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MULTIPLIERS ON SPACES OF FUNCTIONS ON COMPACT GROUPS WITH P-SUMMABLE FOURIER TRANSFORMS

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Let G be a compact abelian group with dual group Γ. For $1 \leqslant p < \infty$, denote by $A_p(G)$ the space of integrable functions on G whose Fourier transforms belong to $\ell_p(\Gamma)$. We investigate several problems related to multipliers from $A_p(G)$ to $A_q(G)$. In particular, we prove that $(A_p, A_p) \subsetneq \bigcap_{2<q<p} (A_q, A_q)$. For the circle group, we characterise permutation invariant multipliers from A_p to A_r for $1 \leqslant r \leqslant 2$.

1. Introduction

Let G be a locally compact abelian group with dual Γ. For $1 \leqslant p < \infty$, the space $A_p(G)$ is defined as:

$$A_p(G) = \{f \mid f \in L^1(G),\ \widehat{f} \in L^p(\Gamma)\}$$

with the norm $\|f\|_{A_p} = \|f\|_{L^1} + \left\|\widehat{f}\right\|_{L^p}$. Then $A_p(G)$ is a commutative semi-simple Banach algebra with maximal ideal space Γ.

A function ϕ on Γ is said to be a multiplier from A_p to A_r if $\phi\widehat{f} \in \widehat{A}_r$ for every $f \in A_p$. The set of multipliers from A_p to A_r is denoted by (A_p, A_r). It is well-known that a continuous linear operator $T\colon A_p \to A_r$ commutes with translations in G if and only if there exists a function ϕ on Γ such that $(Tf)^\wedge = \phi\widehat{f}\ \forall f \in A_p$. We shall denote by $\|\phi\|$ the operator norm of T. For a discussion of (A_p, A_r) multipliers, we refer to the paper by Bloom and Bloom [1].

If G is non-compact, then $(A_p, A_p) = \widehat{M}(G)$ [4]. For a compact group G, if $1 \leqslant p \leqslant 2$, then $(A_p, A_p) = \ell_\infty$, since in this case $\widehat{A}_p = \ell_p(\Gamma)$. Further if $r \geqslant p$, then it is easy to see that $(A_p, A_p) = (A_p, A_r)$. Thus the cases of interest are when $p > 2$ and $1 \leqslant r < p$.

In [10] Tewari and Gupta proved that for $1 \leqslant r < p$ and $2 < p < \infty$

(a) $(A_p, A_p) \cap C_0(\Gamma) \subsetneq (A_r, A_r) \cap C_0(\Gamma)$,
(b) $\bigcup_{r<p} (A_p, A_p) \subsetneq (A_r, A_r)$.

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The following was stated as an open problem in [8].

If $2 < p < \infty$, is $(A_p, A_p) \subsetneq \bigcap_{r<p} (A_r, A_r)$?

In this paper, we provide an affirmative answer to this problem (Theorem 2.1).

In general, the multiplier spaces (A_p, A_r) are not known for all values of p and r, $p > 2$. However it is easy to see that certain sequences spaces are contained in them (see [1]).

If $1 \leqslant r \leqslant 2$, $r < p$, then clearly $\ell_{(rp)/(p-r)} \subseteq (A_p, A_r)$. However, for $p > 2$ it is not known whether this containment is proper. In [10] it was shown that

(i) if $s > (rp)/(p-r)$, then $\ell_s \not\subseteq (A_p, A_r)$, $1 \leqslant r \leqslant 2 < p < \infty$,

(ii) if $s > (2p)/(p-2)$, then $\ell_s \not\subseteq (A_p, A_r)$, $2 < r \leqslant p < \infty$.

Using the idea of the proof of Theorem 2.1, we improve this result in Theorem 2.3.

In the last section we study the space of permutation invariant multipliers $\Pi(A_p, A_r)$ for the circle group T. (A multiplier $\phi \in (A_p, A_r)$ is said to be permutation invariant if $\phi \circ \pi \in (A_p, A_r)$ for all permutations π of Γ). We show that if $1 \leqslant r \leqslant 2 < p < \infty$ then $\ell_{(rp)/(p-r)}$ is precisely the set of permutation invariant multipliers from A_p to A_r.

