triangle ABC from BC is $\frac{1}{3}$ AX. It remains, therefore, to prove that $VV' + WW' - UM = AX$.

From the encyclic quadrilateral AXCV, by application of Ptolemy's theorem, there results

$$AX \cdot CV + CX \cdot AV = VX \cdot AC.$$

Now $\qquad CV = \quad AV = AC/\sqrt{2}$;

therefore $\qquad AX + CX \quad = VX\sqrt{2} = 2VV'$.

Similarly $\qquad AX + BX \quad = 2WW'$.

But $\qquad BC \quad = 2UM$;

therefore $\qquad 2(VV' + WW' - UM) = 2AX + BX + CX - BC$

$$= 2AX.$$

Hence also the distance of the centroid of UVW from either AB or CA is the same as the distance of the centroid of ABC from AB or CA. The two triangles consequently have the same centroid.

§ 40. *O_1 is the centre of a circle which passes through the following* ten points :—V, W, M, X, N, Y, the feet of the perpendiculars from V on WU, WU′, and from W on VU, VU′.

The circle with O_1 as centre and OV or OW as radius is readily seen to pass through the feet of the four perpendiculars from V and W. Also this circle will pass through N and Y, if it can be shown to pass through M and X.

Now O_1M is half of AU, and $AU = VW$; therefore O_1M is half of VW; therefore the circle passes through M.

But since AX and ZM are perpendicular to BC, and $O_1A = O_1Z$, therefore $O_1X = O_1M$; and therefore the circle passes through X.

[The circle on VW as diameter can be proved to pass through X thus: the angle VXC is half a right angle, by § 38, and so is the angle WXB; therefore the angle VXW is a right angle.]

If O_2, O_3 be the middle points of WU, UV then, O_2, O_3 will be the centres of two other ten point circles.

The Potential of a Spherical Magnetic Shell deduced from the Potential of a Coincident Layer of Attracting Matter.

By A. C. Elliott, B.Sc., C.E.

This is the problem of § 670 in Clerk Maxwell's *Electricity and Magnetism.* The author proposes to proceed by another method and to obtain the result in a different form. Let O be the centre of the spherical surface on which the shell lies and Z the point where the

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magnetic potential V_m is to be found. Also let ϕ be the strength of the shell (magnetic moment per unit area), a its internal, and $a+\delta a$ its external radius. To represent the magnetic distribution let a layer of negative magnetic matter of density σ cover the inside face, and a corresponding positive layer the outside face. Finally, let Z be without the matter of the shell and on the positive side.

Since in a magnet the total quantity of magnetic matter is zero, these hypothetical layers are subject to the condition

$$a^2\sigma = \text{const.} \qquad (1).$$

Let V be the potential at Z due to a single layer of density σ and radius a. The magnetic potential V_m is the sum of the potentials due to the two imaginary layers; and hence by Taylor's theorem

$$\begin{aligned} V_m &= V + \frac{dV}{da}\delta a + \frac{dV}{d\sigma}\delta\sigma - V \\ &= \frac{dV}{da}\delta a + \frac{dV}{d\sigma}\delta\sigma \qquad (2). \end{aligned}$$

From the nature of the potential function

$$V = A\sigma \qquad (3),$$

where A is independent of σ—in fact, the potential for unit density.

From (1) $$\delta\sigma = -\frac{2\sigma}{a}\delta a$$

From (3) $$\frac{dV}{d\sigma} = A = \frac{V}{\sigma}$$

Therefore (2) becomes

$$V_m = \frac{dV}{da}\delta a - \frac{2V}{a}\delta a$$

or since δa is an independent variation

$$V_m = a^2\frac{d}{da}\left(\frac{V\delta a}{a^2}\right) \qquad (4).$$

But $$V\delta a = A\sigma\delta a = A\phi.$$

Hence if P be the potential at Z due to a layer of density numerically equal to ϕ

$$V_m = a^2\frac{d}{da}\left(\frac{P}{a^2}\right) \qquad (5).$$

Calling r the distance OZ, Maxwell obtains

$$V_m = -\frac{1}{a}\frac{d}{dr}(Pr) \qquad (6).$$

It appears therefore that the operations denoted by (5) and (6) respectively are equivalent. The first might sometimes be the more convenient to use—for instance, Maxwell, § 695, eqn. 6′.