Hence $\log\left(1-\frac{1}{n}\right) > -\frac{1}{n-1}$.

We have, therefore,

$$\left|\log \prod_{p \leqq x}\left(1-\frac{1}{p}\right) + \sum_{p \leqq x}\frac{1}{p}\right| < \sum_{p \leqq x}\frac{1}{p(p-1)} < \sum_{2 \leqq n \leqq x}\left(\frac{1}{n-1}-\frac{1}{n}\right) < 1.$$

Also $\left|\sum_{p \leqq x}\frac{1}{p} - \log\log x\right| < c_5$.

Hence $\left|\log \prod_{p \leqq x}\left(1-\frac{1}{p}\right) + \log\log x\right| < c_6$, which proves the theorem.

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A note on some networks of polygons

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Given an infinity of polygons which form the boundary of a finite number of polyhedra, we shall consider the complex K consisting of the polyhedra, and of the faces, edges and vertices of the polygons. We consider only those cases in which the Eulerian Characteristic N of K is finite. *Then if the mean number of sides meeting at a vertex is p, and the mean number of sides of a polygon is q, then*

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{2}.$$

The complex K is considered as the limit of a complex K' having a finite number v_0 of points, v_1 of edges, v_2 of polygons, and v_3 of polyhedra, when v_2 tends to infinity in a definite manner. Since $v_2 \leqq \sum_{r=1}^{v_1}\binom{v_1}{r}$, which is finite if v_1 is finite, it follows that v_1 is infinite if v_2 is infinite.

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Now by the definition of N,

$$N = v_0 - v_1 + v_2 - v_3$$

$\therefore \frac{v_0}{v_1} - 1 + \frac{v_2}{v_1} = \frac{N + v_3}{v_1}$ which tends to zero as v_1 tends to infinity.

So for K, $\frac{v_0}{v_1} + \frac{v_2}{v_1} = 1$.

But $v_0 p$ = the number of lines counted twice $= 2v_1$, and similarly $v_2 q = 2v_1$.

Therefore $\quad \frac{1}{p} + \frac{1}{q} = \frac{1}{2}$.

Corrollary. In particular the theorem applies to networks of polygons in a plane, whether the plane be considered as a number-plane, with a point at infinity ($N = 1$) or as a projective-plane with a line at infinity ($N = 0$).

Note. If the theorem is to be applied to nets of polygons on a polyhedron in cases where the network has a boundary (and in particular to plane networks of this kind) the polyhedron must be completed by the addition of a polygon whose boundary is the boundary of the net. Consider for example a circle with n radii $OP_1, OP_2, \ldots, OP_n$ and let n tend to infinity. Then to find p we have n lines meeting at O and 3 lines meeting at each P, *i.e.*, $p = (3n + n)/(n + 1) \to 4$; to find q we have n triangles and one n-gon, *i.e.*, $q = (3n + n)/(n + 1) \to 4$, verifying the theorem.

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