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RODRIGO FIGUEIREDO, *O-minimal de Rham Cohomology*, Universidade de São Paulo, São Paulo, SP, Brazil, 2017. Supervised by Ricardo Bianconi. MSC: Primary 03C64. Secondary 14F40, 58A12. Keywords: De Rham cohomology, o-minimal manifolds, o-minimal structures, Pfaffian closure.

Abstract

O-minimal geometry generalizes both semialgebraic and subanalytic geometries, and has been very successful in solving special cases of some problems in arithmetic geometry, such as André–Oort conjecture. Among the many tools developed in an o-minimal setting are cohomology theories for abstract-definable continuous manifolds such as singular cohomology, sheaf cohomology and Čech cohomology, which have been used for instance to prove Pillay’s conjecture concerning definably compact groups. In the present thesis we elaborate an o-minimal de Rham cohomology theory for abstract-definable C^∞ manifolds in an o-minimal expansion of the real field which admits smooth cell decomposition and defines the exponential function. We can specify the o-minimal cohomology groups and attain some properties such as the existence of Mayer–Vietoris sequence and the invariance under abstract-definable C^∞ diffeomorphisms. However, in order to obtain the invariance of our o-minimal cohomology under abstract-definable homotopy we must work in a tame context that defines sufficiently many primitives and assume the validity of a statement related to Bröcker’s question.

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HENDRICK MAIA, *Quillen Model Categories-Based Notions of Locality of Logics over Finite Structures*, University of Campinas, Brazil, 2019. Supervised by Marcelo Esteban Coniglio. MSC: 03B70. Keywords: locality under k -logical equivalence, locality under isomorphism, Quillen model category-based framework, finite models, descriptive complexity.

Abstract

Locality is a property of logics, whose origins lie in the works of Hanf and Gaifman, having their utility in the context of finite model theory. Such a property is quite useful in proofs

of inexpressibility, but it is also useful in establishing normal forms for logical formulas. There are generally two forms of locality: (i') if two structures \mathfrak{A} and \mathfrak{B} realize the same multiset of types of neighborhoods of radius d , then they agree on a given sentence Φ . Here d depends only on Φ ; (ii') if the d -neighborhoods of two tuples \vec{a}_1 and \vec{a}_2 in a structure \mathfrak{A} are isomorphic, then $\mathfrak{A} \models \Phi(\vec{a}_1) \Leftrightarrow \Phi(\vec{a}_2)$. Again, d depends on Φ , and not on \mathfrak{A} . Form (i') originated from Hanf's works. Form (ii') came from Gaifman's theorem. There is no doubt about the usefulness of the notion of locality, which as seen applies to a huge number of situations. However, there is a deficiency in such a notion: all versions of the notion of locality refer to isomorphism of neighborhoods, which is a fairly strong property. For example, where structures simply do not have sufficient isomorphic neighborhoods, versions of the notion of locality obviously cannot be applied. So the question that immediately arises is: would it be possible to weaken such a condition and maintain Hanf/Gaifman-localities? Arenas, Barceló, and Libkin establish a new condition for the notions of locality, weakening the requirement that neighborhoods should be isomorphic, establishing only the condition that they must be indistinguishable in a given logic. That is, instead of requiring $N_d(\vec{a}) \cong N_d(\vec{b})$, you should only require $N_d(\vec{a}) \equiv_k N_d(\vec{b})$, for some $k \geq 0$. Using the fact that logical equivalence is often captured by Ehrenfeucht–Fraïssé games, the authors formulate a game-based framework in which logical equivalence-based locality can be defined. Thus, the notion defined by the authors is that of *game-based locality*. Although quite promising as well as easy to apply, the game-based framework (used to define locality under logical equivalence) has the following problem: if a logic \mathcal{L} is local (Hanf-, or Gaifman-, or weakly) under isomorphisms, and \mathcal{L}' is a sub-logic of \mathcal{L} , then \mathcal{L}' is local as well. The same, however, is not true for game-based locality: properties of games guaranteeing locality need not be preserved if one passes to weaker games. The question that immediately arises is: is it possible to define the notion of locality under logical equivalence without resorting to game-based frameworks? In this thesis, I present a homotopic variation for locality under logical equivalence, namely a Quillen model category-based framework for locality under k -logical equivalence, for every primitive-positive sentence of quantifier-rank k .

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WILL STAFFORD, *Something Valid This Way Comes: A Study of Neologicism and Proof-Theoretic Validity*, University of California, Irvine, USA, 2021. Supervised by Kai Wehmeier and Sean Walsh. MSC: 00A30, 03B55, 03F99. Keywords: Proof-theoretic semantics, neologicism, inquisitive logic.

Abstract

The interplay of philosophical ambitions and technical reality have given birth to rich and interesting approaches to explain the oft-claimed special character of mathematical and logical knowledge. Two projects stand out both for their audacity and their innovativeness. These are logicism and proof-theoretic semantics. This dissertation contains three chapters exploring the limits of these two projects. In both cases I find the formal results offer a mixed blessing to the philosophical projects.

Chapter 1. Is a logicist bound to the claim that as a matter of analytic truth there is an actual infinity of objects? If Hume's Principle is analytic then in the standard setting the answer appears to be yes. Hodes's work pointed to a way out by offering a modal picture in which only a potential infinity was posited. However, this project was abandoned due to apparent failures of cross-world predication. I re-explore this idea and discover that in the setting of the potential infinite one can interpret first-order Peano arithmetic, but not second-order Peano arithmetic. I conclude that in order for the logicist to weaken the metaphysically loaded claim of necessary actual infinities, they must also weaken the mathematics they recover.

Chapter 2. There have been several recent results bringing into focus the super-intuitionistic nature of most notions of proof-theoretic validity. But there has been very little work