

# MATHEMATICAL PROCEEDINGS

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[2] M. P. FOURAM and D. S. SCOTT. Sheaves and logic. In *Applications of Sheaves* Lecture Notes in Math. vol. 753 (Springer-Verlag, 1979), pp. 302-401.

[3] P. T. JOHNSTONE. *Stone Spaces*. Cambridge Studies in Advanced Math. no. 3 (Cambridge University Press, 1982).

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