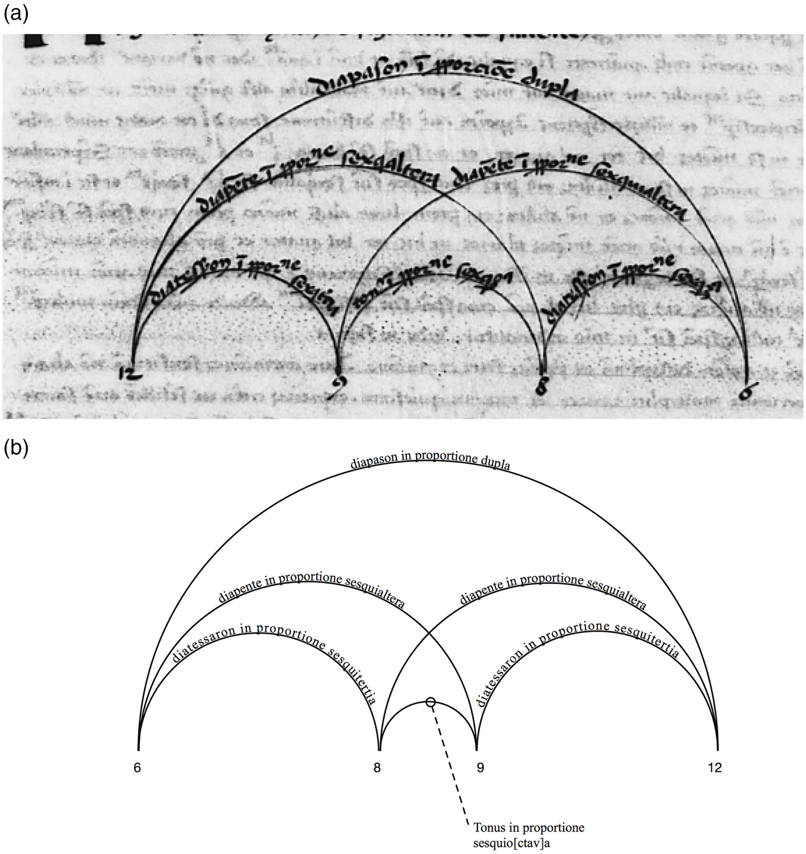

Figure 2 *Figura* A: (a) from Paris, BnF lat. 7207, fol. 294v, courtesy of the Bibliothèque nationale de France; (b) redrawn with modified lengths. The orientation varied in medieval texts; see n. 42 below

Perhaps the easiest way to express this is that a sesquialtera proportion holds between two items (numbers, strings, pitches, for example) when

frequency of the pitches for the other direction: a string twice as long creates a note an octave lower, while a note of twice the frequency is an octave higher.' However, it is not until Part II, *Propositio sexta*, that Jean des Murs notes the inverse connection between frequency and string length (II.91–3 W-Z, p. 201, Meyer, p. 189). 'Et quoniam vult Boetius, quod longior chorda plures partes et maius spatium obtinet breviori, ergo sibi maiorem numerum attribuit et sonum gravem, breviori chordae minorem numerum et sonum acutum, licet fieri aeque bene posset e converso: longiori, eo quod pauciores motus continet breviori, sibi minorem numerum dare, breviori vero plures numeros, eo quod plures motus habet, ut ipse innuit libro suo; quocumque modo fiat, numquam proportio variatur.' (But he does have a related remark in *propositio seconda*, II.28–31 W-Z, p. 182: Meyer, p. 177.)

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the larger is 1½ times the smaller. Here it seems appropriate to add a further note on the language. First, Jean names all the proportions that he uses, and this means he does not need to write down specific numeric ratios. We shall try to adhere to his style; hence terms involving 'sesqui' will be common, where 'sesqui-*n*th' (where *n*th means *n*th part) means the proportion of (1 plus $1/n$) : 1 (or vice versa).[43] Thus he will say, for example, that two lengths 'are in sesquialtera proportion' and this simply means that the ratio of the larger to the smaller is 3 : 2.

Just before the first *Conclusio* Jean says: 'now is the time to extract the mysteries of this figure and the marvels contained, one by one';[44] thus it is appropriate to consider how the figures are constructed. As noted above, they comprise semicircular arcs and straight lines, usually accompanied by words describing the proportion involved. Jean admits the figures look complicated: 'But this figure [Figure 2] can quite reasonably be said to be like a bow', which it is indeed: it has an overarching curved bow. He then goes on to say that many forms lie hidden in it.[45]

Here is one way to understand the construction in which we use the numbers that come from the last figure in the *De institutione arithmetica* of Boethius for ease of navigation.[46] The proportions between 6, 8, 9 and 12 are, successively, sesquitertia, sesquioctava and sesquitertia, corresponding to a fourth, a tone and another fourth.[47] So, to construct the figure, start with the interval of a fourth on the left. Next to it draw a semicircular arc for the tone and then another arc for the second fourth (from numbers 9 to 12). Returning to the left draw the next arc higher in the figure, that for the fifth (going from the number 6 to the number 9). Now go back to the left the distance of a tone and draw an arc from there (at the number 8) to the far

[43] Equivalently $(n+1) : n$. Jean des Murs is unusual in that he happily uses fractions in some of his work, though the aim was always to present proportions by ratios reduced to their lowest terms where the numbers involved were whole numbers. For example, a sesquitertian proportion of 8 : 6 would be presented using the ratio 4 : 3.

[44] I.77 W-Z, p. 176; Meyer, p. 142: 'iam tempus est huius figurae misteria et inclusa mirabilia extrahere sigillatim'.

[45] I.71 W-Z, p. 176; Meyer, p. 142: 'Sed haec figura quasi unum chaos, in quo latitant plures formae, potest satis rationabiliter appellari.' *Chaos* is the Latin version of the Arabic قوس (*qaws*), which means 'arch' or 'bow' or 'arc of a circle' and is also the name of the constellation Sagittarius. (Thanks to Charles Burnett for this elucidation in London and email of 26 June 2018.)

[46] Boethius, *De institutione arithmetica*, II.54, p. 173 in Friedlein's edition.

[47] Bellissima reports that the ancient Greeks used the sequence fourth, tone, fourth. See F. Bellissima, 'Propositions VIII.4–5 of Euclid's *Elements* and the Compounding of Ratios on the Monochord', *BSHM Bulletin: Journal of the British Society for the History of Mathematics*, 30 (2015), pp. 183–99, at p. 193.

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right (number 12). This is another fifth. Finally draw the largest arc, corresponding to the octave (*diapason*).

In manuscripts containing the Boethian-style figures the arcs for the fourths and the tone are usually all of the same size. However, in our redrawn version of *Figura* A in Figure 2 (and subsequent figures except where noted otherwise) the lengths have been modified so that putting the length corresponding to, say, the tone, next to the length corresponding to the fourth gives exactly the same length as the fifth and likewise a fourth plus a fifth gives an octave.

Now we proceed to consider some of the conclusions in order. In his first *Conclusio*, that the proportion for the octave is greater than that for the fifth, des Murs deftly introduces superparticular proportions in the following way, exemplifying as well as defining: a superparticular proportion is less than a multiple and contains one whole (but never two wholes) plus one part,[48] for example, one and a half (sesquialtera), one and a third (sesquitertia), etc.

The first five conclusions are all evident from *Figura* A (see Figure 2 above) and the numbers previously singled out by Boethius: 6, 8, 9 and 12.[49] For this figure he chooses to start with 9, which is in sesquialtera proportion to 6, then add a third part of 9 (namely, 3), giving 12 (forming a sesquitertia proportion), which is the double of 6, 'as shown in the figure'.

