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The interplay of rotation and relaxation in star clusters and galactic nuclei

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Abstract.
The impact of rotation on the relaxational dynamics of dense stellar systems is reviewed; while in the past linear stability analysis and a few N-body models existed only, we report recent advances in the technique of 2D Fokker-Planck models for axisymmetric rotating star cluster, now extending into the post-core collapse phase. It is confirmed that rotating clusters, whether they are in a tidal field or not, evolve significantly faster in pre-collapse than non-rotating ones, while in post-collapse only those in a tidal field keep their larger speed of evolution. Consequences for observed shapes, density distribution, and kinematic properties of young and old star clusters are discussed.

1. Introduction

Five years ago at the IAU Symposium No. 174 here in Tokyo one of the main topics was the existence of gravothermal oscillations in real N-body systems. Gravothermal oscillations had been detected by modelling a star cluster as a heat-conducting sphere (Lynden-Bell & Eggleton 1980, Sugimoto & Bettwieser 1983, Bettwieser & Sugimoto 1984). Makino (1996) then presented the first direct N-body models exhibiting gravothermal oscillations.

Such modelling attempts had been preceded by examinations of the secular stability of self-gravitating isothermal spheres; two methods had been used, one is the linear series, where a sequence of equilibria and their total energies is examined (Antonov 1962, Lynden-Bell & Wood 1968, Magliocchetti, Pucacco & Vesperini 1998), the other is a linear perturbation analysis maximizing the second order variation of the total entropy (Hachisu & Sugimoto 1978, Hachisu et al. 1978, Hachisu 1979, Spurzem 1991). The result is in all cases that gravothermal systems behave counterintuitively: due to the self-gravitation the maximum

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entropy generation goes along with the generation of inhomogeneities (core collapse, gravothermal catastrophe), generation of anisotropy (differences between radial and tangential velocity dispersions or temperature), and increase of angular velocity despite of loss of angular momentum. These phenomena have been dubbed negative specific heat and negative specific moment of inertia. It was only Hachisu (1979) who examined in the linear approximation the negative specific moment of inertia in axisymmetric rotating systems.

Rotation is today one of the most important challenges for dynamicists of gravothermal systems. First, rotation is a natural initial condition at the time of formation of globular clusters, and it is only during their evolution that they would lose angular momentum by diffusive and tidal mass loss. Second, most of our standard modelling tools do not work or have not been properly extended to anything else than spherically symmetric systems. Even in direct N-body modelling little attention has been given to rotating star clusters, though it would be least problematic here, probably because of little other models to compare with (but see Akiyama & Sugimoto 1989, Boily 2000, Boily & Spurzem 2000). Goodman (1983) proposed in an unpublished thesis a way to deal approximately with 2D Fokker-Planck models of axisymmetric rotating star clusters, an approach which has been with much improvements revisited now by Einsel & Spurzem (1999) and Kim et al. (2001). Third, if rotation may not be extremely important for older globular clusters, it is for the case of dense star clusters in galactic nuclei. Very little if anything is known about the relaxational angular momentum interactions between a dense star cluster and massive black holes in nuclei, which are in general axisymmetric or even triaxial.

2. Rotating Star Clusters

Observations show that flattening is a common feature of globular clusters, which has been known since the early work done by Pease & Shapley (1917). Measuring projected ellipticities $e = 1 - b/a$ of large globular cluster samples White & Shawl (1987) derive a mean $\bar{e} = 0.07 \pm 0.01$ for 99 clusters in the Milky Way, and Staneva et al. (1996) find $\bar{e} = 0.086 \pm 0.038$ for 173 clusters in M31, with maximum values 0.27 and 0.24 of individual globulars, respectively. Kinematical data, i.e. radial velocities of large numbers of cluster members, reveal that this flattening may indeed be explained in terms of rotation, and that the minor axes are nearly coincident with the determined rotation axes (Meylan & Mayor 1986). Dust obscuration, anisotropy or tidal distortion are able to explain individual cases of flattening, but can statistically be ruled out as the main mechanism (White & Shawl 1987). Significant ellipticity variations are found within globular clusters (e.g. Geyer et al. 1983), and these partly also coincide with the rotation curves obtained by fits to the radial velocity data with some parametrization specified for the velocity field (Meylan & Mayor 1986). Moreover, Kontizas et al. (1990) show that the outer parts of globulars in the Small Magellanic Cloud are obviously rounder than the parts inside the half mass radius and it is likely that their structure differs from that of the galactic globular clusters because they are younger (in general) and subject to different tidal forces. The importance of age for the interpretation of observed ellipticities has already been emphasized by Frenk & Fall (1982), who undertook eye-estimates of cluster

