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On the Determination of Velocities of Radar Meteors

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Abstract. The results of testing the classical method of velocity determination based on the positions of the extrema of Fresnel diffraction patterns (FDP) in radio echo amplitude, is presented. This technique is compared to the method of nonlinear least squares fitting (NLSF) of the theoretical patterns to the observed those. It is shown that while the classical method can fail in yielding correct values for the velocity of radar meteors, the NLSF method always yields good results.

1. Introduction

One method commonly used to determine the velocity of radar meteors is based on the comparison of the positions of the extrema of the theoretical FDP with those of the observed pattern. The classical FDP method is briefly described in the next section. To the author's knowledge, nobody has yet tried to check this technique's ability to correctly determine the velocity of radar meteors . To test the classical method, we decided to generate 100 theoretical FDP with known starting parameters (see the description of the classical method). These were chosen randomly from within the following intervals: the velocity from 10 to 70 km s^{-1}, the range from 100 to 300 km, the normalizing factor from 20 to 60, the time of passage through the specular point from 0.01 s to 0.02 s, and the ambipolar diffusion coefficient from 0 to 10 $m^2 s^{-1}$. For purposes of illustration the wavelength was set to 11.45 m and the pulse repetition frequency to 379 Hz. Theoretical amplitudes were stored with an accuracy of 3 digits. The method of velocity determination relying on the positions of extrema of FDP, was then applied to generated FDP. To compare to these results, the method of NLSF (e.g. Pecina 1988) of theoretical amplitudes to those generated was also applied. The results given by both methods are summarized in Table 1.

2. Brief description of classical method

The FDP including diffusion are described by the formula (e.g. Šimek 1968)

$$A_i = c_n \sqrt{S_i^2 + C_i^2}, \tag{1}$$

where A_i represents the i-th amplitude oscillation of the FDP, c_n the corresponding normalizing factor, and

$$S_i = \int_{-\infty}^{x_i} \exp[-a(x_i - x)] \sin(\pi x^2/2)\, dx, \quad C_i = \int_{-\infty}^{x_i} \exp[-a(x_i - x)] \cos(\pi x^2/2)\, dx, \tag{2}$$

where $x_i = 2v(t_i - t_0)/\sqrt{\lambda R_0}$ and $a = 8\pi^2 D\sqrt{\lambda R_0}/v\lambda^2$. In the preceding expressions, v designates meteoroid velocity, D stands for the ambipolar diffusion coefficient, R_0 represents the range to the meteor trail at the specular point, and λ is the wavelength of wave transmitted by the radar. The time variable corresponding to A_i is designated as t_i, and the time of meteoroid passage through the specular point as t_0. Both S_i and C_i can be evaluated analytically.

As already stated, the classical method utilizing the FDP for velocity determination compares theoretical extrema positions with those observed. Equating the derivative with respect to x_i of eq.(1) to zero we arrive at the equation

$$S_{ei}\sin(\pi x_{ei}^2/2) + C_{ei}\cos(\pi x_{ei}^2/2) - a\,(S_{ei}^2 + C_{ei}^2) = 0, \tag{3}$$

containing the extrema position variable, x_{ei}, and also a as unknowns. The following iteration process was shown to work. First, we assume $a = 0$. We can then compute x_{ei} from eq.(3). Second, from the ratio of two known amplitudes in the extrema, we get equation

$$A_{ei}\sqrt{S_{ej}^2 + C_{ej}^2} = A_{ej}\sqrt{S_{ei}^2 + C_{ei}^2}, \tag{4}$$

and a new value of a is obtained. Returning back to eq.(3) we get more precise values of x_{ei}. The iteration process can be terminated if two subsequent values of the parameters in question differ by less than some prescribed accuracy. This process converges rapidly, generally only a few iterations are needed in most cases. For our computations we used the ratio of the first maximum to the second minimum. Then the velocity of the meteoroid can be evaluated from the formula

$$v = (\sqrt{\lambda R_0}/2)(x_{ej} - x_{ei})/(t_{ej} - t_{ei}). \tag{5}$$

We used the positions of the first two maxima and minima because these are usually well defined in the observational record. Thus, we have six possible combinations to use in deriving v from eq.(5), as indicated in Table 1. Here $\overline{v}$ was computed from these six quantities while v_{ls} was computed by the NLSF method for which special attention was paid to finding the initial values of parameters entering iteration process.