The sleight of hand is in the choice of starting with 6. Numbers in Boethian texts were usually only whole numbers. To get a sesquitertian proportion the starting number needs to be divisible by 3. However, if we start with 3 and then go, via a sesquitertian proportion, to 4, we cannot go to a whole number by a sesquioctava proportion; instead we would get $4 \times 9/8 = 4½$. So go back to the beginning and start with an even number divisible by 3. The smallest is 6. One can

[48] I.79 W-Z, p. 177; Meyer, p. 144: 'Omnis proportio superparticularis minor est proportione multiplici, quoniam multiplex continet minorem integre, ut bis vel ter vel quater et sic deinceps; sed superparticularis numerus numquam continet bis minorem, sed semel cum aliqua sui parte, ut media, tertia, quarta et cetera, semper augendo partes et minuendo proportiones.' More than one whole plus one aliquot part is a multi-superparticular proportion and if there are more parts it is a multi-superpartient proportion. Murs only deals with one of these latter (in *conclusio quinta decima* near the end of his treatise in its first form; see below). However, other redactions do include definitions of these terms; see e.g. *Musica speculativa*, ed. Falkenroth, p. 87.

[49] At the end of *conclusio septima* (I.175 W-Z, p. 182; Meyer, p. 154) he does simply say: 'Et hoc patet in figura' ('And this is clear in the figure').

then proceed along the sequence 6, 8, 9, 12. Boethius developed this technique, though he did not explicitly explain it, and Jacobus revived it;[50] Jean des Murs explained the technique, as will be seen shortly.

The technique appears to be related to, indeed may have led to, Euclid VIII.4, which is a proposition on how to combine sequences of ratios into a continuing sequence.[51] In this case the sequence of ratios Muris considers is 1 : 2 (octave), fifth (2 : 3), fourth (3 : 4) and tone (8 : 9).[52] So he starts with 6, which is the least number divisible by 2 and 3 (the right-hand numbers of the first two ratios), giving 6 : 9 :12, and the first and last numbers give the ratio for the octave, the first and second for the fifth (*diapente*). Here the fourth (*diatessaron*) is already represented by 9 : 12. Finally, the tone is incorporated by including 8 to give 6 : 8 : 9 : 12. (In fact, the fifth is represented twice, by both 6 : 9 and 8 : 12 and likewise the fourth by 6 : 8 and 9 : 12.) One virtue of these four numbers is that all possible ratios between them correspond to the octave, fifth, fourth or tone.

Conclusio sexta is of a different nature; it comprises a long attempt (following Boethius, *De musica*, III.1) to prove that the tone cannot be divided into two equal parts; we shall deal with this after *Conclusio quinta decima*.

When he comes to *Conclusio septima*, which says that a fourth is two tones plus a semitone, des Murs reveals his hand and gives the reason for starting with the particular numbers he chooses and the key to the general method. He is not only more explicit than Boethius, but he invites the reader, who is addressed in the second person, to join in. First, however, he simply presents the numbers – those in the top line of the table in *Figura* E (see Figure 3).[53] Then he says that these are the smallest whole numbers (*primi numeri*) by which the ratios can be represented.[54] He continues: 'And such is the art of finding such consonances.'[55] Then he explains, and we paraphrase:

> When you want a number of tones or sesquioctava proportions, start from the first octuple, which is 8 and advance one to 9. If you want two, start from the second octuple (i.e. octuple of octuple = $8 \times 8 =$) 64; if three, start from the third ($8 \times 8 \times 8 =$) 192. So, from 64, one tone takes you to $64 \times 9/8 = 72$ and another to $72 \times 9/8 = 81$.

[50] See Crossley, 'Cogitations', p. 18.
[51] See Bellissima, 'Propositions', pp. 192–3.
[52] As mentioned earlier, when dealing with proportions medieval writers did not necessarily specify the order of the numbers; a sesquialtera proportion simply meant that one number was one and a half times the other.
[53] In the text here, following the discussion in the seventh conclusion about dividing the tone, he repeats the Boethian comment that a semitone is called such because it is imperfect, not because it is half a tone. *Figura* E is, in part, on fol. 295^{v} of Paris, BnF lat. 7207, but does not have all the arcs and only contains the top line of the table.
[54] For a discussion of the terminology, see Crossley, 'Cogitations', p. 13.
[55] I.168 W-Z, p. 182; Meyer, p. 154: 'Et est ars inveniendi tales harmonias talis.'

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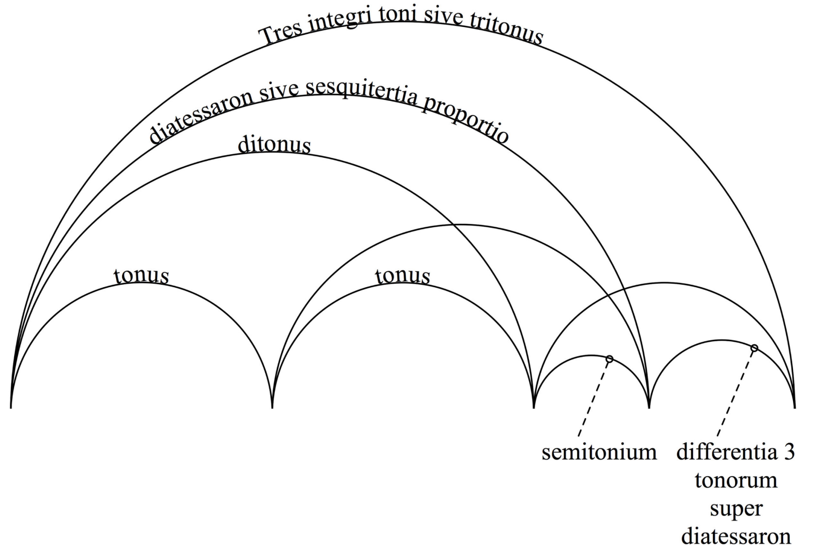
(a)



(b)

192	216	243	256	273⅜
512	576	648	[682⅔]	729
1536	1778	1944	2048	2187

Figure 3 *Figura* E redrawn

The general principle is that if you want to take a sesqui-'something' proportion then you multiply by the 'something' so that the division gives a whole number: thus for sesquioctava: multiply by 8; for sesquitertia, multiply by 3. He then interpolates a remark that if you go up by a sesquitertia proportion from 64 you will get a fourth (*diatessaron*), but 64 does not have a (whole number) third part. So multiply everything by 3 and consider 64 × 3 =192, which does have a third part, and this is shown in the first four numbers in the top line of Figure 3 (*Figura* E).[56] He does not explain the remaining two lines of numbers, but the second line is simply obtained by multiplying the top line by 8/3.[57] The third line is obtained by multiplying the third line by 3 so that all the numbers are whole. *Conclusio*

[56] The fifth number, 273⅜, corresponding to three whole tones, he has calculated earlier, before he gives the explanation (I.164 W-Z, p. 182; Meyer, p. 154).
[57] We do not know why 682⅔ is omitted; perhaps because of the fraction two-thirds.

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octava, which says that a fifth is three tones plus a semitone, is then obvious from the preceding discussion. Putting together parts of the last two arguments then yields *Conclusio nona* that two semitones and five tones comprise an octave.[58]

Conclusio decima simply points out that since a semitone is not half a tone the difference between a tone and a semitone is a different kind of semitone. The former is called a minor semitone or *diesis* (256 : 243) and the latter a major semitone or *apotome* (2187 : 2048),[59] but the calculations only come in *Conclusiones un-* and *duo-decima* respectively. The first calculation is clear from the table in *Figura* F (see Figure 4). For the major semitone one wants to add sufficient to form a tone from a minor semitone. So he starts with 243, but this has no (whole number) eighth part. Applying the method of *Conclusio septima*, he multiplies by 8, getting 1944. Then, multiplying 256 by 8 yields 2048, giving 2048 : 1944 for the minor semitone. For the tone this means taking a sesquioctava proportion from 1944, which is 1944 $\times$ 9/8 = 2187, giving 2187 : 2048 for the major semitone, and he then adds these intervals, as is shown in Figure 4.[60]

The thirteenth conclusion shows that six tones are greater than an octave: the difference is a comma, whose proportion is 531441 : 524288. The technique is again to multiply by 8 the appropriate number of times so that one can take sesquioctava proportions. Jean's treatment is quite succinct.[61]

In establishing *Conclusio quarta decima*, that an interval of five tones is greater than an interval of two fourths, although the respective numbers may be found in *Figura* H they are not mentioned – nor needed – in the text.[62] The explanation is a simple use of the previous result, but following the main conclusion he revisits the result, this time starting from five tones, again using a figure and saying: 'For this see the figure below.'[63] There he shows a fourth ascending from the lowest number (*diatessaron intensa*) and another fourth descending from the highest (*diatessaron remissa*) and these clearly do not fill up the five

[58] Des Murs uses the word *pasodia*, which has caused much discussion but simply seems to be an amalgam of πασῶν and διά with the order reversed from the normal *diapason*. It also occurs in the fifteenth-century MS Ghent, Rijksuniversiteit 70 (71), fol; 103r; see *Cuiusdam Cartusiensis Monachi Tractatus de Musica Plana*, ed. S. Lebedev (Tutzing, 2000), p. 50.

