A special case of the dissection of any two triangles into mutually similar pairs of triangles.

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This note is intended to be supplementary to the paper, by Mr Muirhead, on "The dissection of any two triangles into mutually similar pairs of triangles." The constructions given there, for the general case of this problem, yield no real solution if one angle of one triangle be greater than the sum of any two angles of the other triangle. For this particular case, the following constructions supply the necessary requirements; the first leads to a division of the triangles into three parts, the second to a division into four parts.

Let ABC and DEF be the two triangles and let the angle BAC be greater than the sum of any two angles of the triangle DEF.

FIRST CONSTRUCTION (FIG. 5).

At A make $\angle BAG = \angle DFE$ and $\angle CAH = \angle DEF$,
and at D make $\angle FDK = \angle ABG$ and $\angle EDL = \angle ACH$.

Then $\angle GAH = \angle KDL$ (Euc. I. 32),

and $\angle AHG = \angle CAH + \angle ACH = \angle DEL + \angle EDL = \angle DLK$.

By comparing the angles it is evident that the following pairs of triangles are equiangular and therefore similar:—

ABG and FDK, ACH and EDL, AHG and DLK.

The first two pairs are directly similar, the third pair perversely. By a slightly different arrangement of the angles there may be only one pair directly similar and the other two pairs perversely.

SECOND CONSTRUCTION (FIG. 6).

$\angle BAC > \angle DEF + \angle DFE$, $\therefore$ $\angle EDF > \angle ACB$.

At D make $\angle FDL = \angle ACB$; at A make $\angle BAK = \angle EDL$; then since $\angle KAC < \angle DEL + \angle DFL$, AG may be drawn in such a

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position that $\angle KAG < \angle DEL$ and $\angle GAC < \angle DFL$; at E make $\angle LEM = \angle KAG$; at B make $\angle ABH = \angle DEM$, and at F make $\angle DFN = \angle GAC$. Let the lines meet as in the figure.

Then $\angle HBG = \angle LFN$ (I. 32);

$\angle AKH = \angle BAK + \angle ABK = \angle EDM + \angle DEM = \angle EML$;

$\angle HGB = \angle GAC + \angle GCA = \angle NFD + \angle NDF = \angle LNF$;

The following pairs of triangles are then obviously similar:—
ABK and DEM, BGH and FNL, AKH and EML, CAG and DFN. The two first-mentioned pairs are directly similar, the remaining two pairs perversely.