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ellipticities in the Milky Way and the Magellanic Clouds, the latter being slightly larger than the former, which is explained again in terms of internal globular cluster evolution. This view is supported by studies relating Milky Way globular cluster ellipticities to the cluster concentration parameter $c = \log(r_t/r_c)$ (White & Shawl 1987), where r_t is the tidal radius and r_c is the core radius, or to the half mass relaxation time t_{r_h} (Davoust & Prugniel 1990), both representing an evolutionary status of the respective globular cluster. In these two investigations, the average flattening of the dynamically younger systems is significantly larger as well, indicating that loss of angular momentum originating from diffusion past the escape velocity on relaxation time scales decreases the ellipticity of a cluster.

On the theoretical side there has been until very recently surprisingly little known about rotating star clusters, even in the most idealized case of equal point masses. Unfortunately, rotation, though it is a natural initial condition from collapse of a star–forming cloud, could not be included in most of the existing evolutionary models of star clusters. Monte Carlo and Fokker-Planck techniques (with the exception of the papers by Einsel & Spurzem 1999, henceforth Paper I, and the successor work by Kim et al. 2000, henceforth Paper II) were limited to spherical symmetry, as well as gaseous models. A generalization of such models poses significant challenges, such as what is the effective viscosity scaling describing properly viscous effects due to two-body relaxation (Goodman 1983) in the case of gaseous models. For Fokker–Planck models the main problem is the requirement to neglect a possibly existing third integral of motion on axisymmetric potentials, because it cannot be given analytically. In Paper I diffusion of orbits was considered disregarding the third integral, i.e. in a 2D model only considering E and J_z, orbital energy and z-component of angular momentum of a stellar orbit, and a discussion of possible errors given.

Agekian (1958) suggested a model in which specific angular momentum is lost due to a relatively large fraction of escaping stars residing in the tail of a Maxwell velocity distribution shifted towards the direction of rotation as compared to the fraction residing in the opposite direction having less angular momentum. Considering this effect for every volume element of rotating ellipsoids he obtained a critical ellipticity of $e \leq 0.74$ below which the systems become rounder with time.

Goodman (1983) provided in his unpublished thesis an approach to consider the energy E and the z-component of the angular momentum J_z, as the only isolating integrals and any possible non-ergodicity on the hypersurface in phase space given by E and J_z due to any third integral was neglected. Then an orbit average over the accessible phase space for given E and J_z is possible, and a 2D Fokker-Planck equation can be solved. The orbit average, however, is rather complex and involves a numerical integration in 2D Cartesian coordinate space over a volume bounded by the curved surface accessible for given E and J_z. At the time of Goodman's (1983) thesis the numerical resolution could only be rather poor, and the results were considered not very trustworthy. Paper I revisited such approach with much higher numerical accuracy and resolution. They find that the evolution of the star cluster to core collapse is accelerated by a factor of up to three due to the initial rotation in a tidally limited cluster. Also there is an indication that after a first start-up phase, which could be linked to

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the "gravo–gyro" catastrophe of Hachisu (1979) the evolution is turned into a self-similar collapse solution, as proposed by Lynden-Bell (2001). Subsequent work is being published in Paper II about the post-collapse evolution of rotating globular clusters and a few of the important results will also shortly be described here.