[59] The Greek names are only introduced in *Figura* F; see below.

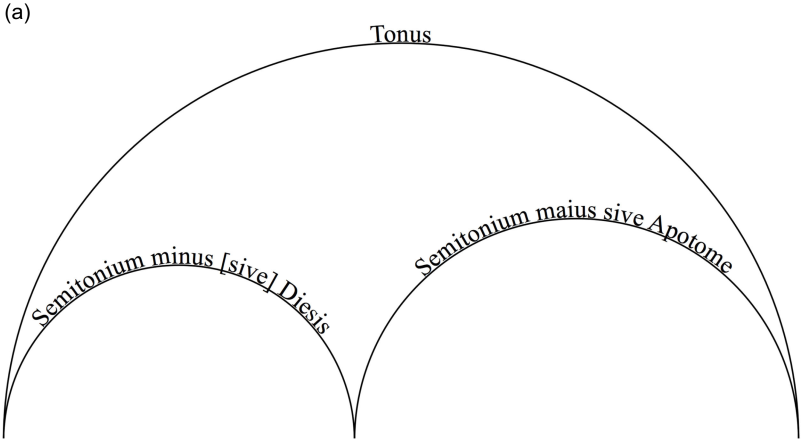
[60] I.217 W-Z, p. 185; Meyer, p. 158: 'Quae autem nunc dicta sunt, praesens figura declarat.' See also below regarding the *differentia*.

[61] A full discussion of the method may be found in Crossley, 'Cogitations', pp. 20–2.

[62] *Figura* H W-Z, p. 188; Meyer, p. 162.

[63] I.244 W-Z, p. 187; Meyer, p. 162: 'Propter hoc vide figuram subscriptam.'

(a)



(b)

	differentia		differentia	
243	13	256	$17\frac{3}{8}$	$273\frac{3}{8}$
1944	104	2048	139	2187

Figure 4 *Figura* F redrawn

tones. He only mentions at the end that the numeric differences may be found in *Figura* I (see Figure 5): ‘as can be seen in this figure’.[64]

This is the first time he uses the terms *intensa* and *remissa*, but they will be more frequent in Part II.[65] The inclusion of the *differentia* is somewhat problematic. It would appear that des Murs simply means the difference between the two components: the five tones and two fourths. However, he associates a number $596\frac{1}{12}$ with this difference. One possible interpretation is as follows. The *differentia* is simply the difference between the two ends of the *diatessara*. It has no other significance. If it were moved to left or right the difference in pitches that it indicates would change.[66] If five tones were equal to two *diatessera* then the two arcs would meet and the *differentia* would be zero.[67]

Conclusio quinta decima shows the comma is ubiquitous: it is the difference between a tone and two minor semitones and the difference between an octave, which comprises five tones and two (minor) semitones, and six tones.

[64] I.249 W-Z, p. 188; Meyer, p. 164: ‘ut videri potest in hac figura’.
[65] Boethius uses them in his Book III.
[66] They would represent different fractions of a tone.
[67] I.249 W-Z, p. 188; Meyer, p. 164: ‘ut videri potest in hac figura’.

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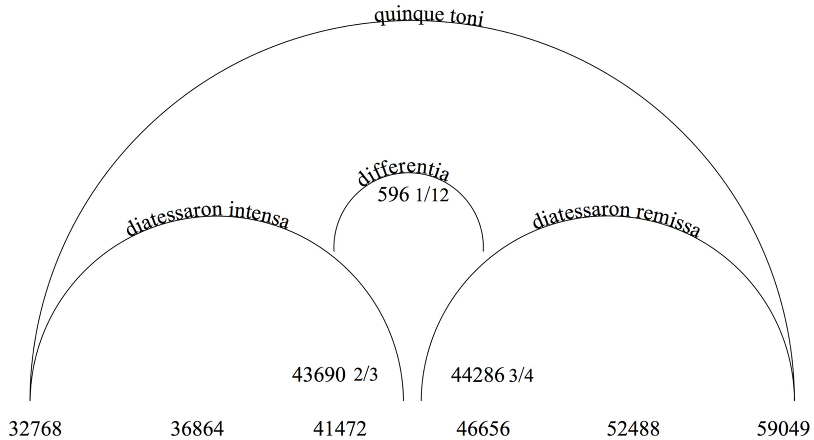

Figure 5 *Figura* I redrawn

Des Murs then has a series of thirteen corollaries, which he claims are evident, which is true of (1) to (5). (6) A true semitone, viz. a genuine half of a tone, does not have a proportion, that is to say, it cannot be expressed by a ratio of whole numbers. From his Pythagorean point of view he therefore infers (7) that a true semitone does not exist in the nature of things.[68] Corollary (8) seems to have been corrupted in many manuscripts, and should probably read 'The comma is not in a multiple or superparticular proportion', which is clear since it is in the proportion 1 and 7153/524288 : 1, roughly 1 7/500: 1, which is a superpartient proportion.[69] Version A indicates that 'the comma does not exist in number but is greater than 75 and less than 74'. This seems to be a corruption of Boethius, III.12, whose chapter heading says: 'In what proportion of numbers the comma may be, and that it is greater than 75 to 74 and less than 74 to 73.'[70] (9) The minor semitone is greater than 20 : 19 and less than 19 : 18. This is curious since Boethius III.13, says it is greater than 20 : 19 and less than 19½ : 18½, which is indeed less than 19 : 18. However, this comes from the sixth conclusion (see Table 3 in Appendix 2). Corollary (10) incorrectly states that the

[68] I.270 W-Z, p. 189; Meyer, p. 166: 'verum semitonium in rerum natura non existere'.

[69] More than one whole plus one aliquot part to one is a multi-superparticular proportion, e.g. 1⅓ : 1, and if there are more parts it is a multi-superpartient proportion, e.g. 2⅔ : 1.

[70] I.271 W-Z, p. 189; Meyer, p. 166, read: 'comma in numero nullo esse, sed maius 75 et minus 74'. Clearly the first words contradict the previous statement that the comma is expressible by the ratio 531441 : 524288 (or 1 and 7153/524288 : 1), but then the rest seems to be taken from Boethius III.12: 'In qua numerorum proportione sit comma, et quoniam in ea quae major sit quam 75 ad 74, minor quam 74 ad 73.' (There are many different versions in the various manuscripts of the *Musica speculativa*.)

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Table 2 The lower right of *Figura* D reconstructed from Paris BnF lat. 7207, fol. 295v

17	18	$19\frac{1}{17}$	$19\frac{1}{8}$
136	144	[$152\frac{8}{17}$][1]	153[2]
289	306	324	$325\frac{1}{8}$
2312	2448	2592	2601

[1] Missing in Witkowska's edition (p. 181).
[2] This row is missing in Paris BnF lat. 7207 (Figure 1).

major semitone is greater than 15 : 14 and less than 14 : 13; in fact both proportions are greater than a major semitone.[71] Corollaries (11) to (13) concern the comma and all require complicated calculations and are perhaps later additions, though we expect Jean des Murs would have been capable of them given his mathematical expertise.