We shall need some results on pointwise multipliers from $\ell_p(\Gamma)$ to $\ell_r(\Gamma)$, were Γ is discrete. The space $M(\ell_p, \ell_r)$ of pointwise multipliers consists of functions ϕ on Γ such that $\phi f \in \ell_r\ \forall f \in \ell_p$. Using the reverse Hölder's inequality it is easy to see that

(i) $M(\ell_p, \ell_r) = \ell_\infty$ if $p \leqslant r$,

(ii) $M(\ell_p, \ell_r) = \ell_{(pr)/(p-r)}$ if $p > r$.

(see [1]).

2. Proper inclusion in (A_p, A_r)-spaces

Theorem 2.1. *Let G be a compact abelian group and $p > 2$. Then*

$$(A_p, A_p) \subsetneq \bigcap_{q<p} (A_q, A_p).$$

The proof of Theorem 2.1 depends on an interesting lemma about sequences spaces, which may be of independent interest and is suggested by the equality

$$M\left(\ell_{(rp)/(p-r)}, \ell_r\right) = \ell_p, \quad 1 \leqslant r < p < \infty.$$

We also use this lemma to improve certain results about (A_p, A_r) multipliers due to Tewari and Gupta [10].

Lemma 2.2. *Let I be an infinite set. Let $1 \leqslant r < p < \infty$ and $\phi \in \ell_p(I)$ be such that $\phi \notin \ell_q(I)$ for every $q < p$. Then there exists $\psi \in \bigcap_{t>(rp)/(p-r)} \ell_t(I)$ such that $\phi\psi \notin \ell_r(I)$.*

PROOF: Clearly, we may assume $|\phi| \leqslant 1$ on I. Fix a positive integer $s > p/r$. Let $q_j = p - 1/j$ and choose $m_0 \in \mathbb{N}$ such that $q_j > r$ for $j \geqslant m_0$. Now define, $\alpha_j = (rq_j)/(q_j - r)$, $j \geqslant m_0$. Then q_j increases to p and α_j decreases to $(rp)/(p-r)$. Let $(a_n)_{n=1}^{\infty}$ be the support of ϕ. Let $n_0 = 0$ and choose $n_1 > 1$ such that

$$\sum_{n=1}^{n_1} |\phi(a_n)|^{q_s} > 1.$$

By induction, construct an increasing sequence $(n_j)_{j=1}^{\infty}$ of integers such that

$$\sum_{n=n_{j-1}+1}^{n_j} |\phi(a_n)|^{q_{js}} > 1, \quad \forall j \geqslant 1$$

Define

$$\psi(a) = \begin{cases} |\phi(a_n)|^{p/\alpha_j} & \text{if } a = a_n, \quad n_{j-1} < n \leqslant n_j, \quad \forall j \geqslant 1 \\ 0 & \text{on } I \setminus \{a_n\}_{n=1}^{\infty}. \end{cases}$$

Then if $k \geqslant m_0$, we have

$$\begin{aligned} \sum_{n=n_k+1}^{\infty} |\psi(a_n)|^{\alpha_k} &= \sum_{j=k}^{\infty} \sum_{n=n_j+1}^{n_{j+1}} |\phi(a_n)|^{\alpha_k p/\alpha_{j+1}} \\ &\leqslant \sum_{n=n_k+1}^{\infty} |\phi(a_n)|^p < \infty, \end{aligned}$$

since α_k is decreasing and $|\phi| \leqslant 1$.