3. Modelling Rotating Clusters

The only approximate model available for large particle numbers based on the Fokker-Planck approximation is the one based on Goodman's (1983) thesis, and redeveloped and improved by Einsel & Spurzem (1999) for the pre-collapse system in a tidal field, and by Kim et al. (2001) extending the models into the post-collapse regime. Fig. 3. displays the main effect, namely a rather significant shortening of core collapse time for systems with a moderate initial amount of rotational energy (compare Table 1, and Einsel & Spurzem 1999).

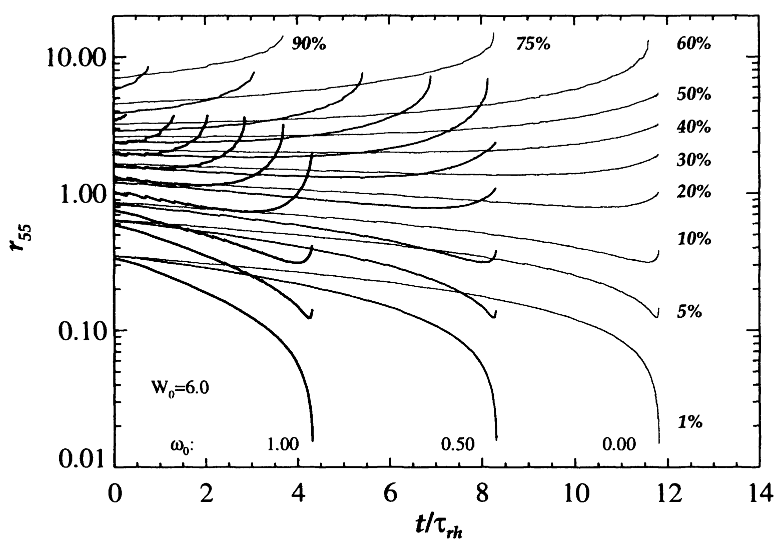

Figure 1. Evolution of mass shells (Lagrange radii) for the model $(W_{0,i}, \Omega_{0,i}) = (6.0, 0.60)$. Shown are the radii for mass columns containing the indicated percentage of total mass in the direction of the $\theta = 54.74°$-angle, the tidal radius $r_{\rm tid}$ determined from $(\phi(R,z) = E_{\rm tid})$, and the core radius $r_c = (9\sigma^2/(4\pi G\rho)^{\frac{1}{2}}$.

In order to extend the evolution beyond the core collapse, we need to add an energy source that drives the post core collapse evolution. Primordial binaries and massive stars can provide energy from very early on, and they delay significantly the core collapse time and affect the details of the binary distribution very much (Gao et al. 1991, Giersz & Spurzem 2000). In order to compare well our models with previous standard results, and due to a lack of any good method to include many hard binaries in our model, we have only considered

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the heating effect due to three-body binaries. The energy generation rate by three-body binaries per mass unit is given as (e.g., Hut 1985)

$$\left(\frac{\delta e}{\delta t}\right)_{3\mathrm{b}} = C_b \frac{\rho^2}{m^2\sigma^2}\left(\frac{Gm}{\sigma}\right)^5 \quad . \tag{1}$$

Here ρ and σ are the local mass density and 1D velocity dispersion, respectively, G the gravitational constant and m the individual stellar mass. We use the parameter of the Coulomb logarithm $\gamma = 0.11$, and $C_b = 90$ (see more detailed discussion in Kim et al. 2001).

3.1. Initial Models and Boundary Conditions

As in Paper I, we employ the rotating King models as initial models following Lupton & Gunn (1987). These models are characterized by two parameters: dimensionless central potential W_0 and the rotational parameter ω_0. We have examined the evolution of clusters with $W_0 = 6$ and W_0=3. The rotational parameters are chosen such that the cluster remains to be stable against the dynamical instabilities. In Table 1, we have listed the global parameters of the initial models used in the present study. The rotation parameters for models with central potential $W_0 = 3$ are chosen such that the ratios of initial rotational energy to initial potential energy should be similar to those for models with central potential $W_0 = 6$ as shown in Table 1. For uniformly rotating systems, secular instability is known to arise if $T_{rot}/|W| > 0.14$ (Ostriker & Peebles 1973), where T_{rot} is the rotational kinetic energy and W is the potential energy. All of our models satisfy the stability criterion.