Now we return to Jean's discussion of the division of the tone into two equal parts and the (misleading) role of the *differentia*. In his *Conclusio sexta* he starts from Boethius I.17, at the end of which he will again say that 'everything is clear in the figures'.[72] The tone is in the proportion of 9 : 8 but there is no (whole) number between 8 and 9, so he considers another representation of the same proportion, namely 18 : 16. Like Boethius he points out, as is clear, that 17 is 1/16th more than 16 but 18 is 1/17th more than 17, so the proportions of 18 : 17 and 17 : 16 are different. For this he presents one of his most complicated figures, which is really two figures: one starting with 16 and looking at a sesqui-16th and the other from 17 with a sesqui-17th proportion. For the latter, where he takes successive sesqui-17th proportions he does not resile from working with fractions, getting successively 17, 18, $19\frac{1}{17}$ and then $19\frac{1}{8}$, the last figure being obtained by taking a tone from 17, giving $9\frac{1}{8} \times 17$. He thus constructs Table 2.

The lower rows of this table are obtained as follows. First, multiply the initial row by 8 to eliminate the fraction in $19\frac{1}{8}$; perhaps the third entry is omitted because it involves the awkward fraction $8\frac{1}{7}$. The row below is obtained from the *first* row by multiplying by 17, which then gives a whole number, 324, in the third place but leaves a fraction of an eighth in the final one. So finally multiply by 8 to resolve this. Since these numbers are not included in the main text one wonders if they were interpolated later.

[71] A major semitone is greater than 16 : 15 and less than 15 : 14. This is correctly presented in a comment in MS Munich, Universitätsbibliothek 4° Cod. ms. 743, and this is the only correct manuscript version cited by Witkowska-Zaremba, p. 231.
[72] I.154 W-Z, p. 181; Meyer, p. 152: 'In figuris haec omnia declarantur.'

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The problem with this approach to trying to show that the tone cannot be equally divided is that one has to show that *all* representations of the sesquioctava proportion, 9 : 8, 18 : 16, 27 : 24, etc., fail to provide an exact halving. Boethius was misled into looking at the additive difference between the numbers in a ratio while compounding of ratios actually requires multiplication. This is similar to the awkwardness noted above at the end of the fourteenth conclusion with the *differentia* (marked as 596½).

The sixteenth and seventeenth conclusions give remarkably simple mathematical results: a double octave corresponds to a quadruple proportion and an octave plus a fifth is in the proportion 3 : 1.

This concludes his abridgement of the mathematical calculations of Boethius, which Bower described as 'relentlessly technical'.[73] The final, eighteenth, conclusion deals with a contentious item in Boethius V. 7–10, namely whether an octave plus a fourth is a consonance. Jean des Murs espouses the Pythagorean view that a fourth plus an octave cannot be a consonance since 2 : 1 and 4 : 3 give the proportion of 8 : 3, a superpartient proportion, and therefore is neither a multiple nor a superparticular proportion since it comprises 2 and two parts (specifically two-thirds) rather than just one extra part. He then puts forward Ptolemy's view that it is a consonance and claims to refute it.[74] He claims that this is similar to whether the fourth below a fifth is a consonance.[75] This, he says, clearly is a consonance (e.g. C–F–C), although a fourth above a fifth (e.g. C–G–C) is better and is sweet-sounding.[76] He then says this revolves around whether the fourth precedes the fifth in the way that a sesquialtera proportion 'precedes' (*priorem esse*) a sesquitertian one, finally appealing to his *Figura* K, which reprises *Figura* A (see Figure 2 above),[77] but now with the number 3 also included to give a double octave, perhaps imitating the way Boethius ends his *De institutione arithmetica* (not the *De institutione musica*). The numbers are 3, 6, 8, 9 and 12, which give rise to all the other intervals (consonances) he has just explained. His argument revolves around the point that in the hierarchy of consonances the fourth only arises out of the octave and the fifth.[78]

[73] Boethius, *Fundamentals of Music*, trans. Bower, p. xxxi.

[74] I.304 W-Z, p. 191; Meyer, p. 170: 'Nec Boetius in sua Musica nec alii musici, quos viderim, hanc quaestionem determinant.'

[75] Cf. Boethius VII.6: 'Utrum diatessaron ante diapente sit consonantia.'

[76] I.305 W-Z, p. 191, Meyer, p. 170: 'non est dubium, quin supra diapente optima sit et dulcis'.

[77] See Witkowska-Zaremba, p. 192; Meyer, p. 173. The diagram is not included here.

[78] I.316 W-Z; Meyer, p. 172: 'Et sic diatessaron non ex se est, sed ex duabus consonantiis actu, scilicet diatessaron et diapente, quibus duobus positis impossibile est illam non

Part II shifts from consideration of music as based on harmonic proportions to the actual divisions of the monochord, 'in which all consonances and their parts and parts of their parts are laid out'. Jean argues that this will provide 'a foundation for constructing a variety of instruments, knowledge of ones yet unknown, and discovery of ones that are new'.[79] This was radically different from Boethius, whose focus was firmly on the past and was little concerned about the sounding music of instruments.

It is often difficult to see such direct connections with the *De institutione musica* of Boethius as one can in Part I. Nevertheless, many of the constructions can be found in IV.5 of Boethius, but there they are involved in the partition of the monochord according to the diatonic *genus*. What Jean does is to show the general method, whereas Boethius constructs notes on the musical scale successively. As Matthieu Husson puts it, Jean's 'contraction produced a systematisation of the content of the original [Boethius]'.[80]

Propositio prima sets up the basic consonances. Jean starts with a string (*chorda*) *ab*, which represents an octave (see Figure 6).[81] Note that the letters are as used in geometry, as for example by Jordanus Nemorarius in his *Arithmetica*, and do not denote pitches.[82] Taking another string that is half the length, up to *c*, but still labelling the other end *a*, means that *ab* is twice the length of *ac* and so *ab* sounds an octave lower than *ac* when plucked. He then takes a third string which he divides into three equal divisions by inserting points *d* and *e*, giving a sequence *a*, *d*, *e*, *b*.[83]

Since the lengths of *ab* and *ae* are in the ratio of 3 : 2, the plucked lengths resound a fifth. Finally dividing *ab* into four equal parts, where the points of division are *f*, *c* and *g*, one can get lengths in the ratio of 4 : 3, yielding a fourth. Finally, since a tone is the difference between a fourth and a fifth, plucking the string lengths *ae* and *ag* resounds a tone.

poni, immo forte non nisi secundum quid differt ab ambabus.' Note that Meyer corrects Witkowska-Zaremba, who interchanges *diatessaron* and *diapason*. This accords with Paris, BnF lat. 7207, fol. 297^{v}.

79 II.1 W-Z, p. 193, Meyer, p. 174: 'in quo omnes consonantiae et earum partes et partes partium denotantur, ac inde per consequens de compositione variorum instrumentorum et cognitione ignotorum, inventioneque novorum maximam praestat fidem'. Further, although he only goes as far as two octaves plus a fifth, he is explicit that the methods can be extended further (see II.127 W-Z, p. 203; Meyer, p. 150).

80 M. Husson, 'Coping with the Tiny: Measures, Computations, and Reasoning with Small Amounts in Jean des Murs's Quadrivial Works', *Erudition and the Republic of Letters*, 4 (2019), pp. 146–65, at p. 153.

81 Paris, BnF lat. 7207 does not include this figure.

82 Jordanus de Nemore, *De Elementis Arithmetice Artis: A Medieval Treatise on Number Theory*, ed. H. L. L. Busard, 2 vols. (Stuttgart, 1991).

83 A simple version, akin to our version of *Figura* L, may be found in Milan, Biblioteca Ambrosiana, H. 165 inf, fol. 11^{v}.