Therefore

$$\psi \in \bigcap_{k=m_0}^{\infty} \ell_{\alpha_k}(I) = \bigcap_{t > (rp)/(p-r)} \ell_t(I).$$

Next, to see that $\phi\psi \notin \ell_r(I)$, consider

$$\begin{aligned} \sum_{n=1}^{\infty} |\phi(a_n)|^r \, |\psi(a_n)|^r &= \sum_{j=1}^{\infty} \sum_{n=n_{j-1}+1}^{n_j} |\phi(a_n)|^{pr/\alpha_j} \, |\phi(a_n)|^r \\ &\geqslant \sum_{j=1}^{\infty} \sum_{n=n_{j-1}+1}^{n_j} |\phi(a_n)|^{q_{js}} = \infty, \end{aligned}$$

since $s > p/r$, and so $r + pr/\alpha_j < q_{js}$ $\forall j \geqslant 1$.

This completes the proof of the lemma. ∎

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Proof of Theorem 2.1: Since $\bigcup_{q<q} A_q \subsetneq A_p$ [9], there exists $f \in A_p$ such that $\widehat{f} \notin \ell_q$ for every $q < p$. Let $\phi = \widehat{f}$. Then ϕ satisfies the conditions of Lemma 2.2 with $r = 2$. Hence there exists $\psi \in \bigcap_{t>(2p)/(p-2)} \ell_t$ such that $\phi\psi \notin \ell_2$. For $2 < q < p$, we have $(2q)/(q-2) > (2p)/(p-2)$, so that $\bigcap_{t>(2p)/(p-2)} \ell_t = \bigcap_{p>q>2} \ell_{(2q)/(q-2)}$. Hence $\psi \in \bigcap_{p>q>2} \ell_{(2q)/(q-2)}$.

Since $\phi\psi \notin \ell_2$, there exists a function ε on Γ whose range is contained in $\{\pm 1\}$ such that $\varepsilon\phi\psi \notin (L^1)^\wedge$ [2, Theorem 1.1]. Then $\varepsilon\psi$ belongs to $\bigcap_{p>q>2} \ell_{(2q)/(q-2)} \subseteq \bigcap_{p>q>2} (A_q, A_q)$, and $\varepsilon\psi \notin (A_p, A_q)$. This completes the proof of the theorem. ∎

We now use Lemma 2.2 to improve some results of [10] mentioned in the introduction.

Theorem 2.3. *Let G be an infinite compact abelian group. Then*

(a) $\bigcap_{s>(rp)/(p-r)} \ell_s \not\subseteq (A_p, A_r)$, $1 \leqslant r \leqslant 2 < p < \infty$.

(b) $\bigcap_{s>(2p)/(p-2)} \ell_s \not\subseteq (A_p, A_r)$, $2 < r \leqslant p < \infty$.

Proof: (a) Since $\bigcup_{q<q} A_q \subsetneq A_p$, there exists $f \in A_p$ such that $\widehat{f} \notin \ell_q$ for every $q < p$. Hence by Lemma 2.2 we get $\psi \in \bigcap_{s>(rp)/(p-r)} \ell_s$ such that $\psi\widehat{f} \notin \ell_r$. Thus $\psi \notin (A_p, A_r)$.

(b) Using (a) for $r = 2$ we get $\phi \in \bigcap_{s>(2p)/(p-2)} \ell_s$ such that $\phi \notin (A_p, A_2)$. Hence there exists $f \in A_p$ such that $\phi\widehat{f} \notin \ell_2$. Now there exists a function ε defined on Γ with range in $\{\pm 1\}$ such that $\varepsilon\phi\widehat{f} \notin (L^1)^\wedge$. Hence $\varepsilon\phi \notin (A_p, A_r)$ and $\varepsilon\phi \in \bigcap_{s>(2p)/(p-2)} \ell_s$. ∎

It was mentioned in the introduction that if $1 \leqslant r \leqslant 2 < p < \infty$ then the proper containment of $\ell_{(rp)/(p-r)}$ in (A_p, A_r) is not known. In Theorem 2.4 below, we give a sufficient condition on a multiplier $\phi \in (A_p(T), A_r(T))$ so that $\phi \in \ell_{(rp)/(p-r)}(\mathbb{Z})$.

