TRANSFORMATION OF THE MEAN PLACE FROM FK4 TO FK5

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ABSTRACT. Transformation procedures from the FK4 reference system of B1950.0 to the FK5 reference system of J2000.0 have been developed by Standish (1982) and by Aoki *et al.* (1983). We review here these procedures and discuss the differences between them. Especially we note that among researchers of this field a misunderstanding still exists in the problem at which stage the equinox correction should be applied. We show that the equinox correction should be applied in the precessing frame as Aoki et al. did. We also show that the epoch of the transfer from the FK4 to the FK5 in the transformation procedure is related to the systematic and individual corrections to the FK4.

1 Introduction

The matrix formulation of the transformation of the mean places and proper motions from the system of the FK4 at B1950.0 to that of the FK5 at J2000.0 has been developed by Standish (1982) and by Aoki *et al.* (1983). The main differences between these transformations are (1) in the application of the equinox correction and (2) in the epoch of the transfer from the FK4 system to the FK5 system.

Murray (1989a, b) claims that the transformation by Standish is correct in both points mentioned above.

In this paper in Sect. 2 we discuss the matter of the equinox correction and show that the transformation by Aoki *et al.* is correct as for the application of the equinox correction. In Sect. 3 we deal with the matter of the epoch of the transfer and show that it is related to the systematic and individual corrections to the FK4.

Details about these matters will be published in a volume of *Astronomy and Astrophysics*.

2 Application of the equinox correction

In the Standish's or Murray's transformation the equinox correction is applied in the fixed frame whereas in the Aoki *et al.*'s transformation it is applied in the frame rotating by the precession. These transformation procedures are expressed as follows at the epoch of the transfer from the FK4 to the FK5:

Standish or Murray (the epoch is B1950.0):

$$\alpha^N = \alpha^O + E, \tag{1a}$$
$$\delta^N = \delta^O, \tag{1b}$$
$$\mu_\alpha^N + m^N + n^N \sin(\alpha^O + E) \tan \delta^O = \mu_\alpha^O + m^O + n^O \sin(\alpha^O + E) \tan \delta^O + \dot{E}, \tag{1c}$$
$$\mu_\delta^N + n^N \cos(\alpha^O + E) = \mu_\delta^O + n^O \cos(\alpha^O + E), \tag{1d}$$

Aoki *et al.* (the epoch is 1984 January 1.0):

$$\alpha^N = \alpha^O + E, \tag{2a}$$
$$\delta^N = \delta^O, \tag{2b}$$
$$\mu_\alpha^N + m^N + n^N \sin(\alpha^O + E) \tan \delta^O = \mu_\alpha^O + m^O + n^O \sin \alpha^O \tan \delta^O + \dot{E}, \tag{2c}$$
$$\mu_\delta^N + n^N \cos(\alpha^O + E) = \mu_\delta^O + n^O \cos \alpha^O, \tag{2d}$$

where m and n are the rates of the general precession in right ascension and declination based on the Newcomb's precession (superscript O) and the IAU 1976 precession (superscript N). Here the time unit is supposed to be the same in both sides of the equations for proper motions. The difference between Eqs. (1) and (2) is in the terms including n^O in the equations for proper motions.

Murray (1989a, b) considers the special case in which there is no change in the precession constant $(m^N = m^O, n^N = n^O)$, and the equinox correction is independent of time $(\dot{E} = 0)$. He supposes that in this case both coordinate systems are inertial, in other words proper motions are the same in the two systems. The Eqs. (1c) and (1d) support his supposition while the Eqs. (2c) and (2d) don't, and he concludes that Standish's procedure is correct. Smith *et al.* (1989, in Note added in proof) and Yallop (1989) support this Murray's inference. But his inference is erroneous, because the precession is dependent on the location of the equinox (mean pole moves always toward the equinox at each instant) and therefore the equinox correction E affects the obtained proper motions even if the equinox motion $\dot{E}$ is zero. Detailed considerations are given in the followings.