Table 1. Input velocities, v_i, of generated FDP, velocities resulting from i-th and j-th extreme, v_{i-j}, their average, $\overline{v}$, and the velocity, v_{ls}, resulting from the application of the NLSF method to generated FDP.

No.	v_i	v_{1-2}	v_{1-3}	v_{1-4}	v_{2-3}	v_{2-4}	v_{3-4}	$\overline{v}$	v_{ls}
1	13.421	13.220	13.243	13.278	13.291	13.337	13.377	13.291	13.421
2	35.791	39.711	36.756	37.456	33.063	35.846	39.556	37.065	35.791
3	66.779	90.349	77.087	69.496	63.824	59.069	54.315	69.023	66.784
4	24.101	25.584	25.541	24.057	25.465	22.869	20.791	24.051	24.103
5	29.971	30.454	30.904	29.906	31.805	29.420	27.512	30.000	29.976
6	54.846	58.911	53.746	57.173	46.859	55.782	69.167	56.940	54.840
7	13.967	13.931	13.872	13.963	13.744	13.996	14.225	13.955	13.968
8	46.901	55.401	44.597	45.698	33.794	39.877	49.002	44.728	46.910
9	65.829	54.966	68.180	30.920	107.821	22.905	12.290	49.514	65.832
10	34.325	35.274	35.825	34.662	36.653	34.203	31.754	34.728	34.324
11	65.654	83.584	71.891	64.701	60.198	55.259	50.320	64.325	65.651
12	30.295	30.709	29.299	29.502	26.762	28.415	30.068	29.126	30.296
13	61.909	59.945	74.429	67.496	117.880	75.048	53.631	74.738	61.913
14	38.907	35.952	24.703	36.576	33.036	37.075	43.132	36.746	38.902
15	15.533	15.438	15.613	15.310	16.029	15.175	14.492	15.343	15.533
16	47.622	47.154	47.588	47.266	48.311	47.359	46.408	47.348	47.620
17	60.706	70.801	69.738	59.375	68.142	52.520	42.104	60.447	60.709
18	22.679	23.814	22.995	23.203	21.521	22.653	23.786	22.995	22.680
19	15.919	15.024	15.393	15.398	16.192	15.771	15.410	15.531	15.919
20	16.217	15.770	15.943	16.566	16.238	17.301	18.542	16.727	16.219
21	56.143	70.498	57.240	58.582	43.982	51.432	62.607	57.390	56.145
22	56.677	48.901	47.391	26.967	45.127	19.655	12.377	33.403	56.687
23	13.405	13.471	13.430	13.566	13.355	13.651	13.948	13.570	13.408
24	63.962	60.967	57.181	60.586	52.132	60.282	72.507	60.609	63.963
25	18.948	17.732	18.635	18.197	20.079	18.569	17.059	18.378	18.944

The first continuation of Table 1.