It is easy to see that for $r \leqslant 2$, $\phi \in (A_p, A_r)$ if and only if $|\phi| \in (A_p, A_r)$. Further $\phi \in (A_p, A_r)$ if and only if $\psi(\gamma) = \phi(\gamma) + \phi(-\gamma) \in (A_p, A_r)$. Therefore it is sufficient to characterise non-negative, even multipliers from A_p to A_r. ∎

Theorem 2.4. *Let $1 \leqslant r \leqslant 2 < p < \infty$ and let ϕ be a non-negative, even sequence on $\mathbb{Z}$ such that $\phi(n+1) \leqslant \phi(n)$, $n > 0$. Then $\phi \in (A_p(T), A_r(T))$ if and only if $\phi \in \ell_{(pr)/(p-r)}(\mathbb{Z})$.*

Proof: If $\phi \in \ell_{(rp)/(p-r)}$, clearly $\phi \in (A_p, A_r)$. For the converse, let $\phi \in$

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$(A_p(T), A_r(T))$. We may assume that $\phi(0) = 0$. Let

$$\psi_m(n) = \begin{cases} (\phi(n))^{r/(p-r)} & \text{on } [-m, m] \\ 0 & \text{otherwise.} \end{cases}$$

Then

$$\|\phi\psi_m\|_{\ell_r} \leqslant \|\phi\| \left(\|\check{\psi}_m\|_{L^1} + \|\psi_m\|_{\ell_p}\right). \tag{2.5}$$

Now by [**3**, 7.3.3]

$$\begin{aligned} \|\check{\psi}_m\|_{L^1} &\leqslant C \sum_{n=1}^{m} \frac{\psi_m(n)}{n} \\ &\leqslant C \sum_{n=1}^{m} \left(\frac{1}{n^{p'}}\right)^{1/p'} \left(\sum_{n=1}^{m} (\phi(n))^{pr/(p-r)}\right)^{1/p}. \end{aligned} \tag{2.6}$$

Also,

$$\begin{aligned} \|\phi\psi_m\|_{\ell_r} &= \left(2 \sum_{n=1}^{m} (\phi(n))^r (\phi(n))^{r^2/(p-r)}\right)^{1/r} \\ &= 2^{1/r} \left(\sum_{n=1}^{m} (\phi(n))^{pr/(p-r)}\right)^{1/r}. \end{aligned} \tag{2.7}$$

Hence, combining (2.5)–(2.7) we get

$$2^{1/r}\left(\sum_{n=1}^{m} (\phi(n))^{pr/(p-r)}\right)^{1/r} \leqslant \|\phi\| \left\{ C' \left(\sum_{n=1}^{m} (\phi(n))^{pr/(p-r)}\right)^{1/p} + \left(\sum_{n=1}^{m} (\phi(n))^{pr/(p-r)}\right)^{1/p} \right\}.$$

It follows that

$$\left(\sum_{n=1}^{m} (\phi(n))^{pr/(p-r)}\right)^{1/r-1/p} \leqslant \|\phi\| (1 + C') 2^{-1/r},$$

where the constant C' is independent of m. Hence $\phi \in \ell_{pr/(p-r)}(\mathbb{Z})$. This completes the proof of the theorem. ∎

3. Permutation invariant multipliers from A_p to A_r

In this section we study permutation invariant multipliers from A_p to A_r on the circle group. In general, as we have already seen, if $p \leqslant 2$, then $(A_p, A_r) = \ell_{rp/(p-r)}$ if $r < p$ and $(A_p, A_r) = \ell_\infty$ if $r \geqslant p$. Therefore we assume that $p > 2$. The following theorem completely characterises $\Pi(A_p(T), A_r(T))$, $1 \leqslant r \leqslant 2 < p < \infty$.

