Cosmic Strings and Galaxy Formation

Neil Turok

Department of Theoretical Physics
Imperial College
London SW7 2BZ, England

The hot big bang theory of the early universe is rather well established. Among its successful predictions are the Hubble expansion, the microwave background radiation and the abundances of the light elements. It also fits in rather nicely with ideas from particle physics. According to these ideas (which are firmly based on experiment) at high energies particle interactions become more symmetrical and the apparently complicated particle spectrum today becomes very simple. It is an appealing notion that such a state of high symmetry was actually realised in the very early universe at very high temperatures, and the symmetry was broken as the universe expanded and cooled[1].

However we know that the hot big bang theory is incomplete without a source of perturbations. We know from the observed isotropy of the microwave background the the universe was very isotropic and (unless we are very special) homogeneous early on, but obviously some perturbations were essential to produce the structure we see today.

The recent observations have underlined this fairly dramatically. For example there appear to be giant "filaments" i.e. roughly linear overdense regions in the distribution of galaxies about 100 h^{-1} Megaparsecs long and 5 h^{-1}Mpc across[2], large "voids" i.e. regions nearly empty of bright galaxies 60 h^{-1} Mpc in diameter[3] and in more complete deep surveys most galaxies appear to lie on the surfaces of "bubbles" 20-30 h^{-1} Mpc across[4]. For comparison the Hubble radius H_0^{-1}(the length scale characterising the expansion rate of the universe) is 3000 h^{-1} Mpc. The most dense clusters of galaxies, called Abell clusters, are defined to be regions smaller than 1.5 h^{-1} Mpc in radius

J. Audouze et al. (eds.), Large Scale Structures of the Universe, 281–288.

containing more than 50 bright galaxies. For comparison the mean separation of bright galaxies (i.e. the inverse of the cube root of the number density) is 5 h^{-1} Mpc. Observations[5] indicate that these are significantly clustered on scales of at least 50 h^{-1} Mpc. Their mean separation is about 55 h^{-1} Mpc. Here h is of course Hubbles constant in units of 100 km s^{-1}.

What makes these observations interesting is that it does not seem possible to have formed such large scale structure by moving galaxies around since the big bang. Peculiar velocities (velocities relative to the Hubble flow) grow as $t^{1/3}$ in an expanding universe under gravity. In fact in the linear regime there is a precise relation that the peculiar displacement $\delta r = H_0^{-1} \delta v$ where δv is the peculiar velocity. Now galaxies today only rarely have velocities greater than 600 km $s^{-1}=2.10^{-3}c$ relative to the observed structures and H_0^{-1} is 3000 h^{-1} Mpc so we have quite a strong upper bound on the distances they have moved of only 6 h^{-1}Mpc ! Thus galaxies have not moved very far since the big bang and not nearly far enough to produce the large scale structure we see. Of course explosions could also have moved the matter around but it is difficult to move it further than about 10 h^{-1} Mpc with these unless one invokes exotic high energy phenomena like superconducting strings. Thus there are good reasons to believe that in the large scale structure we are looking very directly at the primordial density perturbations!

There is and will always be the problem that statistics are poor for the very largest scale surveys but they are certainly improving quickly and there is every hope for very good statistics on structures up to 100 h^{-1} Mpc or so in the next few years.

Perhaps the most direct evidence on the primordial perturbations would come from observations of anisotropy in the microwave background radiation - the present observational sensitivities are within an order of a few of the levels predicted in all current theories and have already ruled out many theories. Needless to say a picture of the primordial density perturbations would give a unique window on fundamental physics and the very early universe.

Cosmic strings are one idea as to the origin of the primordial density perturbations. The basic idea of the cosmic string theory is very simple. We know that the universe at very early times was nearly homogeneous and isotropic, but also very hot. If our ideas of unification are correct, then symmetry breaking processes occurred as well. Now in a certain class of unified theories when this symmetry breaking occurs topologically stable line defects form called strings or vortex lines[6]. They have a direct analogue in the flux lines formed in superconductors when the U(1) symmetry of electromagnetism is broken by the Cooper pair condensate.