Table 1. Properties of Initial Models

W_0	ω_0	r_t/r_c	r_h/r_t	$T_{rot}/\lvert W\rvert$	e_{dyn}	r_h [pc]	τ_{rh} [yrs]
	0.0	18.0	0.15	0.000	0.000	4.19	1.64×10^8
6	0.3	14.5	0.18	0.035	0.107	5.02	2.15×10^8
	0.6	9.6	0.24	0.101	0.285	6.70	3.32×10^8
	0.0	4.7	0.26	0.000	0.000	7.25	3.73×10^8
3	0.8	4.2	0.29	0.035	0.102	8.89	4.40×10^8
	1.5	3.3	0.35	0.097	0.267	9.77	5.84×10^8

e_{dyn}: dynamical ellipticity as defined in Paper I.
$T_{rot}/|W|$: rotational over potential energy.
$m = 1M_\odot$ assumed to compute dimensional quantities. We have used fixed value of $r_t = 27.9$ pc to obtain r_h.

3.2. Velocity Dispersion and Angular Speed

In Fig. 2, we have shown the time evolution of central velocity dispersion (σ_0) and central rotational angular speed (Ω_0). Both σ_0 and Ω_0 increase with time rather slowly until core collapse and decrease afterward. The increase of Ω_0 is a consequence of the gravo-gyro instability. During the post-collapse, these quantities drops rapidly with time. The general behavior of σ_0 does not seem

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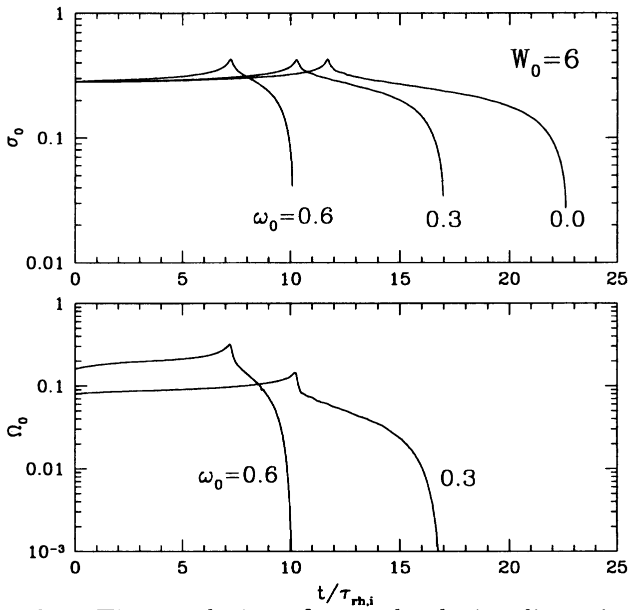

Figure 2. Time evolution of central velocity dispersion and central angular speed for models with $W_0 = 6$ and $\omega_0 = 0.0$, 0.3 and 0.6.

to be affected by the presence of rotation. Rotational energy is only a small fraction of the total kinetic energy near the center throughout the evolution.

The relationship between the central density and σ_0, and Ω_0 are shown in Fig. 3. For σ_0 versus ρ_0, we have plotted all six models of Table 1, while only rotating four models are shown for Ω_0 versus ρ_0 figure. The velocity dispersion is nearly independent of initial rotation: all models with the same W_0 fall on nearly single lines. The power-law behavior of σ_0 on ρ_0 during the pre collapse phase is a consequence of self-similarity of collapsing core. During this phase, it is well known that $\sigma_0 \propto \rho_0^{0.1}$ (e.g. Cohn 1980). During the post-collapse phase, we can again apply the energy balance argument to obtain $\sigma_0 \propto \rho_0^{1/6} M^{1/9}$. For isolated clusters, M = const. and $\sigma_0 \propto \rho_0^{1/6}$. Since both M and ρ_0 decrease with time, we expect that $\beta > 1/6$ if we express $\sigma_0 \propto \rho_0^{\beta}$. Our result in Fig. 3 shows that $\beta \approx 0.23$.