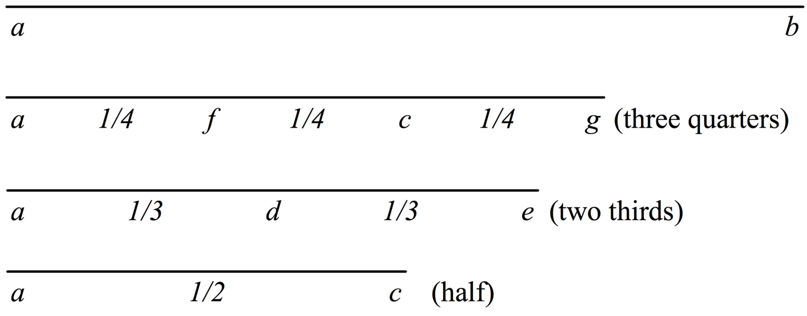

Figure 6 Redrawn from the lower part of *Figura* L

Des Murs also includes a bow diagram showing all the fourths and fifths and differences of a tone between them, so the figure becomes very complicated.[84] With the four strings corresponding to the lengths, he links them with curves to indicate the various intervals. Then, striking the strings will generate the basic consonances 'so that you [singular] can judge the consonances, which you did not know of before, by your ears'.[85]

Jean recalls that the four strings, taken together, reconstruct the Tetrachord of Mercury, as recorded in Boethius I.20, who, referring to Nicomachus, says that in the beginning there was 'simple music' performed on four strings. These provided only consonances: the outer two resounding an octave and the middle two a fourth or fifth up or down from the ends.[86] This of course corresponds to *Figura* A (Figure 2 above). Des Murs recommends listening to the strings being struck many times so that 'from the information of the intellect and the senses, you will marvel at the appearance of the consonances and judge the natural consonances as marvellous'.[87]

Propositions 2–4 show how to use fourths and fifths, and later tones, to create ascending or descending intervals. Note that the diagrams have the higher notes to the left, the opposite way to present practice. The second proposition concerns creating a tone at the top or bottom of an interval on the monochord (see Figure 7). The process obviously

[84] Meyer, p. 175, n. 63, notes that understanding this caused difficulties and, in certain manuscripts, phrases had been added to clarify matters.
[85] II.14 W-Z, p. 194; Meyer, pp. 174: 'ut auris habeat consonantiam iudicare, quam prius ignorabas'.
[86] For an interesting discussion of a fourteenth-century explanation of this tetrachord see J. Bates and S. McCoy, 'Mercury's Tetrachord', *Early Music*, 10 (1982), 213–15.
[87] II.14 W-Z, p. 194; Meyer, p. 174: 'et informatione intellectus et sensus, miraberis circa sonorum consonantias apparentes, tunc mirabiles consonantias naturales iudicabis tunc mirabiles consonantias naturales iudicabis'.

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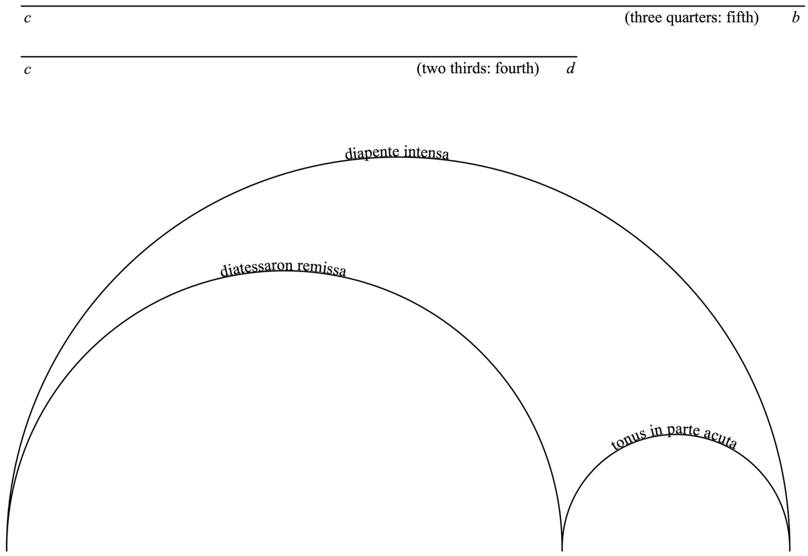

Figure 7 Redrawn from the left part of *Figura* M

follows Boethius III.9.[88] To obtain a high (*acutus*) tone one starts with a note *b* and goes up a fifth to *c* (i.e. to the left), and then goes down a fourth to *d*. The interval between *b* and *d* then yields the desired high tone.[89]

The third proposition is to construct a high minor semitone (*Figura* N). The principle is the same: one adds or subtracts the appropriate intervals. In this construction he follows Boethius III.10 to the letter.[90] After this the construction of a low (*gravis*) minor semitone follows, but now in a much-abbreviated fashion. This is again following Boethius III.9 exactly; even the letters are the same.[91] It seems odd that des Murs should follow Boethius so closely rather than simply use the second technique for both high and low minor semitones.

Next comes the construction of a low minor semitone (which is much simpler) followed by the construction of major semitones, both high and low, and then the same for the comma, this time using the result of the fifteenth conclusion in the first part of the work. This

[88] Boethius, ed. Friedlein, p. 279. Cf. n. 66 above.
[89] In Paris, BnF lat. 7207, fol. 298v, the numbers have been inserted: 192 for *b*, 288 for *c* and 216 for *d*, which correspond to the relevant proportions.
[90] Boethius, ed. Friedlein, pp. 283–5.
[91] *Ibid.*, p. 281.

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latest (*propositio quarta*), because of the thoroughness of Part I, is easily accomplished by subtracting a minor semitone from a major one.[92]

After this he omits the discussion of the impossibility of the equal division of the tone (or indeed of any superparticular proportion) into two that Boethius includes in his III.11: Jean has already dealt with this in Part I. Nor does he revisit questions such as the relative sizes of the semitone versus three commas (cf. Boethius III.14 and 15). Instead he proceeds at once to the discussion of the monochord (cf. Boethius, Book IV) and how it embraces all harmonic *genera*, in order to satisfy the wishes of his readers, whom he now addresses as 'you' [plural] with his fifth proposition: 'Having done these preliminaries you may wish to know about the monochord.'[93] In Part I he had demonstrated the theory of consonances (*speculatione consonantiarum*) and their reduction to recognisable figures (*sensibiles figuras*) 'that greatly please mathematicians, since by them truth, which is in the intellect, is appropriately reduced to sight and sound'.[94] Having broken down consonances into tones and major and minor semitones, they can now be represented by intervals on the monochord,[95] but, before he gives the details, he asks the reader to note three preliminaries.

First, every division of the monochord, *which implicitly and virtually contains every kind of instrument*, comes from the tetrachord.[96] The tetrachord is a group of four pitches (strings) bounded by the fourth (*diatessaron*); in the diatonic *genus* this is filled in with two tones and a minor semitone.[97] In proceeding to break down consonances into smaller components he considers a double octave, which reduces to two octaves; a single octave, which reduces to a fifth and a fourth; but then says that the fifth

[92] The diagram, *Figura* P, in Witkowska-Zaremba (p. 198, repeated in Meyer, p. 182) is at best confusing since the semicircles are identical, which means that the high and the low commas both occur at the left, rather than one on the left and one on the right as in Paris, BnF lat. 7207, fol. 298v.

[93] II.48 W-Z, p. 198; Meyer, p. 182: 'His expeditis monochordum scire velitis.'

[94] II.50 W-Z, p. 198; Meyer, p. 182: 'de earum reductione ad sensibiles figuras, quae multum placent mathematicis, quoniam veritas, quae est in intellectu, per eas ad iudicium visus et auditus conformiter reducta est'.

[95] II.51 W-Z, p. 198; Meyer, p. 182: '[cum] . . . omnis consonantia composita sit ex tonis et semitoniis maioribus vel minoribus aut eorum partibus, ut visum est, infertur iam conveniens esse ad divisionem monochordi accedere'.

[96] II.54 W-Z, p. 198; Meyer, p. 182: 'Omnis divisio monochordi, quod in se continet implicite et virtualiter omnia genera instrumentorum, vadit per tetrachorda.' Italics have been added for later reference. Grocheio says virtually the same as this but about the vielle in *De musica,* [12.2]: 'Ita viella in se uirtualiter alia continet instrumenta.'