The condition for such strings is that the vacuum manifold (the states of least energy in the theory) possess noncontractible loops. The vacuum automatically has a lot of degeneracy in unified theories because it must be invariant under the full symmetry group of the theory, and is in fact equal to the coset space G/H where G is the original symmetry group and H is the subgroup it is broken to. The occurrence of noncontractible loops is a purely group theoretic question and has been answered affirmatively in a wide range of simple theories including those based on superstring theories[7]. Unlike magnetic monopoles however, strings are not forced on you by unification but are simply an option. They are generic enough however for us to take seriously the possibility of their formation at some stage in the early universe.

A nice feature when comparing strings to quantum fluctuations during inflation as a source of density perturbations is that no fine tuning of coupling constants is needed to obtain strings with the right mass per unit length to form galaxies - the grand unification scale emerges naturally. By contrast theories based on inflation generally require extra "singlet" fields added by hand to the GUT theory with very tiny self-couplings to work at all. You can of course have inflation first and then form strings but so far the models constructed to do this are even more contrived than those for inflation.

The potential strings have as density perturbations is easily seen as follows. In a radiation or matter dominated universe the total density $\rho \sim 1/Gt^2$ where G is Newtons constant. If the network of strings evolves in such a way that there is a fixed number of strings of mass μt crossing each horizon volume t^3 where μ is the mass per unit length of the string then the string density $\rho_s \sim \mu/t^2$ and the fractional density perturbation $\rho_s/\rho \sim G\mu$ = constant. In a gauge theory $\mu = 2\pi\upsilon^2 f(\lambda/e^2)$ where υ is the value of the symmetry breaking Higgs field in the vacuum and f is a dimensionless function of order unity . λ is the self coupling of the Higgs field and e is the gauge coupling constant[8].

Without any fine tuning of parameters υ is of the order of m_{gut}, typically about 10^{16} GeV in GUT theories predicting strings , and $\mu \approx m_{gut}^2$. Since we do not know the

https://doi.org/10.1017/S0074180900136150 Published online by Cambridge University Press

theory let alone the couplings we shall treat μ as a free parameter. In fact since the simplest strings do not couple strongly to anything except via gravity μ only enters as $G\mu \sim m_{gut}^2/m_{planck}^2$ which we shall parametrise as $G\mu = 10^{-6}\mu_6$. It is also useful to write Newtons constant as a line density : then $\mu = 2.1\ 10^7\ \mu_6 M_\odot\ \mathrm{parsec}^{-1}$

String formation may be understood heuristically as follows[6]. At a temperature $T_c \sim m_{gut}$ the Higgs field Φ begins to notice the potential and tends to fall towards its mimima. At this stage Φ fluctuates on a scale equal to the correlation length $\sim T_c^{-1}$. We may imagine the universe as broken up into domains of roughly this size where the direction θ in which Φ points on the vacuum manifold is chosen at random in each domain but matches on smoothly at the boundaries. Now as the system cools θ will vary from domain to domain, causing defects to form on the edges common to certain domains. For if θ varies by 2π as we encircle such an edge then Φ must vanish on that edge. Where it does so $V(\Phi)$ is nonzero and a thin tube of vacuum energy is stored there. In fact these lines where Φ vanishes cannot have any ends. So the strings are either in the form of closed loops or infinitely long.

String formation was originally understood numerically by simply throwing down phases θ at random on a lattice of domains, with a prescription for smoothly varying the phases from one domain to the next[9]. Most of the string is in one string as large as the box in which the simulation is performed. The remainder is in the form of a scale invariant distribution of loops. Recently we have understood how these results may be understood analytically by counting states in the quantised closed bosonic string, an intriguing connection[10].

After the strings form we have to evolve them. First the strings are damped by collisions with particles until the temperature falls to $\sim (G\mu)^{1/2} m_{gut}$. In this stage, and later on, the typical curvature scale of the string increases rapidly while the width remains constant.

