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THE SOLAR NEBULA PRESSURE GRADIENT AND ITS EFFECT ON PLANETESIMAL MOTIONS

S. J. WEIDENSCHILLING

In a centrally condensed solar nebula, the gas is partially supported by a pressure gradient, and rotates at less than the Keplerian velocity. Solid bodies lack this support, and spiral inward due to drag. The radial velocities developed can be significant, even in low-mass nebular models. Possible effects include fractionation of particles by size or density, rapid accumulation of planetesimals, and production of regions of anomalous (non-solar) composition in the nebula.

In most models of the solar nebula, the pressure and density decrease in the direction away from the axis. The nebular gas is partially supported by this pressure gradient, and therefore rotates more slowly than the Kepler orbit velocity. Solid bodies in the nebula, from dust grains to protoplanets, are not supported by the pressure gradient. In the absence of gas, they would pursue Kepler orbits. They move with respect to the gas, and their motions are affected by drag in ways which depend on their sizes and the nebular structure. Whipple (1972, 1973) examined the limiting cases of very small and large bodies, for two particular drag laws. Cameron (1973) applied his results to the nebular models of Cameron and Pine (1973). The results presented here cover the full range of particle sizes and drag laws for more generalized nebular models.

Using Whipple's notation, P = gas pressure, T = temperature, ρ = density, μ = molecular weight, $\bar{v}$ = mean termal velocity, λ = mean free path, $\eta = \rho\bar{v}\lambda/2$ = viscosity, s = radius of particles (assumed spherical), ρ_s = particle density, and r the distance from the nebular axis. The central gravity is

$$g = V_k^2/r \qquad (1)$$

where V_k is the velocity of a circular Kepler orbit. In a low-mass nebula, $g = GM_\odot/r^2$, where G = gravitational constant, and $M_\odot$ is the solar mass. In a frame rotating with the gas, the residual gravity is

$$\Delta g = \rho^{-1}\, dP/dr \qquad (2)$$

for the gas to be in hydrostatic equilibrium ($\Delta g < 0$ for a centrally condensed nebula). The rotational velocity of the gas, V_g, is given by

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$$\frac{V_g^2}{r} = \frac{V_k^2}{r} + \Delta g \tag{3}$$

from which, when $\Delta g/g << 1$,

$$\Delta V = V_k - V_g \simeq \left(\frac{-\Delta g}{2g}\right) V_k. \tag{4}$$

Suppose P can be described by a power law: $P = P_o \cdot (r/r_o)^{-n}$. For an ideal gas, $P = \rho RT/\mu$. Then

$$\Delta g = \rho^{-1}\, dP/dr = -nRT/\mu r, \tag{5}$$

which does not depend on the magnitude of P or ρ. Hence, ΔV is independent of the nebular mass, if the nebula's self-gravitation is small (M(nebula) << $M_\odot$), but depends on the exponent n and the nebular temperature structure.

When a particle moves through gas with velocity v, the drag force is

$$F_D = C_D\, \pi s^2\, \rho v^2/2 \tag{6}$$

where C_D, the dimensionless drag coefficient, depends on another dimensionless parameter, the Reynolds number, $Re = 2\rho vs/\eta$. From experiments, it is known that

$$C_D \simeq 24\ Re^{-1} \text{ for } Re < 1 \tag{7a}$$

$$C_D \simeq 24\ Re^{-.6} \text{ for } 1 < Re < 800 \tag{7b}$$

$$C_D \simeq 0.44 \text{ for } Re > 800 \tag{7c}$$

where (7a) is the Stokes drag law. When $\lambda > s$, the Epstein drag law applies:

$$F_D \simeq 4\pi\rho s^2\, v\bar{v}/3. \tag{8}$$

Whipple defines the "stopping time," t_e as

$$t_e = mv/F_D. \tag{9}$$

The type of motion depends on the ratio of t_e to the orbital period, t_p. A "small" particle, with $t_e/t_p << 1$, is carried with the gas ($v << \Delta V$). It feels the residual gravity, Δg, and, in a frame rotating with the gas, falls radially at a terminal velocity

$$dr/dt = t_e \Delta g. \tag{10}$$

A "large" body, with $t_e/t_p >> 1$, moves in a Kepler orbit, experiencing a transverse "wind" of velocity ΔV. The drag causes the orbit to decay at a rate

$$dr/dt = r\Delta g/t_e g. \tag{11}$$

From (10) and (11), it can be seen that $dr/dt \to 0$ in the limits of large and small s. The radial velocity is greatest when $t_e \simeq t_p/2\pi$. In the strongly-perturbed case, the equations of motion must be solved numerically. However, it can be shown analytically (Weidenschilling 1976, in preparation) that $|dr/dt|_{max} = |\Delta V|$. This limit is independent of the drag law or particle properties, and depends only on the nebular structure. Figure 1 shows dr/dt schematically as a function of s; the shape of the curve is determined by the drag laws.