No.	v_i	v_{1-2}	v_{1-3}	v_{1-4}	v_{2-3}	v_{2-4}	v_{3-4}	$\overline{v}$	v_{ls}
26	68.147	83.988	70.417	38.224	56.845	26.873	16.762	48.836	68.140
27	47.989	37.075	41.530	42.253	50.442	47.431	44.420	43.858	47.986
28	42.530	47.972	45.494	43.300	42.189	40.185	38.181	42.887	42.532
29	40.025	38.495	37.452	39.410	36.063	40.143	46.262	39.638	40.019
30	49.701	43.541	44.079	48.137	44.975	52.734	64.372	49.640	49.704
31	49.073	41.203	48.461	48.287	70.234	56.787	47.822	52.132	49.074
32	46.534	36.871	41.116	41.868	49.606	46.864	44.123	43.408	46.533
33	52.455	41.504	47.210	47.882	58.620	54.260	49.900	49.896	52.454
34	55.311	51.204	57.996	55.566	74.977	59.929	49.897	58.262	55.315
35	46.356	39.665	44.171	44.990	53.184	50.316	47.447	46.629	46.353
36	53.267	43.784	47.149	48.291	53.879	52.799	51.718	49.603	53.271
37	37.017	37.417	38.723	38.307	40.899	39.049	37.198	38.599	37.018
38	61.077	63.579	57.548	61.270	49.507	59.423	74.297	60.937	61.073
39	15.973	15.838	15.892	16.140	15.990	16.420	16.850	16.188	15.974
40	57.740	58.095	55.781	42.372	52.310	34.511	25.611	44.780	57.730
41	54.060	50.327	49.561	54.300	48.283	58.273	73.259	55.667	54.058
42	64.960	63.281	60.852	46.215	57.207	37.682	27.920	48.860	64.954
43	17.610	17.285	17.575	17.794	18.107	18.262	18.417	17.907	17.609
44	26.463	27.630	28.127	26.952	29.021	26.398	24.211	27.057	26.462
45	13.534	13.429	13.539	13.705	13.714	13.938	14.186	13.752	13.533
46	27.792	29.154	27.538	29.014	25.115	28.893	33.931	28.941	27.791
47	57.207	56.498	60.252	54.925	67.761	53.667	44.270	56.229	57.201
48	26.175	26.425	25.791	25.933	24.680	25.502	26.325	25.776	26.172
49	29.164	25.751	28.168	27.855	33.607	29.958	27.039	28.730	29.171
50	69.730	72.478	62.198	24.002	51.918	15.923	8.723	39.207	69.743
51	27.351	27.038	28.104	27.794	30.502	28.550	26.988	28.163	27.351
52	54.013	48.521	62.599	56.338	104.835	64.156	43.816	63.377	54.025
53	23.940	25.793	24.273	23.403	22.145	21.730	21.314	23.110	23.942
54	33.662	31.563	32.597	33.517	34.665	35.471	36.277	34.015	33.658
55	60.995	65.333	66.003	63.928	67.009	62.874	58.739	63.981	60.992
56	24.361	24.762	24.164	24.406	23.089	24.087	25.084	24.265	24.365
57	65.621	63.235	61.902	46.895	59.904	38.725	28.135	49.799	65.619
58	46.720	46.165	50.251	51.386	58.424	56.607	54.791	52.938	46.714
59	67.386	78.648	75.403	73.655	70.535	69.911	69.287	72.907	67.388
60	60.893	65.245	64.995	63.118	64.619	61.523	58.428	62.988	60.890
61	51.344	45.209	48.518	49.717	55.136	54.225	53.313	51.020	51.343
62	67.867	74.921	61.830	48.086	48.738	37.352	29.761	50.115	67.881
63	39.419	36.202	37.819	38.862	41.052	41.523	41.994	39.575	39.422
64	46.785	44.204	45.344	43.043	47.624	42.049	37.868	43.355	46.785
65	53.728	51.270	49.062	51.798	46.118	52.220	61.373	51.974	53.726
66	45.004	37.728	41.420	42.296	48.806	46.865	44.924	43.673	45.000
67	48.864	51.153	51.663	50.041	52.428	49.207	45.987	50.080	48.864
68	67.376	59.431	75.570	68.220	123.989	77.009	53.519	76.290	67.384
69	43.068	42.070	40.205	40.707	37.874	39.733	42.212	40.467	43.069
70	28.820	30.639	29.475	29.726	27.380	28.904	30.429	29.426	28.818

3. Discussion and conclusions

A look at Table 1 clearly shows that the classical FDP method relying on the extrema positions works satisfactorily only for low velocity meteors while for medium and particularly high velocity meteors the agreement of v_i with $\overline{v}$ is very poor. We can see that values of $\overline{v}$ can be both lower and higher than v_i (e.g. cases numbers 2, 3, 6, 22, 26, and many others). Also, nonhyperbolic meteors can become hyperbolic, e.g. cases nos. 13, 59, 68. This situation exists even though our generated FDP were not subject to any noise. Therefore, one cannot expect this method to work properly given the presence of noise in practical observations. Thus, the classical method of velocity determination making use of the extrema positions of the FDP should be rejected in practice. Moreover, it produces fictitious deceleration in the latter half of trails in most cases, and complete deceleration in case of meteors nos. 3, 11, 28, and 53. Furthermore, if we consider only the sequence of velocities v_{1-2}, v_{2-3}, and v_{3-4}, we will infer deceleration for almost all cases. This indicates that the de-

The second continuation of Table 1.