Quite quickly it becomes a very good approximation to treat the strings as infinitely thin relativistic lines or "Nambu-Goto" strings the action for which is simply the area of the two dimensional worldsheet they trace out in spacetime. Note however that the Nambu action is only valid if there is no structure along the string. This is not true for superconducting strings[11] where the current-carrying fields do vary along the string.

In this case there is a significant local modification to the action and in fact the "positive pressure" contributed by the current can cancel the string tension entirely, leading to strings behaving more like shoelaces than relativistic string[12]. The nicest thing about the Nambu-Goto Action is that it is completely geometrical - the parameter μ does not enter in the equations of motion which depend solely on the background spacetime metric. The characteristic velocity of the string is simply the speed of light. In a given universe (and we know that to a very good approximation our universe was flat FRW radiation dominated i.e. $a \propto t^{1/2}$ at early times) the string evolution has no free parameters at all.

The Nambu action breaks down where two strings collide. In this case on has to solve the full nonlinear field equations. This was done by Shellard and others[13] who found that when two strings collide they reconnect the other way for centre of mass velocities $\lesssim .95$ i.e. essentially always. This a very nice result because the string interactions are also fixed and cannot be adjusted. Again for strings with more complex internal structure like superconducting strings this may not be the case.

How does a string network evolve? The result of the numerical simulations is that a network of strings in an expanding universe formed according to the above prescription rather quickly i.e. in a few expansion times, approaches a "scaling solution". In the scaling solution there is only one length scale, the Hubble radius, which grows as t. The distribution of strings can be separated into two components. Strings longer than the Hubble radius have a curvature scale of the order of t and several such strings cross each horizon volume. Unless these strings chop off a constant fraction of their length each expansion time they quickly come to dominate the total energy, since their energy remains roughly constant while the energy in radiation decreases as the inverse of the scale factor. They do apparently manage to do this in the simulations, and this is now supported by analytic calculations for strings in flat spacetime which show that there is a lot more phase space available to small loops than long strings and thus a strong imbalance favouring the chopping off of loops over their reconnecting onto long strings[10]. Analytic approaches to string networks in expanding universes have been developed by Kibble and Bennett[15].

The production of loops by the string network is a very important feature. Since these loops only decay very slowly into gravitational radiation, and their energy remain roughly constant until they do, their density scales as matter so the smallest loops actually dominate the energy density in string.

string. Our simulations show that typically when a loop is chopped off a long string it self intersects several times, breaking up into several smaller loops but then the process terminates. In other words a large fraction of the phase spac available to a chopped off loop consists of non-self-intersecting trajectories[14].

Loop production may be described in more detail as follows. If $n(r,t)dr$ is the number density of loops of radius r to r+dr at time t then in the scaling solution n obeys

$$\partial n/\partial t = -3\,\dot{a}/a\; n + f(r/t)/t^5 \qquad (2)$$

where the scale factor $a \propto t^{1/2}$ in the radiation era and $f(X)$ is a dimensionless function. We cut $f(X)$ off by definition a $X=X_c \gtrsim 1$ and if loop self-intersection ceases soon after loops are produced then f cuts off for $X \ll 1$ also. If any intersection happens it has to happen rapidly - the loops motion is periodic with period one half of its length in the centre of mass frame, and the length is some number β times r with $r \sim t$ for loops produced at time t. So any intersection must be completed in an expansion time or so. (2) yields

$$n(r,t) = \nu\, r^{-5/2}\, t^{-3/2} \quad ; \quad \nu = \int_0^{X_c} f(X)\, X^{3/2} dX \qquad (3)$$

According to numerical simulations[14] $\beta \sim 10$ and $\nu \sim .01$ and both are uncertain by a factor of 2-3.