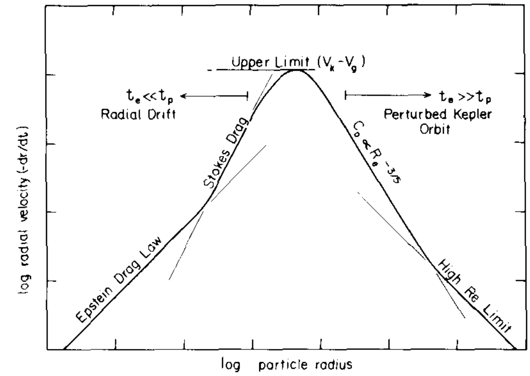

Figure 1. Radial velocity vs particle size (schematic). The shape of the curve is determined by the drag laws, but the peak value depends only on the nebular structure.

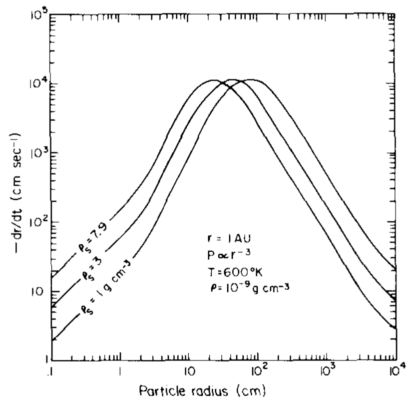

Figure 2. Radial velocity vs size in the model nebula at r = 1 AU, for different particle densities.

Consider a model nebula with $P \propto r^{-3}$ and $T \propto r^{-1}$, similar to that suggested by Lewis (1974). I take T = 600° K, $\rho = 10^{-9}$ g cm^{-3} in the central plane at 1 AU. This is a "low-mass" nebula ($\simeq 0.05\ M_\odot$). Figure 2 shows dr/dt for particles of different densities. Radial velocites exceed 10^4 cm sec^{-1} for s ~ 10-100 cm. Characteristic lifetimes (r/(dr/dt)) in this size range are very short, < 100 yr. Note that for "small" particles, larger and/or denser ones have larger radial velocities, while for "large" particles, the opposite is true. Fractionation by size or density can occur in either direction, depending on the sizes of the particles.

Figure 3 shows dr/dt for particles of various sizes as a function of r. Bodies of the appropriate sizes may have large radial velocites, even in a low-mass nebula. They can be transported long distances in times shorter than the nebular lifetime. Meteorites often contain inclusions of anomalous chemical or isotopic composition; they may have been brought from different heliocentric distances in larger bodies, broken up in collisions, and accreted by the meteorite parent bodies.

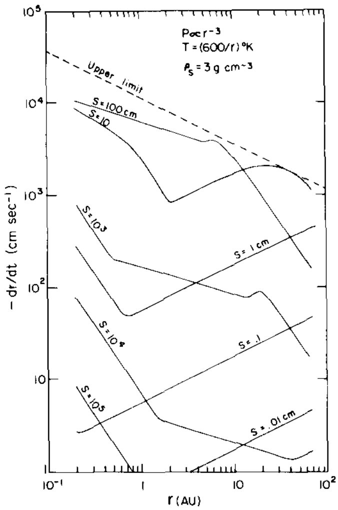

Figure 3. Radial velocity vs r in the model nebula, for different particle sizes.

It is often assumed that gas drag would quickly damp relative velocities of solid bodies in a non-turbulent nebula. However, since dr/dt depends strongly on s, bodies of different sizes maintain appreciable relative velocities of a magnitude to promote accretion. Most collisions would be between particles of very unequal sizes, in which the smaller could become embedded in the larger. Bodies of nearly equal size would automatically have low relative velocities, preventing disruptive collisions.

Besides mixing solid condensates, the pressure gradient might produce another type of chemical anomaly. Suppose that large, icy bodies formed in the outer part of the nebula. They would spiral inward until they passed the region of ice stability, and evaporate. For "large" bodies, $dr/dt \propto s^{-1}$, so the volatiles would be released in a narrow zone, whatever their sizes. Bodies composed of H_2O ice would cause local enhancement of O; NH_3 and CH_4 hydrates could increase N and C abundances also. The chemical effects would be equivalent to depletion of hydrogen. Analogous effects may have been produced by other components in different parts of the nebula.

REFERENCES

Cameron, A.G.W. 1973, *Icarus, 18,* 407.
Cameron, A.G.W., and Pine, M. R. 1973, *Icarus, 18,* 377.
Lewis, J. S. 1974, *Science, 186,* 440.
Whipple, F. L. 1972, in *From Plasma to Planet,* A. Elvius, ed. (Wiley, New York).
Whipple, R. L. 1973, in *Evolutionary and Physical Properties of Meteoroids,* NASA SP-319.