The angular speed also appears to follow power law on ρ_0 during the pre collapse phase, except for the early stage of evolution for models with central potential of $W_0 = 3$. Although the behavior of Ω_0 during the post-collapse phase depends on the amount of angular momentum loss by the end of the core-collapse, it still has a power-law relation with central density, ρ_0.

3.3. V_{rot}/σ

We try to give some evidence what could be the consequences of our model simulations for the interpretation of velocities (rotational and dispersion) in globular star clusters. It has to be emphasized, however, that due to the idealized nature of our numerical studies (e.g. equal point masses, no tides, no stellar

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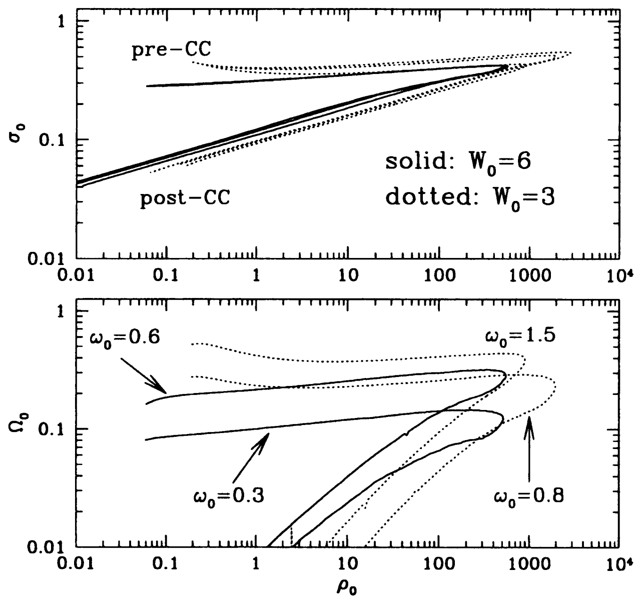

Figure 3. The evolution of σ_0 and Ω_0 as a function of ρ_0 for all six models shown in Table 1. The velocity dispersion follows power-laws during pre and post collapse phase, and is nearly independent of rotation. The angular speed also follows near power-law on ρ_0 during the collapsing phase.

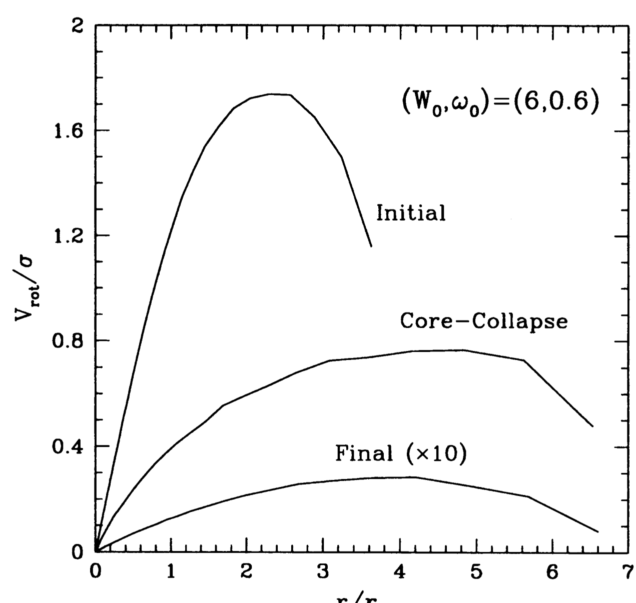

Figure 4. V/σ as a function of radius in units of half-mass radius for models with initial $W_0 = 6$, and $\omega_0 = 0.6$ at three epochs as indicated in the figure. The location of maximum V_{rot}/σ moves outward.