[97] II.55 W-Z, p. 198; Meyer, p. 182: 'Omne autem tetrachordum vocant musici spatium duorum tonorum cum minori semitonio: et hoc est diatessaron.'

presupposes the fourth while the fourth presupposes no other consonances, just the tone and semitone.[98] This seems to suggest that the fourth has primacy, which would contradict his argument in *Conclusio octava decima* that 'a fourth does not exist as a proportion in itself, but only arises as a consonance out of an octave and a fifth'.[99]

The second preliminary concerns putting together fourths: if they are joined together (i.e. conjunct, as in G–C then C–F) they do not make an octave but if juxtaposed (i.e. are disjunct, as in G–C then D–g) then they do make an octave.[100]

This is a prelude to the third point, which is that, although a fourth contains two tones and a semitone, the ancients arranged the order of these three components in different ways and he presents a diagram (*Figura* R) showing the three ways. These different ways arose from the three *genera*: diatonic, chromatic and enharmonic. Des Murs, however, is surprised that, of the three ancient modes, only the first was widely used 'throughout the world by the faithful'.[101] Despite him not knowing of these modes being used anywhere in the world, they can still be represented on the monochord, but he does not consider these further, thereby allowing himself to omit any account of the ancient Greek *genera* as treated in Boethius IV.3–17.

Propositio sexta signals his agreement with Boethius on how the divisions of the monochord should be made, and he restricts himself, at this point, to the double octave. He then includes a figure for the double octave that Boethius considers, complete with the relevant numbers and Greek note names (see Figure 8).[102] This is the only place where he uses the Greek names that feature so strongly in *De musica,* I.24 and IV.5–11.

He concludes *Musica speculativa* by talking about his own version of what he calls the monochord.[103] After explaining how his instrument is similar to that of Boethius, he describes how he has changed the

[98] II.56 W-Z, p. 198; Meyer, p. 182: 'nam ut in praecedentibus est ostensum, bis diapason ad semel diapason reducitur, diapason autem ad diapente cum diatessaron, diapente autem diatessaron praesupponit; et haec omnia prius sunt manifesta. Diatessaron autem nullam consonantiam praesupponit, sed tonum et semitonium.'

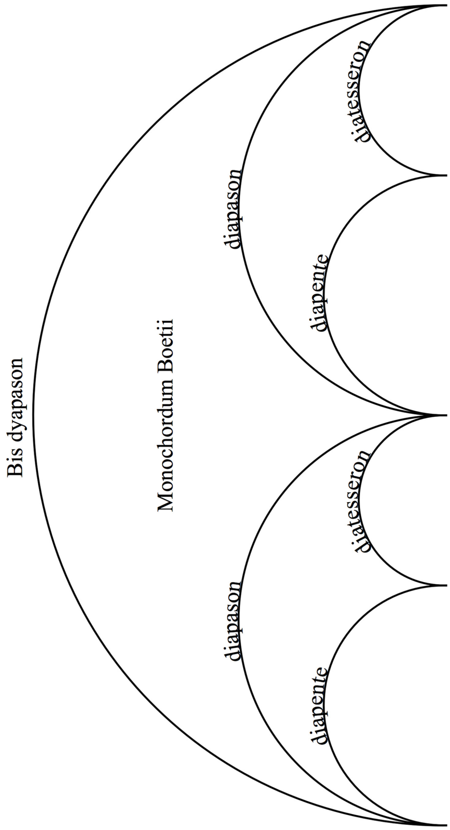
[99] See n. 78 above.

[100] II.67 W-Z, p. 199; Meyer, p. 184: 'Notandum est, quod in duobus tetrachordis coniunctis sunt septem chordae, sed in disiunctis octo.'

[101] II.78 W-Z, p. 200; Meyer, p. 186: 'ubi viget religio catholica fidem in orbe terrarum'.

[102] This is the unlabelled figure after II.77 W-Z, p. 202.

[103] Jean's use of the word 'monochord' is confusing. Throughout Book II he discusses the division of a string or strings but concludes with a diagram of a nineteen-stringed instrument that covers over two octaves, which he still seems to refer to as a monochord. This could, however, be interpreted in terms of pitches produced by moving a bridge on a single string.



nete hyperboleon	2304
paranete hyperboleon	2592
trite hyperboleon	2916
nete diezeugemon	3072
paranete diezeugemon	3456
trite diezeugemon	3888
paramese	4096
mese	4608
lichanos meson	5188
parhypate meson	5832
hypate meson	6144
lichanos hypaton	6912
parhypate hypaton	7776
hypate hypaton	8192
proslambanomenos	9216

Figure 8 Redrawn from the unlabelled figure <*Bis diapason. Monochordum Boetii*>. Notice that the ends of the arcs coincide with the appropriate notes

placing and number of the strings in his monochord, 'because the way of singing in vogue in the time of Boethius has now changed' and 'a more refined form of music is now delighting not just the learned, but the common crowd, particularly the young, and women also'.[104] But he then goes on to say that he does not know how this has happened unless it is by 'natural industry regulated by some higher circle' but

[104] II.99–101 W-Z, p. 202, Meyer, p. 190: 'quoniam illa maneries canendi quae suo viguit tempore, super numeros, tempore nostro sic est solum accidentaliter variata, quod nostra placentior est auditui quam antiqua. Subtiliataque multum est musica per exercitium modernorum non solum litteratorum hominum in hac arte studentium auxilio vel inventione, sed et vulgus commune et specialiter iuvenes ac etiam mulieres ad hoc moventur . . . '.

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things are continually changing and perhaps at some time they will come full circle and things will be as they were before.[105]

He then turns to the monochord and shows how to determine the lengths of the strings relative to each other for all the consonances and for the tone and semitone, with these last two extending the treatment of Boethius IV.18. This section seems largely to reprise *propositio prima*.

Jean explains that his own instrument, with nineteen strings, covering two octaves and a fifth (although he says it could be more), is in the shape of a triangle, in which the third side is not strictly a line, but more like the circumference of a circle, which in Euclid IV.5 is defined by three points (though here the curve is determined by nineteen points).[106] In manuscripts with diagrams there then follows an image (see Figure 9) of the strings on the monochordal instrument (presented in the edition of Witkowska-Zaremba, p. 204, from a manuscript of Version A, MS Prague, National Library V.F.6, copied in 1431; Pr3 in her edition).[107] In Paris, BnF lat. 7378A, the text (Version B) concludes with a diagram of this roughly triangular monochordal instrument on fol. 45v.[108] Whereas Boethius had confined himself to abstract proportions and a single string, Jean explains how a monochordal instrument can take many shapes, and does not even have to be flat, yet virtually contains within itself all other instruments (a variation of what Grocheio had said about the primacy of stringed instruments, namely the psaltery, the cithara, the lyre, the Saracen guitar and the vielle, which he claimed contains other instruments virtually in itself).[109] He concludes by declaring 'I want to describe to you the different shapes in which this monochord can

[105] II.101–2 W-Z, p. 202, Meyer, p. 190: 'nescio qua forte nisi naturali industria nunc a superiori circulo regulata. Mutantur enim haec continue et illa revoluto forte circulo aliquo redibunt et erunt sicut prius.'

[106] II.127 W-Z, p. 203, Meyer, p. 192: 'Continet autem hoc instrumentum 19 chordas, scilicet bis diapason cum diapente, licet sit possibile ulterius augmentari. Et est in figura trianguli orthogoni, quantum ad duo sui latera; sed tertium latus non sub una linea cadere potest, sed maxime ad circumferentiam accedit, super tria puncta descripta.' Witkowska-Zaremba includes a diagram on p. 204 showing nineteen strings. This diagram is slightly different from that in Paris, BnF lat. 7207, fol. 300r, which does not show the strings.

[107] Elżbieta Witkowska-Zaremba informed us (email of 10 March 2018) that this diagram was drawn in a similar way to that 'in Pr3 but is also transmitted in MSS Pr1, Pr2, K1, K4. Transmissions S and W differ a little bit in shape.' (The sigla are those of Witkowska-Zaremba, pp. 148 ff.)