Theorem 3.1. *Let $1 \leqslant r \leqslant 2 < p < \infty$, then*

$$\ell_{pr/(p-r)}(\mathbb{Z}) = \Pi(A_p(T), A_r(T)).$$

The proof of the above theorem depends on the following lemma:

Lemma 3.2. *Let $2 < p < \infty$ and $(a(n)) \in \ell_p(\mathbb{Z})$ be such that $a(n) \geqslant 0$ and $a(n) = a(-n)\ \forall n \in \mathbb{N}$. Then there exists a permutation π of $\mathbb{Z}$ such that $a \circ \pi(n) \in (L^1(T))^\wedge$.*

Proof: Let π be a permutation of $\mathbb{N}$ such that $a \circ \pi$ is decreasing on $\mathbb{N}$. Extend π to $\mathbb{Z}$ by defining $\pi(-n) = -\pi(n)\ \forall n \in \mathbb{N}$ and $\pi(0) = 0$. We show that $a \circ \pi \in (L^1(T))^\wedge$. Clearly, $a \circ \pi(n) \geqslant 0$, $a \circ \pi(n) = a \circ \pi(-n)\ \forall n \in \mathbb{N}$ and $a \circ \pi(n)$ decreases to zero on $\mathbb{N}$. Also,

$$\sum_{n=1}^{\infty} \frac{a \circ \pi(n)}{n} \leqslant \left(\sum_{n=1}^{\infty} (a \circ \pi(n))^p\right)^{1/p} \left(\sum_{n=1}^{\infty} 1/n^{p'}\right)^{1/p'} < \infty.$$

Therefore by [3, 7.3.3] $a \circ \pi \in (L^1(T))^\wedge$.

This completes the proof of the lemma. □

Proof of Theorem 3.1: It is clear that $\ell_{pr/(p-r)}(\mathbb{Z}) \subseteq \Pi(A_p(T), A_r(T))$. Conversely, suppose $(a(n)) \notin \ell_{pr/(p-r)}(\mathbb{Z}) = M(\ell_p(\mathbb{Z}), \ell_r(\mathbb{Z}))$, then there exists a sequence $(b(n)) \in \ell_p(\mathbb{Z})$ such that $(a(n)b(n))$ does not belong to $\ell_r(\mathbb{Z})$.

Define

$$c(n) = \max\left(\frac{1}{|n|} + |b(n)|, \frac{1}{|n|} + |b(-n)|\right).$$

Then $(c(n)) \in \ell_p(\mathbb{Z})$, and $(a(n)c(n)) \notin \ell_r(\mathbb{Z})$. Also $(c(n))$ satisfies the conditions of Lemma 3.2, hence there exists a permutation π of $\mathbb{Z}$ such that $(c \circ \pi(n)) \in (L^1(T))^\wedge$. Therefore $(c \circ \pi(n)) \in \widehat{A}_p(T)$. It follows that $a \circ \pi \notin (A_p(T), A_r(T))$ as $(a \circ \pi(n)\, c \circ \pi(n)) \notin \ell_r(\mathbb{Z})$.

This completes the proof of the theorem. □

In the case $2 < r \leqslant p < \infty$ we are not able to characterise $\Pi(A_p(T), A_r(T))$. Observe that if $p > 2$, then $\ell_{2p/(p-2)} \subsetneq \Pi(A_p, A_p)$ since the constant function 1 on Γ belongs to $\Pi(A_p, A_p)$. We prove the following theorem, characterising a subclass of $\Pi(A_p(T), A_r(T))$.

THEOREM 3.3. *Let $2 < r \leqslant p < \infty$. If $(a(n))$ is a sequence on $\mathbb{Z}$ such that $(a(n)\varepsilon(n)) \in \Pi(A_p(T), A_r(T))$ for every sequence $(\varepsilon(n))_{n\in\mathbb{Z}}$, $\varepsilon(n) = \pm 1$, then $(a(n)) \in \ell_{2p/(p-2)}(\mathbb{Z})$.*

