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BOOK REVIEWS

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BROWN, K. A. AND GOODEARL, K. R. *Lectures on algebraic quantum groups* (Birkhäuser, Basel, 2002), 3 7643 6714 8 (paperback), £25.

If you have ever wondered what a quantum group is, this book is a must have, with its price tag of £25. According to the title, the book is a collection of lectures. As such, it usually contains informal motivations, but proofs of difficult theorems are often avoided. Instead, the authors point a reader to a proof in the literature. In fact, with a bibliography of 218 titles, the book is a valuable source of references. The detailed index makes the book easy to navigate.

The word *algebraic* in the title is pivotal. It actually means *ring theoretic* as opposed to *representation theoretic*, *Hopf algebraic*, *quantizational*, *topological*, or any other aspect of quantum groups. The book is written by algebraists and devoted to algebraic aspects of the theory. If you are an algebraist and want to move to quantum group theory, the book may prove a useful guide. If you need some algebraic insight into quantum groups occasionally, you will find the book handy.

The deepest insights the book provides are related to prime ideals. For instance, consider the following question (once posed by James Humphreys).

Let U be the universal enveloping algebra of a finite-dimensional semi-simple Lie algebra $\mathfrak{g}$. In positive characteristic, it admits a finite-dimensional quotient algebra U_χ, $\chi \in \mathfrak{g}^*$, a reduced enveloping algebra of $\mathfrak{g}$. Take two simple U_χ-modules that have the same central character on U. Do they belong to the same block of U_χ? While representation theory provides an insight that this is true, the only known proof is ring-theoretic (due to Ken Brown and Iain Gordon). It is essentially explained (in the context of quantum groups at roots of unity) in § III.9.2 of the book.

Going back to the topic of the book, quantum groups is an unusual subject in mathematics. Despite the absence of a good definition, the area comprises plenty of examples, and rich and beautiful theory as well as breathtaking applications. In short, a quantum group is a Hopf algebra if mathematicians agree to call it a quantum group. Let G be a complex semi-simple algebraic group with Lie algebra $\mathfrak{g}$. The two Hopf algebras that merit the title of 'quantum group' are the quantized universal enveloping algebra $U_q = U_q(\mathfrak{g})$ and the quantized function algebra $O_q = O_q(G)$. To be precise, U_q also depends on a choice of a lattice, which is irrelevant for this review.

The book consists of three parts. Part I contains the definitions of U_q and O_q. There are eight main chapters and eight appendices. The main chapters are essentially an expressway towards the definition of O_q and its basic properties. The appendices, with miscellaneous topics (Hopf algebras, the diamond lemma, PI-algebras, blocks), are very useful for filling in a reader's background in algebra.

Part II contains the theory of O_q at generic q. The focus of study here is prime and primitive ideals. The topics covered include the quantum double Bruhat decomposition, the Dixmier–Moeglin equivalence, and the catenarity theorem.

Part III is devoted to the root-of-unity case. It covers both U_q and O_q. When q is a root of unity, these are polynomial identity algebras. Thus, PI-theory plays a central role in this part. The topics covered include Poisson structures on the centres, the Azumaya loci and Müller's theorem.

Both Parts II and III contain 10 chapters. The final chapter of each is devoted to open problems. There is a fine difference between the parts.

The final chapter of Part II is called *Problems and conjectures*. There a *problem* is usually a rather vague question, definitely important, but probably too general to be answered to everybody's satisfaction. On the other hand, a *conjecture* is a very precise question. *Conjectures* will provide work and important benchmarks for algebraists while *problems* will offer subjects for numerous tea (coffee, cacao, wine, vodka, sake, etc.)-time discussions across the globe.

The final chapter of Part III is called *Problems and perspectives*. With this slightly misleading title, it contains 11 *questions*, some of which are *problems*, while others are *conjectures* (in the above sense). It also contains a short exposition (a *perspective*?) of Lusztig's conjecture.

The book fills a gap in the quantum-groups literature that will be appreciated by many students and researchers in physics and mathematics. It can be used as a foundation to an advanced course on quantum groups. It may be also given to a postgraduate student for independent reading. Finally, the book is an important source of references and information for specialists.

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EHRENPREIS, L. *The universality of the radon transform* (Oxford University Press, 2003), 0 19 850978 2 (hardback), £80.

Let us start, as this 700-page monograph rather oddly does not, by saying what we mean by a Radon transform. The basic set-up is that we are interested in how an integral of a function restricted to a submanifold varies as we deform the submanifold. Consider then a family of smooth submanifolds L_x ($x \in X$) of a smooth manifold M, parametrized by a smooth manifold X. Then for suitable (i.e. sufficiently smooth and decaying appropriately at infinity) functions f on M we define the *Radon transform* to be the function Rf on X defined by integration,

$$Rf(x) = \int_{L_x} f,$$

with respect to some suitable measure. Thus we have a map R from functions on M to functions on the parameter space X and one can study this map (injectivity, smoothness properties, characterization of the range, etc.) and, where it is injective, try to find an explicit inverse. The name comes from Radon's work of 1917, where he showed that one can recover a suitably nice function on the plane if one knows its integral over every line.

The other thing that is missing is some explanation of the scope and aims of the book, whose title rather suggests that the intention is to survey the whole range of the Radon transform in modern mathematics. In fact, the book begins with 100 or so pages of introduction that I suppose are intended to illuminate the philosophy of the remainder but which I found disconnected, rather vague and hard to get to grips with. And the subject matter of the book is centred around the case of submanifolds of $\mathbb{R}^n$, mainly the case of families of affine subspaces, although there is some discussion of what the author calls 'nonlinear Radon transforms', which just means integrating over other sorts of submanifold.

Within these limitations, the reader will find a lot of interesting material, much of it pleasingly concrete, explicit and example-based and considerably more readable than the introduction. There is a lot on harmonic functions, on Cauchy problems and 'Watergate problems' (i.e. situations like specifying data for the wave equation on the time axis, so that an infinite number of