[108] The strings are drawn at right angles to those in the figure in Witkowska-Zaremba, p. 204, and Meyer, p. 193, and it is difficult to see the ends of them, but their lengths appear to correspond to the pitches immediately above them.

[109] II.130 W-Z, p. 203, Meyer, 192; cf. Grocheio, *Ars musice* 12.2, ed. Mews et al., p. 72: 'Ita viella in se virtualiter alia continet instrumenta.' Cf. n. 97 above.

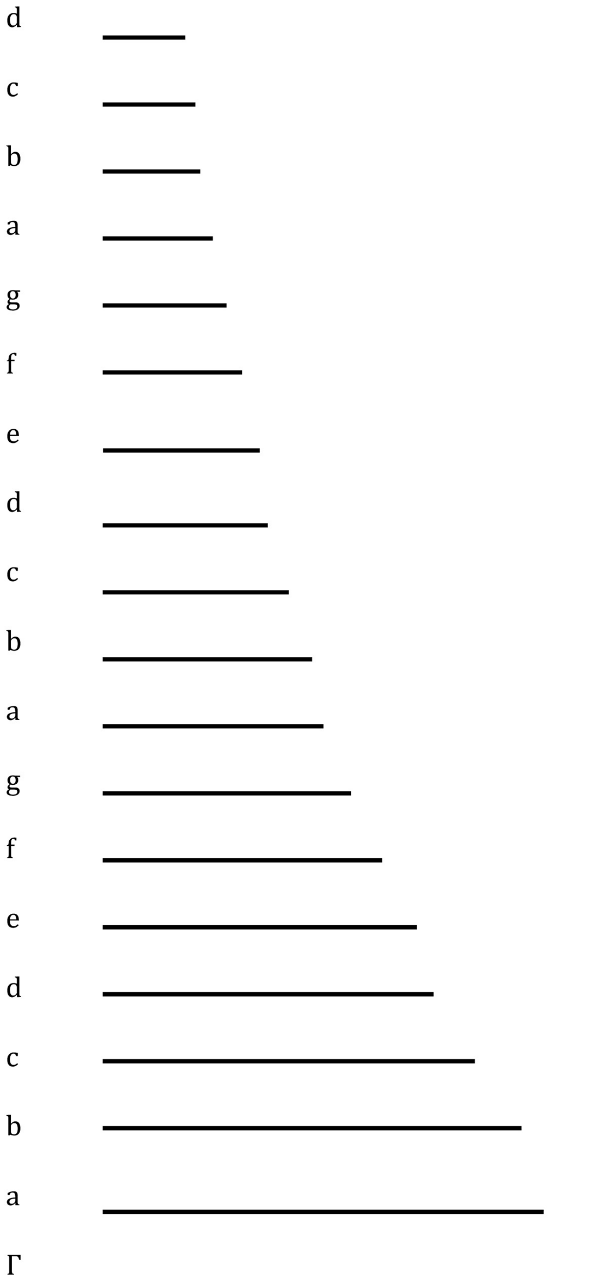

Figure 9 The strings of the monochord-style instrument of Jean des Murs drawn to scale (it resembles a psaltery rather than a conventional monochord)

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be drawn by means of an example. Their shapes follow in this order.'[110] The final sentence of the text (about different shapes of instruments) seems to refer to other images found on folio 45^v of Paris, BnF lat. 7378A, but missing from all other manuscripts, namely images of a vielle, a *lira*, *psalterium*, *canon*, *guiterna* and *chitara* – very close to the instruments mentioned by Grocheio.[111] It may be that they were too difficult for most subsequent scribes to reproduce. They demonstrate how Jean des Murs wanted to show that an actual instrument of the monochord could be reconfigured in many different ways, and how the core mathematical insights of Boethius could apply to a range of new instruments that he never imagined.

Throughout his text Jean des Murs relies heavily on diagrams but also reveals the methods that Boethius (and his predecessor Nicomachus) used. His diagrams put before the eyes of his readers a clear vision of the way musical intervals work together. With the exception of the discussion of the equal division of the tone, des Murs progresses systematically through the standard consonances, putting in just enough explanation to be clear while including nothing superfluous; he has made an epitome in both senses of the word. He makes the essence of Boethius as clear as it can be, without ever committing himself to the assumptions about *musica* as cosmic, human and instrumental that open that treatise. At the same time, he shows much more awareness of the way musical harmonies were always the product of experience, and how new musical instruments reconfigure the core principles of the monochord.

CONCLUSION

The *Musica speculativa* of Jean des Murs, initially completed in Paris in June 1323, marked a major step forward from the *Notitia artis musicae*, the treatise by which Jean first established his reputation as a music theorist, but which drew on Boethian ideas in only a limited way. In the thirteenth century, Boethius had tended to be referred to only briefly, mainly for what he said about the three different types of *musica* (cosmic, human and instrumental), conceptions that were challenged by the increasing awareness that Aristotle placed on the experience of music as sound. While Jean des Murs had started to

[110] II.130 W-Z, p. 203, Meyer, p. 192: 'Diversas tamen figuras, ad quas potest hoc monochordum transferri, exempli causa tibi describere volo. Quorum figurae sunt in hoc ordine consequentes.'

[111] Cf. also Lawrence Gushee, review of Falkenroth (see n. 2 above), *Music & Letters*, 76 (1995), pp. 275–80, at p. 279.

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present his ideas about the theory of music in the *Notitia* as resting on experience, he had drawn mainly on the first two books of the *De institutione musica* of Boethius. By contrast, in the *Musica speculativa* he condenses the arguments of the entire work, utilising a series of diagrams. He also replaces the brief discussion of types of *musica* by Boethius with his own reflections, shaped by Aristotle, on how all knowledge rests on experience. In the revised version of the Prologue (1325), he refines core arguments about the relationship between sound and number that he had raised in the *Notitia artis musicae*. He shows his readers, increasingly steeped in an Aristotelian perspective, that there is still much of value in the detail of what Boethius had to say about proportions.

In his *Speculum musicae*, a treatise that asserts a much more strongly Platonic version of Boethius than does Jean, Jacobus made specific hostile allusions to both the *Notitia artis musicae* and the *Musica speculativa* as reflecting what he considered perspectives not faithful to what he thought to be the original intention of Boethius. Jacobus refers to a 'quidam doctor modernus' who is clearly Jean, since part of his *Conclusio octava decima* in the *Musica speculativa* is quoted almost verbatim.[112] Around 1300 it appears it was common not to name contemporaries who were being criticised, which suggests that Jean and Jacobus were contemporaries. It also seems plausible that it was the criticisms of Jacobus himself (before his writing of at least some of Book VII of the *Speculum musicae*) that forced Jean des Murs to take full account of what Boethius had to say. The *Speculum musicae* offered the response of Jacobus, not just to new tendencies in musical notation, but to ways of reading Boethius. Both men were fascinated by the need to come to grips with what Boethius was arguing in the *De musica*. Yet it was des Murs who better succeeded in showing to future generations how core ideas transmitted by Boethius still had value, even within an Aristotelian perspective of how all knowledge ultimately rested on experience. For Jean des Murs, this was the true significance of Pythagoras in his discoveries about music.

112 *SM* VII.6.

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APPENDIX 1

Translation of the Prologue of Version A (1323)

Based on the Latin text as edited by Witkowska-Zaremba, pp. 171–2 and Meyer, pp. 134–6.

Although the excesses of bestial pleasures, through which taste and touch overthrow the intellect with their unrestrained assaults, are not undeservedly censured (according to the first book of Aristotle's *Ethics*:[113] 'indeed many bestial people therefore choosing the life of sheep . . . '), the ordered and moderated delights of sight and hearing, which serve the intellect in due measure in clearer and fuller ways, are not condemned because of this (as Aristotle says in the fourth book of the *Ethics* concerning the temperate man:[114] 'but he moderately pursues whatever there is for health and good habits, as he ought'); further, sight is much praised over hearing, in that it makes us come to know things as much as possible and shows us the many differences between things.[115] Likewise, as witnessed in experience, voices and sounds completely created by the subtlety of human artifices are a means to lead to the sweetest delights of the intellect. And, after the labours of serious work, which cannot be endured by human nature unceasingly, it grants the benefit of well-deserved rest to the listener. And perhaps also Ulysses wished to indicate that poetically (according to Aristotle's account in the eighth book of the *Politics*[116]), when he said that it is the noblest demonstration when those gathered together with men rejoicing hear a nightingale on the roof.