Proper motions of stars (μ_α, μ_δ) are not directly determined from observations but are determined from the observed variations $(d\alpha/dt, d\delta/dt)$ of the mean places (α, δ) from the following expressions:

$$\frac{d\alpha}{dt} = m + n \sin \alpha \tan \delta + \mu_\alpha,$$
$$\frac{d\delta}{dt} = n \cos \alpha + \mu_\delta.$$

The right ascension α appearing in the right sides of the above equations is, of cource, the observed right ascension in the catalog (FK4 or FK5) system and not in the dynamical system. Therefore if the catalog includes an error E in the right ascension, it also affects the obtained proper motions. From this consideration the equations (2c) and (2d) are derived.

The correctness of the equations (2c) and (2d) can also be confirmed using the formulae of spherical triangles.

Thus we can conclude that as for the application of the equinox correction the procedure by Aoki *et al.* is correct.

Murray insists that the equinox corrections determined from analyses of observations of the Sun, Moon and planets are corrections to the right ascension system of FK4 *in the B1950.0 frame*, but this statemant is not correct. In fact, the equinox corrections so determined are corrections to the right ascension system of FK4 at the epoch of observation *in the frame of date*. For example, in the Eq. (5) given by Fricke (1985) and applied to observations, for example, by Hughes and Scott (1982):

$$\Delta\alpha = -E + \cos\epsilon\sec^2\delta\Delta L - \cos\alpha\tan\delta\Delta\epsilon$$
$$+ 2\sin\alpha\sec\delta\Delta h - 2\cos\alpha\sec\delta\cos\epsilon\Delta k,$$

the quantity E is the equinox correction at the epoch of observation in the frame of date. In the above equation $\Delta\alpha$ is the $O - C$ in the apparent right ascension of the Sun, ϵ is the obliquity of the ecliptic and ΔL is the correction to the mean longitude of the Sun.

It is obvious that Fricke (1982) assumed that the equinox correction $E(t)$ at any epoch *in the frame of date* can be expressed as

$$E(t) = E_{1950} + \dot{E}t,$$

and obtained the values of E_{1950} and $\dot{E}$ from analyses of observations of the Sun, Moon and planets. Because $\dot{E}$ in the above equation is theoretically equal to the value obtained from an analysis of FK4 proper motions in the frame of 1950.0, Fricke (1982) compared these values and obtained the final value of $\dot{E}$.

3   Epoch of the transfer from the FK4 system to the FK5 system

In the transformation by Aoki *et al.* the transfer from the FK4 system to the FK5 system is performed at the epoch of 1984 January 1 when the FK5 came into effect. Motivation of their transformation is that observed values such as stars' positions, UT1, etc. are not changed when the system of the catalog is changed, except stars' right ascensions and proper motions in right ascension, which are intentionally changed by the values obtained by Fricke (1982). Especially we note that their transformation is consistent with the new expression for the relationship between UT1 and GMST (Aoki *et al.*, 1982). On the other hand Murray insists that the transfer should be performed at the epoch of B1950.0 as Standish did. The two 6×6 transformation matrices are given in Tables 1 and 2. In calculating these matrices the application of the equinox correction is changed from Murray for the reasons mentioned in Sect. 2 and for the values of the equinox correction to the FK4 we adopt the values $E = 0\overset{s}{.}035$ at J1950.0 and $\dot{E} = 0\overset{s}{.}085$ / Julian century as Aoki *et al.* did in order to be consistent with the new expression for the relationship between UT1 and GMST. For the precession formulae based on the Newcomb's precession constant we use those given by Kinoshita (1975). The formulae for obtaining the position and velocity vectors from the mean place, proper motion, radial velocity and parallax, and vice versa, are given by Aoki *et al.*

The role of fundamental catalogs such as FK4 or FK5 is to define the equator and the equinox among stars on the celestial sphere *at any epoch* (see e.g. Woolard and

Table 1. Transformation Matrix (Transfer at 1984 January 1)

$$
\begin{pmatrix}
+0.9999256782 & -0.0111820610 & -0.0048579477 & +0.0000024239502 & -0.0000000271066 & -0.0000000117766 \\
+0.0111820609 & +0.9999374784 & -0.0000271765 & +0.0000000271066 & +0.000002423 9788 & -0.0000000000659 \\
+0.0048579479 & -0.0000271474 & +0.9999881997 & +0.0000000117766 & -0.0000000000658 & +0.0000024241017 \\
-0.00055 & -0.23854 & +0.43574 & +0.99994704 & -0.01118251 & -0.00485767 \\
+0.23849 & -0.00267 & -0.00854 & +0.01118251 & +0.99995883 & -0.00002718 \\
-0.43562 & +0.01225 & +0.00212 & +0.00485767 & -0.00002714 & +1.00000956
\end{pmatrix}
$$