PROOF: If on the contrary $(a(n)) \notin \ell_{2p/(p-2)}(\mathbb{Z}) = M(\ell_p(\mathbb{Z}), \ell_2(\mathbb{Z}))$, then there exists a sequence $(d(n)) \in \ell_p(\mathbb{Z})$ such that $d(n) \geqslant 0$, $d(n) = d(-n)$, $d(n) \neq 0\ \forall n \in \mathbb{N}$ and $(a(n)d(n)) \notin \ell_2(\mathbb{N})$. By Lemma 3.2, there exists a permutation π of $\mathbb{Z}$ such that $(d \circ \pi(n)) \in \widehat{A}_p(T)$. Since $(a \circ \pi(n)\, d \circ \pi(n)) \notin \ell_2(\mathbb{Z})$, therefore there exists a sequence $(\varepsilon(n))_{n\in\mathbb{Z}}$, $\varepsilon(n) = \pm 1$, such that $(\varepsilon(n) a \circ \pi(n) d \circ \pi(n)) \notin \left(L^1(T)\right)^\wedge$. Hence $(a(n)\varepsilon(n)) \notin \Pi(A_p(T), A_r(T))$, a contradiction.

This completes the proof of the theorem. □

REMARK 3.4. Lemma 3.2 can be viewed as an intermediate result between the following:

(i) (Helgason [5]): Let G be a compact abelian group and ϕ a function on Γ. Then $\phi \in \ell_2$ if and only if $\phi \circ \pi \in \widehat{L}^1$ for every permutation π of Γ.

(ii) (Kahane [7]): There exists a sequence $\phi \in C_0(\mathbb{Z})$ such that $\phi \circ \pi \notin \left(L^1(T)\right)^\wedge$ for any permutation π of $\mathbb{Z}$.

The proof of Helgason's result gives the following result about (A_p, A_p)-multipliers:

THEOREM 3.5. *(a) Let $p > 2$ and $\phi \in \Pi(A_p, A_p) \cap C_0$. Then there exists a permutation π of Γ such that $\phi \circ \pi \in (A_p, A_2)$.*

(b) $\Pi(L^1, L^1) \cap C_0 = \ell_2$.

PROOF: (a) Let $E = \{\gamma \in \Gamma \mid \phi(\gamma) \neq 0\}$. Since $\phi \in C_0$, E is countable. If E is finite then $\phi \circ \pi \in (A_p, A_2)$ for every permutation π of Γ. So we assume that E is infinite. Let E_1 be an infinite subset of E such that

$$\sum_{\gamma \in E_1} |\phi(\gamma)|^{2p/(p-2)} < \infty. \tag{3.7}$$

Let $E_2 = E \setminus E_1$. If E_2 is finite then for every permutation π of Γ $\phi \circ \pi \in \ell_{2p/(p-2)} \subseteq (A_p, A_2)$. So we assume that E_2 is infinite. Choose a countably infinite Λ_2 subset F_1 of Γ [6] and define $F_2 = \Gamma \setminus F_1$. Let π be a permutation of Γ mapping F_1 onto E_2. We claim that $\phi \circ \pi \in (A_p, A_2)$. Let $f \in A_p$. Since $\phi \circ \pi \in (A_p, A_p)$, $\widehat{g} = (\phi \circ \pi)\widehat{f} \in \widehat{A}_p$. Using Hölder's inequality and (3.7), we get

$$\sum_{\gamma \in F_2} |\phi \circ \pi(\gamma)|^2 \left|\widehat{f}(\gamma)\right|^2 < \infty,$$

since for $\gamma \in F_2$ $\phi(\pi(\gamma)) \neq 0$ only if $\pi(\gamma) \in E_1$. Hence there exists an $h \in L^2$ such that

$$\widehat{h}(\gamma) = \begin{cases} \phi \circ \pi(\gamma)\widehat{f}(\gamma), & \gamma \in F_2 \\ 0, & \text{otherwise.} \end{cases}$$

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Now $g - h \in L^1_{F_1}$. Since F_1 is a Λ_2 set, we have $g - h \in L^2$ [6]. Therefore, $g \in L^2$.

(b) In this case we choose the set $E_1 \subset E$ such that

$$\sum_{\gamma \in E_1} |\phi(\gamma)|^2 < \infty.$$

Then proceeding as above we see that $\phi \circ \pi \in (L^1, L^2) = \ell_2$ [6]. Hence $\phi \in \ell_2$. □

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