In addition, Boethius elegantly teaches in the prologue of his *De institutione musica* that 'harmonic sounds may hold much efficacy over human appetites in directing them to different kinds of habits'.[117] Inspecting them will be of interest for those who turn themselves [to it]. In the same place one will even be able to see how much men of the polis ought to care about the preservation of well-moderated music.[118] Indeed, since in these days the books of

[113] Aristotle, *Nicomachean Ethics*, 1095^{b} 20 = Aristoteles latinus, XXVI.1–3 Fasc. 3: 'Multi quidem igitur omnino bestiales videntur esse, pecudum vitam eligentes.'

[114] Aristotle, *Nicomachean Ethics*, 1119^{a} 15 = Aristotelis latinus XXVI.1–3 Fasc. 3: 'Quecumque autem ad sanitatem sunt vel ad bonam habitudinem delectabilia existencia, haec appetet mensurate, et ut oportet, et alia delectabilia non impedimenta hiis existencia, vel preter bonum, vel super substanciam.'

[115] Aristotle, *Metaphysics*, 980^{a} 27 = Aristotelis latinus XXV, 2: 'Causa autem est quod hic maxime sensuum cognoscere nos facit et multas differentias demonstrat.'

[116] Aristotle, *Politics*, 1338^{a} 28 (= ps.-Thomas Aquinas [Peter of Auvergne], *In Politicorum*, L. VIII.1. I 1115): 'Quapropter Homerus in poemate suo ita inquit. Sed quale est vocare ad mensam opipare paratam. Deinde nominas quosdam alios, subdit et citharoedum, qui omnes demulceat. et alio loco ait Ulyxes, optimam esse degendi rationem, quando laetis hominibus convivae audiunt citharoedum sedentes per ordinem.'

[117] Cf. Boethius I.1 (ed. Friedlein, p. 178): 'Musica naturaliter nobis esse conjunctam, et mores vel honestare vel evertere.'

[118] Cf. Boethius I.1 (ed. Friedlein, p. 180): 'Unde Plato etiam maxime cavendum existimat, ne de bene morata musica aliquid permutetur.'

the ancient philosophers, not just about music, but also [those] of the rest of the mathematicians, are not read, and because of this it happens those are abhorred as unintelligible or much too difficult, it seemed to me a good thing that, from the *De institutione musica* of Boethius, which, according to the power given to me by God, I have studied closely and understood to some extent with God's favour, I have been devising a short treatise in which I shall try to make manifest the more beautiful and essential conclusions pertaining to the very art of music with clarity of discussion and vividness of meaning.

APPENDIX 2

Translation of the Prologue of Version B (1325)

Based on the Latin text of Version B, ed. Falkenroth, available at http://www.chmtl.indiana.edu/tml/14th/MURMSPE.

Since music is about sound related to numbers or vice versa it is necessary that music consider both, namely number and sound. For before anything is numbered it is necessary for it to exist.

Therefore, it is necessary for sound to be generated before it is numbered.

Three things are necessarily required for the generation of sound: a beater, the thing beaten and the medium of beating. The first is rapidly breaking up the air, the second is a naturally resonant body, the third is air violently broken up. There cannot be a beat without motion, therefore neither can there be a sound without motion. Therefore, a sound exists by the breaking up of the air from the impact of the one beating on the thing beaten. For it is impossible for there to be sound when there is only one of these.

Of sounds, one is low, another high. The low is what is generated from slower and more sparse motions, the high indeed is what is generated from faster, more concentrated motions. Although there may be several separate movements in both, making several sounds in number (as is evident in the beating of gut strings). Yet they are heard without audible interruption as if it were like one continuous sound because of the speed of contiguous movements, just as colour existing on the surface of the tip of the moving crocus seems to be a circular line although it is only a point.

A high sound indeed comes from a low sound through the addition of motions; likewise, a low sound comes from a high sound by subtraction. Therefore, in high sounds there are more motions, in low ones fewer.

But all paucity is related to plurality by a certain number. Therefore, it is necessary for a low sound to be related by comparison to a high one, just as the number of motions of the one is compared proportionately to the number of motions of the other.

Since it has been shown that music exists from sounds that are proportional to each other in a certain way according to the number of motions found in them, it is not pointless to enquire generally into the proportions of numbers.

But every number compared to another is either equal or unequal to it for they correspond to a certain quantity.[119] If equal, they are one and the same in quantity. Therefore, they are not varied. A consonance is not made from equal sounds; they are either greater or lesser. If, however, unequal, they are either greater or lesser. But one being greater than the other is described in five ways, as multiple, superparticular, superpartient, multiple superparticular and multiple superpartient. It is said to be less in just as many ways by the assigned phrase.[120]

A multiple is when the greater number contains the whole of the lesser in itself, twice, thrice or fourfold and whose kinds are the proportions double, triple and so on to infinity.

A superparticular [proportion] is when the greater contains the lesser in itself and some part of it; its kinds are sesquialtera, sesquitertia, and so on.

A superpartient is when the greater contains the lesser and some parts of it; its kinds are superbipartient, supertripartient and so always.

A multiple-superparticular is when the greater contains the lesser twice or three times [and so one] and some part of it; its kinds are double sesquialtera, double sesquitertia, triple sesquialtera and thus of whichever part to infinity.

A multiple-superpartient is when the greater number contains the lesser twice or three times [and so on] and whatever of its parts; its kinds are double superbipartient, double supertripartient and in adding whatever kind of multiple to whatever of the superpartient.

Examples of the various kinds [of proportion] are shown in numbers. [Jean places his version of Table 3 here.]

That all teaching and every discipline arise from pre-existing knowledge.
That no other [knowledge] is found prior to sensory knowledge.
That it [sensory knowledge] rests on manifold experience as its endpoint.
That experience of sensible things creates art.

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119 See Boethius I.3, who takes this classification from Nicomachus. The only 'equal' comparison is of equal numbers, e.g. 1 : 1, 4 : 4, etc.; all other comparisons are called 'unequal'.

120 Thus 15 is greater as a superparticular of 12, since it is one whole (12) plus one-fourth (3) and conversely 12 is less as a superparticular of 15, since one has to add one part (a fourth of 12) to 12 to get 15. Cf. the first sentence of n. 42 above.

Table 3 Des Murs on proportions

Basis of the relation			12	24	36	48
Multiple	triple	3×	36	72	108	144
	double	2×	24	48	72	96
Superparticular	sesquitertia	1⅓×	16	32	48	64
	sesquialtera	1½×	18	36	54	72
Superpartient	supertripartient	1¾×	21	42	63	84
	superbipartient	1⅔×	20	40	60	80
Multiple superparticular	double sesquialtera	2½×	30	60	90	120
	triple sesquitertia	3⅓×	40	80	120	160
Multiple superpartient	double superbipartient	2⅔×	32	64	96	128
	triple supertripartient	3¾×	45	90	135	180

NOTE: We have moved the last row to the top, since des Murs starts with the numbers in this row. Then he gives two examples of each kind of proportion. We have also added an explicatory column showing the factor by which the number at the very top should be multiplied in order to get a number in the appropriate proportion. For example, to get a number in double sesquialtera proportion to 12 one multiplies by 2½, yielding $24 + 6 = 30$; similarly for the other numbers in this row: 24, 36, 48, yielding 20, 60, 90 respectively. The reason des Murs starts with 12 is that he wishes to get whole numbers in the appropriate proportion and, since he uses ⅔ and ¾ he needs to have numbers that are divisible by 3 and 4, and clearly 12 is the smallest such number. The other numbers are then simple multiples of 12 going up to 12×4.