diagrams of the countable ω-models of Kripke–Platek set theory (KP). We show that, given a path f through T_{KP}, representing a model $\mathcal{M}$ of KP, and another computable ill-founded ω-branching tree T, if f fails to compute a path through T, then $\mathcal{M}$ assigns to T a nonstandard ordinal tree rank. Furthermore, we indicate some circumstances in which, given computable ω-branching trees T_0 and T_1, a fixed path through T_{KP} helps the paths through T_0 compute paths through T_1.

In a different line of work, we consider *effective forcing notions*. In particular, we define a class of effective forcing notions that are similar to versions of Mathias forcing and Cohen forcing defined in the literature, and prove some results about how these notions relate. As a consequence, we see that the generics for an effective version of Mathias forcing compute generics for an effective version of Hechler forcing, and vice-versa. Later, we focus on a notion of Mathias forcing over a countable Turing ideal, defined by Cholak, Dzhafarov, and Soskova. We show that there are nested Turing ideals for which the Mathias generics for the larger ideal do not all compute Mathias generics for the smaller ideal.

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PURBITA JANA, ***A Study of the Interrelation between Fuzzy Topological Systems and Logics***, University of Calcutta, India, 2016. Supervised by Mihir K. Chakraborty. MSC: 03B52, 03B60, 97H50. Keywords: geometric logic, fuzzy topological system, graded frame.

Abstract

The ultimate objective of the thesis is to develop fuzzy geometric logic and fuzzy geometric logic with graded consequence. The motivation mostly comes from the main topic of Vickers' book "Topology via Logic," where he introduced the notion of topological system and indicated its connection with geometric logic. A topological system is a triple $(X, \models, A)$, where X is a nonempty set, A is a frame, and $\models$ is a binary relation between X and A. A frame is a lattice which is closed under arbitrary join and finite meet together with the property that binary meet distributes over arbitrary join. The relationships among topological space, topological system, frame and geometric logic play an important role in the study of topology through logic (geometric logic). Generalizations of topological space to fuzzy topological space were introduced by C. L. Chang and R. Löwen, and these concepts have been studied extensively and intensively. Naturally the question "from which logic can fuzzy topology be studied?" comes to mind. If such a logic is obtained what could be its significance?

To answer these questions, as basic steps, we first introduce some notions of fuzzy topological systems and establish the interrelation with appropriate topological spaces and algebraic structures. These relationships are studied in a categorical framework. As a matter of fact, study of duality takes place as one of the important parts of the thesis.

Geometric logic has been discussed in various works by P. T. Johnstone, S. Mac Lane & I. Moerdijk, S. J. Vickers. However for our purpose the reference point is Vickers' books and articles. The formulae of geometric logic are based on two propositional connectives viz. $\wedge$, the binary conjunction and $\bigvee$, the arbitrary disjunction over arbitrary set of formulae including null set. As a special case the binary disjunction $\vee$ is obtained. In addition, the logic has an existential quantifier $\exists$. It is noteworthy that geometric logic does not have negation, implication or universal quantifier. Also in this logic sequents of the form $\alpha \vdash \beta$ are derived from a set (possibly null) of sequents. These special sequents have exactly one formula on either side of the symbol $\vdash$ (turnstile), the intention of the symbol being, as usual, that β follows from α.

Some motivations behind generalizing the geometric logic to multivaluedness are the following:

- Geometric logic is endowed with an informal observational semantics: whether what has been observed does satisfy (match) an assertion or not. Now, observations of facts and assertions about them may corroborate with each other partially. It is a fact of reality

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and in such cases it is natural to invoke the concept of "satisfiability to some extent or to some degree." As a result the question whether some assertion follows from some other assertion might not have a crisp answer "yes"/"no." It is likely that in general the "relation of following" or more technically speaking, the consequence relation turnstile ($\vdash$) may be itself many-valued or graded. Thus, we have adopted graded satisfiability as well as graded consequence in the thesis.

- It has been imperative to link with fuzzy topological systems (and fuzzy topological spaces as a result), a many-valued logic similar to the link between classical topological systems and geometric logic. This goal has been achieved here with the introduction of a general fuzzy geometric logic. In our case, of course, a further generalization has been made by taking the consequence relation as many-valued also and this in turn gives rise to a generalization of the algebraic structure frame to graded frame.

The thesis is organised as follows. The first chapter includes introduction and almost all possible ground notions which are essential to make the thesis self contained.

In Chapter 2 the notion of fuzzy topological system is introduced and the categorical relationship with fuzzy topology and frames is discussed in detail. Also this chapter contains some methodology for making new fuzzy topological systems from old ones.

Chapter 3 provides a generalization of fuzzy topological system which shall be called $\mathscr{L}$-topological system and categorical relationships with appropriate topological space and frame. Furthermore, two ways of constructing subspaces and subsystems of an $\mathscr{L}$-topological space and an $\mathscr{L}$- topological system are provided.

Chapter 4 deals with the concept of variable basis fuzzy topological space on fuzzy sets and contains a new notion of *variable basis fuzzy topological systems* whose underlying sets are fuzzy sets. In this chapter the categorical relationship between space and system is established.

Chapter 5 contains a different proof of one kind of generalized stone duality, which was done directly by Y. Maruyama, introducing a notion of $\bar{n}$-fuzzy Boolean system.

The last two chapters, Chapters 6 and 7 deal with the ultimate objective. Chapter 6 deals with the question "From which logic can fuzzy topology be studied?" To answer this, the notion of fuzzy geometric logic is introduced. In addition, further generalized notions such as fuzzy geometric logic with graded consequence, fuzzy topological spaces with graded inclusion, graded frame, and graded fuzzy topological systems come into the picture.

Chapter 7 provides categorical relationships among fuzzy topological spaces with graded inclusion, graded frames, and graded fuzzy topological systems.

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Francesco Parente, ***Regularity of Ultrafilters, Boolean Ultrapowers, and Keisler's Order***, University of East Anglia, UK, 2019. Supervised by Mirna Džamonja and David Asperó.
MSC: Primary 03E05, Secondary 03C20, 03C45. Keywords: ultrafilter, Boolean algebra, Keisler's order.

Abstract

This thesis investigates combinatorial properties of ultrafilters and their model-theoretic significance. Motivated by recent results on Keisler's order, we develop new tools for the study of Boolean ultrapowers, deepening our understanding of the interplay between set theory and model theory.

The main contributions can be summarized as follows. In Chapter 2, we undertake a systematic study of regular ultrafilters on Boolean algebras. In particular, we analyse two different notions of regularity which have appeared in the literature and compare their model-theoretic properties. We then apply our analysis to the study of cofinal types of ultrafilters; as an application, we answer a question of Brown and Dobrinen by giving two examples of complete Boolean algebras on which all ultrafilters have maximum cofinal type.

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