A loop produced with radius r_0 has a mass $\beta\mu r_0$ and loses energy to gravity waves[16] at a rate $\dot{E} = -\Gamma G\mu^2$ with $\Gamma \sim 50$. Thus the radius at a later time is given by $r_0 - \gamma G\mu t$ with $\gamma = \Gamma/\beta \sim 5$ and we find for the final loop distribution

$$n(r,t) = \nu\,(r + \gamma G\mu t)^{-5/2}\, t^{-3/2} \qquad (4)$$

In the cosmic string theory we may identify loops of a given mean separation with objects of the same mean separation (in comoving coordinates) today[17]. Remarkably, simulations of string evolution show that loops are produced with a correlation function which closely matches that observed for Abell clusters, with no adjustable parameters[17]. However to calculate the required value of $G\mu$ one needs to know exactly which sized loops gave rise to galaxies, clusters, etc. This part of the calculation also depends on the type of dark matter one assumes present. Now loops with radius greater than r have a number density $n_>(r,t) = \int_r dr\, n(r,t) \equiv d^{-3}$. Following (4) through to the present and ignoring loop decay we find for bright galaxies $d = 5\ h^{-1}$Mpc gives $r = 4\ h^{-2}$ pc whereas for clusters $d = 55\ h^{-1}$Mpc and $r = 0.5\ h^{-2}$ kpc. This is just smaller than the Hubble radius at t_{eq}, so cluster loops were produced just before t_{eq}[17].

https://doi.org/10.1017/S0074180900136150 Published online by Cambridge University Press

Now in order to accrete an object of mass M with an overdensity (ρ/ρ_b) by today, with cold dark matter one requires a seed mass

$$m_s = M\ (\rho/\rho_b)^{1/3}\xi/(5(1+Z_{eq})) \quad (5)$$

where ξ equals 1 for a seed mass laid down long before t_{eq} and represents the loss in growth for a seed mass laid down later on[17]. For example $\xi \approx 4$ if accretion begins at t_{eq}.

Cluster loops have masses $m_c = \beta\mu r_c = 10^{11}h^{-2}\mu_6 M_0$. However clusters have masses $5\ 10^{14}\ \sigma^2\ h^{-1}M_0$ and overdensities of $130\sigma^2$ in an Abell radius where σ is their velocity dispersion in units of 700 km s^{-1}, so from (5) they required a seed mass of $10^{11}h^{-3}\ \sigma^{8/3}M_0$. Thus we require $\mu_6 \approx h^{-1}\ \sigma^{8/3}$, just about the value predicted in GUTS. The total uncertainty in μ_6 is probably about an order of magnitude given our still fairly crude numerical simulations and the uncertainty in σ and $d_{cluster}$. For galaxies we find just by scaling that the total mass of comparable overdensity $M_g = 4\ (d_g/d_c)^2\ M_c$ $= 10^{13}\ M_0 h^{-1}\ \sigma^2$ ($\xi \approx 1$ for galaxy loops) and a rotation velocity $v = \sqrt{3}\ \sigma_g = \sqrt{3}\ 4^{1/3}\sigma \approx 400$ km s^{-1}. This is on the large side but is improved in the neutrino scenario.

Brandenberger will describe in his talk how the scenario changes if the dark matter is hot[18]. Suppression of growth on small scales leads to $M_g \approx 1.5\ 10^{12}\ M_0 h^5\ \sigma^8$. If $h \approx .5$, as is required from the age of the universe then we require a large value of σ i.e. cluster velocity dispersions of ≈ 1000 km s^{-1} for galaxies to be as massive as observed. This requires a larger value of $\mu_6 \approx 4$. This leads to larger observed peculiar velocities[19]. In fact the neutrino scenario looks from many points of view the more attractive. Notice that strings cure the main problem of the conventional neutrino models where free streaming erases strucure on small scales. The string loops survive free streaming and are able to accrete galaxies, albeit less efficiently than with CDM.

I have dealt in some detail in this lecture with the normalisation of the cosmic string theory, as this of considerable importance to people now beginning to look for more direct evidence. I hope I have brought out the many uncertainties and their sources[20]. Nevertheless the most hopeful feature of the scenario is that if strings exist they should be detectable fairly soon. Recently Cowie and Hu have found a candidate object for lensing by a string loop[21], and several groups are considering the problem of detecting strings through their effect on the microwave background[22].

