

CORRESPONDENCE

DAMAGED LIVES

To the Editor of the Transactions of the Faculty of Actuaries,

SIR,—I learn from the correspondence which you have printed in the *Transactions*, that Mr. Low is of opinion that “we cannot by means of a Select-Mortality Table determine at all the number of select lives contained in a body of mixed lives”; whereas this, as I showed in my paper on Select-Mortality Tables, is one of the simplest problems that such Tables can solve. I see also that my son, Dr. Ernest Sprague, has written to you defending my theory, of which he had made use in a paper lately submitted to the Faculty. In these circumstances I think it is right that I should myself say something in defence of the proposition that Mr. Low has attacked, and not leave the whole burden of the defence to fall on others, however willing they may be to support me. At the same time I cannot refrain from mentioning, that I find it extremely irksome to take up again and reconsider a subject that I laid aside so long ago; and that, therefore, whatever reply Mr. Low may make to my present remarks, I shall not be tempted to pursue the matter any further.

It is now more than twenty-six years since my Select-Mortality Tables were published (*J. I. A.* for January 1879 and January 1881). The principles on which they were based were entirely novel, and it was not surprising that a good many years elapsed before there was any evidence that they were received with favour by my professional brethren; but in the year 1896 such evidence was furnished by the resolution of the Council of the Institute of Actuaries, to publish a volume containing my tables, along with others calculated on the same principles. At the request of the Council of the Institute I wrote an Introduction to that volume, in which I described briefly the construction and properties of my tables; and I think that Mr. Low would do well to study that Introduction, if he has not already done so. Further and still more conclusive evidence of the esteem in which the principles of my tables were held by those most competent to judge, was the publication in the year 1903 of two volumes of “Select Tables, computed and published on the authority and under the superintendence of the Institute of Actuaries and the Faculty of Actuaries in Scotland.” No writer has hitherto expressed any doubt as to the soundness of the principles on which my tables were based, or of the truth of the conclusions I drew from them. Has Mr. Low, then, discovered a flaw in my reasoning, which has been overlooked by all other actuaries for so many years? I think I shall be able to show that this is not the case, and that the proposition which Mr. Low calls in question, is perfectly true.

The fundamental assumption on which a Select-Mortality table is based, is that the effect of selection wears off after a limited term of years; that is to say, that if the effect of selection lasts for n years and no more, a body of persons who have been insured for n years, will be subject to the same rate of mortality as persons of the same age who have been insured for $n+1$ or more years. I think that in the passage which Mr. Low has quoted from my paper, I have satisfactorily proved that it necessarily follows from the above assumption, that all the damaged lives in a body of mixed lives that have been insured for n years, die off in

the following n years. As Mr. Low does not admit this conclusion, I will endeavour to make the point clearer by expanding the reasoning. Adopting Mr. Low's notation, we have A, or $l_{(30)}$, lives entering at age 30; these after five years are reduced by death to B, or l_{35} , mixed lives of 35; and after five years more they are reduced to D, or l_{40} , mixed lives of 40. Mr. Low now proceeds: "We have also C (or $l_{(35)}$) lives entering at the age of 35." But this is not correct; for these lives do not enter at 35, but are a section of the A select lives that entered at 30, and also of the B mixed lives that were still insured at the age of 35; and their number, C, is calculated from the known number D, by means of the probabilities of living a year, as found from the probabilities of dying in a year contained in my table of the values of q_x , for different ages at entry and different periods since entry (*J. I. A.*, xxii, 394). We thus get successively from D (or l_{40}) the values of

$$l_{(35)+4}, l_{(35)+3}, l_{(35)+2}, l_{(35)+1}, l_{(35)} \text{ or C.}$$

This number C, or $l_{(35)}$, that we thus obtain, is the number of select lives of 35 which, after being insured for five years, will give us D, or l_{40} , survivors. Mr. Low appears to ignore altogether the process by which we have obtained C from D; but if he will consider what is the effect of it, I think he must admit that the C select lives of 35 correspond to the D survivors of 40, just in the same way as the A select lives of 30 correspond to the B survivors of 35. From A select lives we calculate, by means of the above-mentioned probabilities, the number of survivors B; and from the D survivors we calculate, by an inverse process, the corresponding number of select lives, C. It follows that C is the number of select lives contained in B; or in other words, $C=B'$. Also $B''=B-B'=B-C$. Making these substitutions in Mr. Low's equation $B'p'+B''p''=Cp'$, we get $(B-C)p''=0$; and since $B-C$ is not $=0$, we have $p''=0$; and we learn that none of the $B-C$ lives (or $l_{35}-l_{(35)}$ lives) will survive for five years. Since C is the number of select lives in B, $B-C$ is the number of the damaged lives; and we see that it is a necessary consequence of our fundamental assumption, that all the damaged lives die off within five years.

