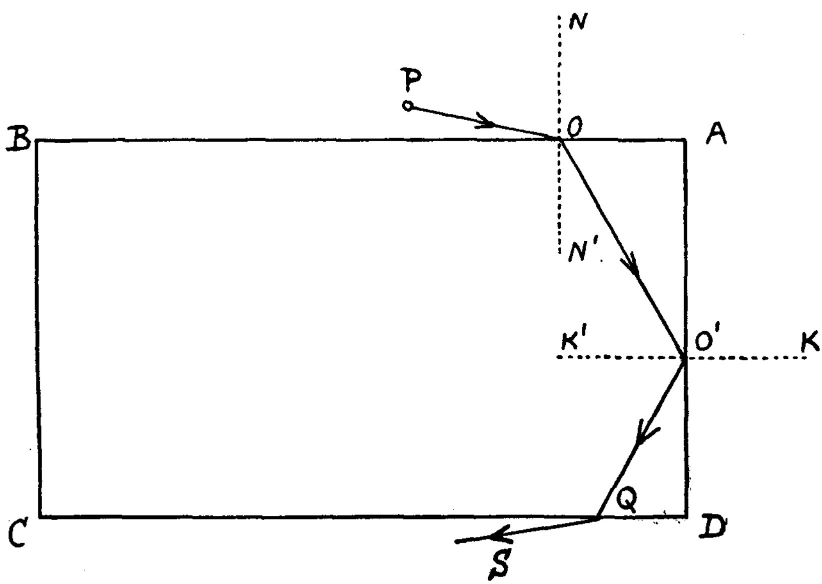
numerically equal. For if $T_k = T_{k+1} = -T_{k+2}$, then $\dfrac{|x|}{|x|+1}(n+1)=k$ and $\dfrac{|x|}{|x|-1}(n+1)=k+1$; solving these equations simultaneously we obtain the above values of $|x|$ and n. Thus

$$(1+5)^{\frac{7}{5}}=1+7+7-7+14-\ldots$$

Finally, for all values of n the first term is the greatest if $|nx|<1$.

D. M. Y. SOMMERVILLE

An Experiment in Light.—Let ABCD be a horizontal section of a rectangular slab of glass. A pin is set up vertically at P, close to the face AB, or at a short distance from it. It is possible to see the pin P through the glass if we look through the face CD; but the pin is invisible if we look through the face AD. If, however, we look through the face CD, *into the face* AD, we shall see an image of the pin in AD, which acts like a mirror. The experiment illustrates total reflection of light; the explanation is easy. Let PO be a ray of light which after passing into the glass will be incident on AD at O′. Let NON′ be the normal to AB at O,



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and KO′K′ be normal to AD at O′. The critical angle for glass being 41° 45′ approximately,

the $\angle N'OO' \not> 41°45'$;

$\therefore$ the $\angle OO'K' \not< 48°15'$;

i.e. the $\angle OO'K' > 41°45'$;

$\therefore$ the ray OO′ will not emerge through AD, but will be reflected in the direction O′Q, and will emerge at Q in the direction QS.

W. A. LINDSAY

Note on Fermat's Theorem.—The Theorem that $a^n - a$ is exactly divisible by n, if n be a prime number, may be established as follows from the Binomial Theorem.

We have

$$(a+b)^n = a^n + C_n^1 a^{n-1} b + C_n^2 a^{n-2} b^2 + \dots + C_n^r a^{n-r} b^r + \dots + b^n,$$

where $$C_n^r = \frac{n(n-1)(n-2)\dots(n-r+1)}{\lfloor r};$$

$$\therefore\ (a+b)^n - a^n - b^n = C_n^1 a^{n-1} b + C_n^2 a^{n-2} b^2 + \dots + C_n^r a^{n-r} b^r + \dots + C_n^{n-1} a b^{n-1}. \quad (1)$$

Now since C_n^r is an integer, the product

$$n(n-1)(n-2)\dots(n-r+1)$$

must contain as a factor the product $1 . 2 . 3 \dots r$; but if n is a prime number it cannot contain as a factor any one of the integers $2, 3, 4, \dots r$, each of which is $< n$;

$\therefore$ the product $\lfloor r$ must be contained in $(n-1)(n-2)\dots(n-r+1)$, and $\therefore$ n is a factor of C_n^r.

This is true for all values of r from 1 to $n-1$.

It follows from (1) that $(a+b)^n - a^n - b^n$ is exactly divisible by n.

Hence $(a+1)^n - a^n - 1$ is exactly divisible by n. (2)

i.e. $(a+1)^n - (a+1) - (a^n - a)$ is exactly divisible by n; which shows that if $a^n - a$ is exactly divisible by n so will $(a+1)^n - (a+1)$.

Now from (2) it follows that $2^n - 2$ is exactly divisible by n, (putting $a = 1$);

$\therefore$ $3^n - 3$ is exactly divisible by n,

$\therefore$ $4^n - 4$ is exactly divisible by n, and so on.

We might also reason as follows:—

To show that $a^n - a$ is exactly divisible by n, let $a = y + 1$.

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