No.	v_i	v_{1-2}	v_{1-3}	v_{1-4}	v_{2-3}	v_{2-4}	v_{3-4}	$\bar{v}$	v_{ls}
71	59.169	51.812	56.434	57.703	65.679	63.594	61.509	59.455	59.161
72	53.939	59.585	59.611	50.614	59.651	45.232	35.619	51.719	53.938
73	36.558	32.208	33.722	34.647	36.749	37.085	37.422	35.305	36.556
74	42.545	36.320	40.564	43.234	51.173	51.876	52.578	45.957	42.550
75	46.855	38.299	43.065	45.876	54.980	55.348	55.715	48.880	46.857
76	60.697	59.279	55.884	59.161	51.357	59.067	70.631	59.230	60.694
77	38.881	39.892	38.230	38.404	35.321	37.101	38.882	37.972	38.877
78	26.613	27.989	28.991	28.675	30.660	29.247	27.833	28.899	26.614
79	23.544	23.544	24.530	23.996	26.108	24.358	22.609	24.191	23.541
80	53.115	49.920	53.611	48.827	60.993	47.953	39.259	50.094	53.114
81	24.517	24.452	24.592	24.830	24.842	25.169	25.497	24.897	24.515
82	10.423	10.306	10.362	10.353	10.442	10.390	10.329	10.364	10.423
83	40.564	34.848	39.728	38.037	51.928	41.225	34.090	39.976	40.571
84	34.847	30.349	35.441	32.385	50.715	34.421	26.274	34.931	34.846
85	39.026	41.685	42.165	38.377	42.964	36.014	30.802	38.668	39.030
86	29.205	28.875	29.096	28.195	29.427	27.686	25.944	28.204	29.209
87	30.229	29.172	29.937	30.787	31.468	32.401	33.335	31.183	30.227
88	35.182	34.451	35.058	35.156	36.121	35.773	35.424	35.331	35.184
89	67.852	68.071	66.995	65.198	65.383	63.043	60.703	64.899	67.846
90	54.270	53.486	53.316	51.770	53.061	50.484	47.907	51.671	54.271
91	12.481	12.555	12.583	12.315	12.631	12.112	11.645	12.307	12.481
92	62.678	49.350	62.190	33.745	100.708	27.057	14.782	47.972	62.690
93	37.831	44.140	42.063	39.999	39.294	37.239	35.183	39.653	37.831
94	41.471	40.228	45.086	43.237	57.231	46.247	38.924	45.159	41.471
95	55.742	49.326	56.120	56.917	69.709	64.508	59.307	59.314	55.739
96	69.186	92.061	77.485	52.536	62.909	39.362	27.588	58.657	69.192
97	56.438	51.448	55.931	57.206	64.899	62.964	61.029	58.913	56.442
98	18.709	18.132	18.318	18.412	18.742	18.711	18.683	18.500	18.709
99	20.159	20.558	20.192	19.867	19.618	19.325	19.033	19.766	20.164
100	30.947	31.324	31.140	31.591	30.883	31.799	32.945	31.614	30.943

celeration resulting from the application of the classical method is false, a result which can also be understood from the fact that the classical FDP has been derived under the assumption of **constant meteoroid velocity**. This could indicate that the decelerations following from the quasi-simple processing of diffraction pattern as reported by Baggaley et al. (1994) should be treated with care. On the other hand, the NLSF method yields quite satisfactory results as can easily be seen from Table 1. Only the task of obtaining the standard deviation of the parameters it supplies remains open. This is beyond the scope of this work and will be treated in another article. This method is capable of yielding not only the velocity of the meteoroid, but also other parameters such as the time of passage of the specular point by the meteoroid and the ambipolar diffusion coefficient. The results are not biased by aliasing effects and it works quite properly within the whole velocity range considered even though we generated the FDP for a pulse repetition frequency of 379 Hz. The situation of very high speeds where only a few extrema are present should also be investigated. The NLSF method appears to produce reliable results within the usual meteor speed range. It should be used for determination of the velocity of radio meteors, at least at the first approximation stage.

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