It is quite true that, as Mr. Low points out, experience shows that all damaged lives do not die off in five years; but the proper conclusion from this is that the effect of selection does not wear off in five years. This is a fact which I have repeatedly mentioned in my paper, and which has long been well known. In the Select Tables published by the Institute of Actuaries and the Faculty of Actuaries, the effect of selection is traced for ten years; and it probably lasts, though in a greatly diminished degree, for many years longer. It must therefore be admitted that the numerical conclusions drawn from tables in which selection is traced for five or ten years, are only approximately true; and I agree with Mr. Low in thinking that they should be employed with caution.

Mr. Low professes to apply my "line of reasoning" to a supposed case, in which, he says, selection operates in another direction; but in which there is really no selection at all. He supposes that a Life Office invites persons of a certain class to become members, without any inquiry being made as to their fitness for insurance. In these circumstances the lives admitted, whom he calls sub-standard lives, would be subject to a heavy rate of mortality at first; but "it is reasonable to suppose" that after five years the worst of them would have died and the survivors would be subject to the $H^{(6)}$ mortality. He proceeds:—"Suppose we have a table constructed on a plan similar to that of a Select Table, and we endeavour "by means of it to ascertain how many of the ultimate lives of a given age are "still of sub-standard quality. In this case the rate of mortality of ultimate

"lives of 35 will be lower than that of sub-standard lives entering at the same age, and hence in correspondence with a given number of ultimate lives of the age of 40, the number of sub-standard lives at 35 will be larger than the number of ultimate lives at that age"; that is, C is greater than B . There is, I think, a good deal of confusion here. It is, in fact, impossible to make out what is the exact problem which Mr. Low here proposes for solution. What does he mean by "still of sub-standard quality"? Does he mean at the age of 40? Apparently not, for he presently speaks of the ultimate lives of 35. By his supposition the entrants, taken as a whole, are of sub-standard quality; but they are a body of mixed lives, containing a certain number of what he calls the "worst" lives, who all die off in five years; and others, whom he calls ultimate lives. It is therefore incorrect to speak of sub-standard lives of any age as different from the entrants at that age. Nevertheless, Mr. Low says that, reasoning by my method, the ultimate lives of 35 will consist of C sub-standard lives and *minus* ($C - B$) others. (It is to be noticed that he deals here with only one age—35, or the age at entry; whereas my problem deals with the number of survivors at 35, out of entrants at the age of 30.) The proper conclusion from the facts supposed by Mr. Low, is that in C , the number of sub-standard lives who enter at 35, there are included B ultimate lives, and $C - B$ "worst" lives who all die within five years; and the absurd conclusion drawn by Mr. Low, is not got by any reasoning at all analogous to mine, as explained above. The "danger" is not due to my method of reasoning, but to the application of my formula to a totally different problem, to which it is not applicable.

To sum up:—the assumption that the benefit of selection wears off after n years, is a convenient one to make in the investigation of the problems of selection; but, when we suppose that n is equal to 5 or even 10, this is only approximately, and not strictly true. If it were strictly true, then all the damaged lives would, as I have proved above, die off in the five or ten years after entry; but this, as Mr. Low points out, is not the case. I think, however, that Mr. Low greatly exaggerates when he says that "many" of the damaged lives become select, and are admitted at the ordinary rate of premium on applying a second time for insurance. In forming a Select Table and in drawing conclusions from it, we are entitled to use any facts that observation and experience furnish us with; and one of the most important of these, to the use of which Mr. Low, strangely enough, objects, is the fact that, after the lapse of any number of years, a body of select lives will cease to be select, not in consequence of any deterioration that affects them all, but because a comparatively small number of them have been attacked by various diseases and infirmities which lessen their chances of life, while the majority are still select lives. In conclusion, although a select table cannot be trusted to give us the exact number of damaged lives after the lapse of any number of years, the number given by the table is approximately correct; and this is all that can be said about any conclusion deduced from even the best mortality tables.

I am, etc.,

T. B. SPRAGUE.

EDINBURGH, 26th May 1907.

To the Editor of the Transactions of the Faculty of Actuaries,

SIR,—Mr. Low, in his letter of 14th March, refers to the D^{MF} experience as affording formal proof that damaged lives do not all die in five years, and