

there is a discussion of p -bases and differential bases, coefficient rings, Jacobian criteria re openness of loci, Marot's theorem on the preservation of the Nagata property under power-series extension, Falting's proof of Grothendieck's theorem on the equivalence of formal smoothness and geometric regularity, and Kunz' theorems on Noetherian rings of characteristic p . Finally, there is a brief introduction to André cohomology. All these topics display the beautiful interrelationship between the properties of excellent rings (or, more generally, G -rings or Nagata rings), smoothness, differentials, and field extensions.

The appendix also contains some possible source of confusion. The result on p. 248 needs to be stated for the semilocal case, and the footnote on p. 252 should refer to Th. 77 as well. It is only implicitly given that a semilocal G -ring is a Nagata ring, since it is a J_2 ring, yet this fact is used on p. 260. The reference on p. 258 should be to (24.B), while the second example on p. 260 needs to be supported by a reference to (37.8) of Nagata's "*Local Rings*". (A reference to p. 64 of Nagata's book would have been of interest in the result on Zariski rings on p. 176.) Recalling the method of proof of Th. 65 might have been of help in the section on formal étaleness. On p. 291 it would perhaps be better to refer to the *proof* of Th. 73 (3). The implications of Marot's theorem could have been made explicit. In the discussion of the second of Kunz' theorems, the expression " $= K \otimes B^*$ " needs a little interpretation, while part of the subsequent lemma is taken for granted in the proof of the theorem.

As regards misprints, on p. 248 the reference should be to Lemma 1(ii); on p. 273 "formally étale" is meant, not "formally smooth"; on p. 300 we should have $A \simeq A^q$; while on p. 303, $q = Q \cap B^*$, on p. 305, κ not δ (twice), and on p. 270, K' not K^p are meant. There are one or two other minor misprints and lacunae as well.

However, set against page after page of beautiful mathematics, these are very minor cavils. Professor Matsumura has once again given us a marvellous and engrossing book—it's a must.

L. O'CARROLL

DAVIES, E. B., *One-Parameter Semigroups* (Academic Press, London, 1980), viii + 230 pp., £19.80.

The title refers to semigroups $\{T_t\}$ ($0 \leq t < \infty$) of bounded linear operators on a Banach space, and the book is primarily concerned with the relationship between the semigroup and its generator Z ($Zf = \lim_{t \rightarrow 0(+)} t^{-1}(T_t f - f)$, the limit existing for all f in some dense subspace of the Banach space). The author's stated aim is to provide an up-to-date treatment concentrating on the abstract theory rather than applications. This may seem a bold strategy in view of the current tide, but it is justified in terms of both the existing state of the literature and the need to keep the book to a manageable size. It should be mentioned in this respect that the preface contains a careful list, with references, of several related topics and applications which are not covered in the sequel. Also, various concrete examples throughout the text, often involving semigroups generated by differential operators, serve both to illustrate the general theory and to give some indication of the range of applications.

In Chapter 1 the basic properties of generators are established and it is quickly shown that the Cauchy problem for the differential equation $f'_t = Zf_t$ is uniquely soluble if Z is the generator of a one-parameter semigroup and f_0 lies in the domain of Z . Chapter 2 deals with spectral theory for Z and T_t , and then covers a range of results of Hille–Yosida type giving conditions for an operator Z to be the generator of a (possibly contraction) semigroup. The core of the book is completed by the perturbation theory in Chapter 3 (if Z is a generator when is $Z + A$ also a generator, and what is the relationship between the generated semigroups?).

Both Chapters 4 and 6 deal with operators on Hilbert space. Chapter 4 concentrates on self-adjoint operators; it covers Stone's theorem on one-parameter unitary groups (though curiously the result is nowhere referred to under this name), self-adjoint contraction semigroups, and quadratic forms. This chapter might benefit if the functional calculus for an unbounded self-adjoint operator were dealt with a little more fully instead of being relegated to the problems. The main feature of Chapter 6 is the theorem concerning the dilation of a one-parameter contraction semigroup to a unitary group on a larger Hilbert space. Chapter 5 is concerned with asymptotic

analysis, for example the behaviour as $\lambda \rightarrow 0$ of the semigroup with perturbed generator $\lambda^{-1}Z + A$. The later sections of this chapter form perhaps the most technical part of the book. Chapter 7 deals with the notions of positivity and irreducibility for semigroups acting on (for simplicity) L^p -spaces and $C(X)$. Finally, Chapter 8 covers the theory of spectral subspaces for an (unbounded) Hermitian operator H on a Banach space (iH is the generator of a one-parameter group of isometries), culminating in the important result that H is uniquely determined by its family of spectral subspaces.

This book should be much appreciated by anyone wishing to work steadily through a systematic exposition. Proofs are kept reasonably short which has the two-fold advantage of encouraging the reader and then providing the beneficial but not over-taxing task of filling in some details. One criticism here is that results from earlier sections are frequently used without mention. This is fine for someone who is working through the book but may cause difficulty for the casual reader. The same applies to the sudden reintroduction of earlier notation (see, for example, the reappearance of P_1 on p. 128 after its use on p. 84). Occasionally the treatment is a little too abrupt: for example, "spectral radius" is defined without even a passing reference to the spectrum, and the term "dissipative" is introduced with no reference to the heat equation or any other physical example which would explain why this particular adjective is chosen.

The text contains several problems, mainly on the general theory, and many concrete examples. The former are for the most part straightforward while the latter require something of a change of gear. Filling in the details may provide a lengthy but rewarding tussle, involving techniques in integration, differentiation, and Fourier theory. Notes at the end of each chapter indicate original sources, and show that much of the material in the later chapters comes from the last decade and appears in book form for the first time. Some of the notes might have been better placed in the preceding text (e.g. the reference for Polya's criterion on p. 94 would be more helpful if it came on p. 85), and there are one or two other places where better sign-posting would help. For instance, when discussing relative boundedness in Chapter 3 it would have been very helpful to give a forward reference to the illuminating Example 4.21. However, these are minor points and do not affect the general conclusion that this is a most valuable addition to the literature in this field.

R. J. ARCHBOLD

ASCHBACHER, MICHAEL, *The Finite Simple Groups and Their Classification* (Yale Mathematical Monographs 7, Yale University Press, 1980) ix + 61 pp., £4.40.

The complexity of the problem of classifying the finite simple groups has fascinated many mathematicians and the quantity of the work carried out has necessitated the production of a number of surveys. These range from Daniel Gorenstein's book, *Finite Simple Groups* (Harper & Row, 1968), which can be regarded as the finite simple group theorist's bible, to the most recent comprehensive survey "The classification of finite simple groups", also by Daniel Gorenstein, part I of which appeared in *Bull. American Math. Soc. (New Series)* 1 (1979), 43–199 and the remainder of which has still to appear. However, most of the surveys have been for the use of experts and there has been a distinct lack of exposition suitable for the wide range of mathematicians who are interested in the subject.

I was therefore pleased to read that Michael Aschbacher's book, which is based on four special lectures delivered at Yale, is "intended for a general mathematical audience". Unfortunately this intention has not been carried out. The author has not thought carefully enough about the existing knowledge of his reader. The result is a rather uneven set of assumptions. Thus the reader finds that he is told more than once the definitions of the terms involution and elementary abelian p -group but is expected to cope without help with a variety of terminology and notation concerning normalizers, commutators and conjugates and to absorb quickly some rather complex ideas. So I disagree with the claim on the back cover that the contents are "accessible to most mathematicians".