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evolution) such discussion must be preliminary and can certainly not be done on a quantitative, but rather on a more qualitative level.

V_{rot}/σ has been analyzed in our model clusters, where V_{rot} denotes the rotational velocity, and σ the 1D velocity dispersion, as a function of time and radius. V_{rot}/σ describes the relative importance of rotational versus pressure support in the local cluster kinematics and dynamics; while it has been very successfully used in observational studies of elliptical galaxies and bulges of spiral galaxies this quantity recently has become available also from globular cluster observations (Gebhardt 2001).

In Fig. 4, we have shown the runs of V_{rot}/σ as a function of radius in units of half-mass radius for the model with initial $W_0 = 6$ and $\omega_0 = 0.6$ at three epochs: initial model, at core-collapse, and near final disintegration. We notice from this figure that the position of the maximum V_{rot}/σ relative to r_h moves outwards with time. This is another indication of angular momentum transport. Also note that $(V_{rot}/\sigma)_{\max}$ for the initial model is much larger than the late values of an evolved cluster, it decreases monotonically with time. For this particular model, the $(V_{rot}/\sigma)_{\max}$ becomes around 0.8 at the core collapse, and further decreases in the post-collapse phase to very small values smaller than 0.1. Towards the centre all models show a decrease of (V_{rot}/σ) to zero.

Note, that multi-mass models obtained with an earlier version of our code (Spurzem & Einsel 1998) exhibit a much more pronounced gravo-gyro instability effect (Hachisu 1979) as compared to the equal mass model. Heavy masses go into core bounce with a strong acceleration of rotational velocity and a decrease of dispersion velocity, so they would exhibit a high V_{rot}/σ in the centre. We will further study this effect in future work.

Though many of the Galactic globular clusters do not rotate significantly, there have been rotation curves measured for some clusters earlier (Meylan & Mayor 1986, Lupton, Gunn & Griffin 1987, Gebhardt et al. 1994, 1995). The clusters with measured rotation includes ω Cen, 47 Tuc, M13, M15, and NGC6397. For most of these clusters, the rotation measurements are done in a rather limited range and rotation curves are hardly known. Also in earlier papers, Gebhardt et al. (1995) made very careful measurements of stellar velocity field using Fabry-Perot Spectrophotometer and found that the projected rotation velocity rises or remains nearly flat from $30''$ to $10''$ for M15, 47 Tuc, and NGC 6397. As was discussed above this cannot be reproduced in our equal mass models here yet, but we think a multi-mass model could be able to explain the effect. Also in the case of 47 Tuc the rotation curve seems to be consistent with the solid body rotation up to $\sim 2'$ corresponding to $\sim 0.7 r_h$, although again the very central part appears to deviate from the solid body rotation.

4. Conclusions

We have shown that rotating star clusters accelerate their evolution due to rotation even in post-collapse, provided they are immersed in a tidal field. Although their total angular momentum is strongly and irreversibly reduced in post-collapse, the structure near the half-mass radius differs as a function of initial rotation. Therefore it is theoretically possible to distinguish pre- and post-collapse clusters by their monotonically decreasing V_{rot}/σ values near the

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half-mass radius. Though this needs further exploration, in particular because we expect that multi-mass systems exhibit some of the effects discussed here (gravo-gyro instability, kinematical signature of initial rotation) in a much more pronounced way. Further work on multi-mass systems is under way.

It has been demonstrated that our models embark on the path of a self-similar solution in which dispersion and rotational velocity follow the same power-laws, in accord with recent suggestions by Lynden-Bell (2001).

Finally we have demonstrated that in pre-collapse rotation accelerates the evolution both in isolated and tidally limited systems. This is interpreted as the effect of mass loss by diffusion and tidal boundary, respectively. In post-collapse however, the amount of rotation is small enough that only clusters in a tidal field keep their accelerated evolution, which is due to the different structure of the system as a function of initial rotation at the half-mass radius and further outside. Isolated star cluster in post-collapse evolve very similar to non-rotating ones.

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