REFERENCES

1. D.A. Kirzhnits and A.D. Linde, Ann. Phys.N.Y. 101 (1975) 195.
2. R. Giovanelli and M.P. Haynes, Astron. J. 87 (1982)1355.
3. R.F. Kirschner, A. Oemler, P.L. Schecter and S.A. Schectman in "Early Evolution of the Universe and its Present Structure", I.A.U. Symposium 104 (1983) 197.

4. V. de Lapparent, M. Geller and J. Huchra, Ap. J. Lett. 302 (1986) 11.
5. N.A. Bahcall and R.M. Soneira, Ap. J. 270 (1983) 20.
6.T.W.B. Kibble, J.Phys. A9 (1976) 1387; Phys. Rep. 67 (1980) 183: Ya. B. Zel'dovich, MNRAS 192 (1980) 663: A. Vilenkin, Phys. Rev. Lett. 46 (1981) 1169, 1496 (E); Phys. Rep. 121 (1985) 263.
7. T.W.B. Kibble, G. Lazarides and Q. Shafi, Phys. Lett. 113B (1982) 237; D. Olive and N. Turok, Phys. Lett. 117B (1982) 193, E. Witten, Phys. Lett. 149B (1984) 351.
8. e.g. H. Nielsen and P. Olesen, Nuc. Phys. B61 (1973) 45; L. Jacobs and C. Rebbi, Phys. Rev. B19 (1979) 4486.
9. T. Vachaspati and A. Vilenkin, Phys. Rev. D30 (1984) 2036, A. Albrecht and N. Turok, ref.14, J. Frieman and R. Scherrer, Phys. Rev. D33 (1986) 3556.
10. D. Mitchell and N. Turok, Phys. Rev. Lett, 58 (1987) 1801, Nuc. Phys. B to appear.
11. E. Witten, Nuc. Phys. B249 (1985) 557.
12. E. Copeland, M. Hindmarsh and N. Turok, Phys. Rev. Lett. 58 (1987) 1910, and to appear.
13. P. Shellard, Nuc. Phys. B283 (1987) 624.
14. A. Albrecht and N. Turok, Phys. Rev. Lett. 54 (1985) 1868, and in preparation; D. Bennett and F. Boucher, in preparation.
15.T.W.B. Kibble, Nuc. Phys. B252 (1985) 227; D. Bennett, Phys. Rev. D33(1986)872, D34(1986),3592,1235(E),3932(E).
16. N. Turok, Nuc. Phys. B242 (1984) 520; T. Vachaspati and A. Vilenkin, Phys. Rev. D31 (1985) 3052; C. Burden, Phys. Lett. 164B (1985) 277.
17. N. Turok, Phys. Lett. B126 (1983) 437; Phys. Rev. Lett. 55 (1985) 1801; N. Turok and R. Brandenberger, Phys. Rev. D33 (1986) 2175; H. Sato, Mod. Phys. Lett. A1 (1986) 9; A. Stebbins, Ap. J. Lett. 303 (1986) L21; for a recent review see N. Turok, Lectures at Erice Winter Scool 1987, Imperial College preprint 1987.
18. R. Brandenberger, N. Kaiser, and N.Turok, DAMTP preprint 1987 and R. Brandenberger, N. Kaiser, D. Schramm and N. Turok, DAMTP preprint 1987; E. Birtschinger, MIT preprint, 1987.
19. A. Dressler, S. Faber, D. Burstein, R. Davies, D. Lynden-Bell, R. Turlevich and G. Wegner, 1986, preprint. P. Shellard, R. Brandenberger, N. Kaiser and N. Turok, Nature, 1987.
20. P.J.E. Peebles, preprints, 1986, 1987 ; N. Turok, in "Nearly Normal Galaxies: from the Planck Time to the present", proceedings of the Santa Cruz Summer Workshop, Ed. S. Faber, Springer-Verlag, 1987.
21. L. Cowie and D. Hu, Ap. J. Lett, in press
22. N. Kaiser and A. Stebbins, Nature 310 (1984) 391, A. Stebbins, Fermilab preprint 1987.