

MATHEMATICS OF QUANTIZATION AND QUANTUM FIELDS

Unifying a range of topics that are currently scattered throughout the literature, this book offers a unique and definitive review of some of the basic mathematical aspects of quantization and quantum field theory. The authors present both elementary and more advanced subjects of quantum field theory in a mathematically consistent way, focusing on canonical commutation and anti-commutation relations. They begin with a discussion of the mathematical structures underlying free bosonic or fermionic fields, such as tensors, algebras, Fock spaces, and CCR and CAR representations (including their symplectic and orthogonal invariance). Applications of these topics to physical problems are discussed in later chapters. Although most of the book is devoted to free quantum fields, it also contains an exposition of two important aspects of interacting fields: the diagrammatic method and the Euclidean approach to constructive quantum field theory. With its in-depth coverage, this text is essential reading for graduate students and researchers in departments of mathematics and physics. This title, first published in 2013, has been reissued as an Open Access publication on Cambridge Core.

JAN DEREZIŃSKI is a Professor in the Faculty of Physics at the University of Warsaw. His research interests cover various aspects of quantum physics and quantum field theory, especially from the rigorous point of view.

CHRISTIAN GÉRARD is a Professor at the Laboratoire de Mathématiques at Université Paris-Sud. He was previously Directeur de Recherches at CNRS. His research interests are the spectral and scattering theory in non-relativistic quantum mechanics and in quantum field theory.

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JAN DEREZIŃSKI

University of Warsaw

CHRISTIAN GÉRARD

Université Paris-Sud





Shaftesbury Road, Cambridge CB2 8EA, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

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