Table 2. Transformation Matrix (Transfer at B1950.0)

$$
\begin{pmatrix}
+0.9999256782 & -0.0111820602 & -0.0048579481 & +0.0000024239502 & -0.0000000271054 & -0.0000000117789 \\
+0.0111820602 & +0.9999374784 & -0.0000271632 & +0.0000000271054 & +0.0000024239788 & -0.0000000000658 \\
+0.0048579481 & -0.0000271607 & +0.9999881997 & +0.0000000117789 & -0.0000000000659 & +0.0000024241017 \\
-0.00055 & -0.23803 & +0.43551 & +0.99994704 & -0.01118172 & -0.00485911 \\
+0.23802 & -0.00266 & -0.00023 & +0.01118172 & +0.99995884 & -0.00002716 \\
-0.43549 & +0.00395 & +0.00212 & +0.00485911 & -0.00002717 & +1.00000955
\end{pmatrix}
$$

Clemence, 1966, pp. 376−377; or Fricke and Kopff, 1963, p.1). Whether the equator or the equinox defined by a fundamental catalog is coincident with the real equator or equinox is another question (the differences are called the systematic error of the catalog); absolute or fundamental observations will answer that question. In fact because we now know that the adopted precession constant in the FK4 has an error, if the equator defined by the FK4 is coincident with the real equator at *some* epoch, the equator defined by the FK4 at any other epoch is not coincident with the real equator. This fact suggests that the systematic error of the fundamental catalog depends on time. (The error of the first power of time can be eliminated by adjustment of proper motions, but the error of the second and higher power of time cannot be eliminated.) Besides systematic error there is an individual error for each star. This depends also on time. Murray implicitly assumes that the systematic and individual errors in the FK4 are zero at the epoch of B1950.0, but this cannot be justified, because positions and proper motions in the FK4 were not determined from observations only at the epoch of B1950.0.

If one uses the transformation matrix given in Table 1, one must apply the systematic correction to the FK4 system at the epoch of 1984 January 1, and if one uses the transformation matrix given in Table 2, one must apply the systematic correction to the FK4 system at the epoch of B1950.0. We can show that the matrix given in Table 2 can be derived from the matrix given in Table 1 by applying the systematic and individual corrections to the FK4 system.

If one ignores the systematic and individual corrections to the FK4 in the transformation to the FK5 system, the matrix given in Table 1 (Aoki *et al.*'s procedure) is recommended, because it is consistent with the definition of UT1. Also in the transformation of precise positions obtained by VLBI, the matrix given in Table 1 is recommended, because most of those observations are performed in the 1980's.

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Discussion

SEIDELMANN: The 1984 date is only an arbitrary date introducing the change. It is involved in the UT1 equation to maintain continuity across the change. It is not a part of the transformation of the star catalog mean position. It was not part of the determination of the equinox correction in the definition of the J2000 system of the FK5.

SôMA: The catalog positions have a close relation to the sidereal time and UT1. Therefore, it is important that the catalog positions should be determined in a consistent way with the equation for UT1 and sidereal time. But, as we have shown in this paper, the positions in the FK5 system do not depend on the epoch of transfer, if one applies the systematic correction to the FK4 properly.

MURRAY: In the first part of your paper you gave a fair description of the difference between us, but I still maintain that Fricke's motion of the equinox must be interpreted as being in the *fixed* frame B1950.

My motivation in entering into this subject was to determine the transformation to galactic coordinates for J2000. Using the transformations by Aoki *et al.*, the galactic coordinates of an object calculated from its equatorial coordinates in the J2000 frame differ from those calculated from the B1950 frame, which is absurd!

SôMA: As shown in our paper, the equinox corrections must be applied in the moving frame. Aoki *et al.*'s transformation gives no inconsistency even in the galactic coordinates.