introduction of the first paraconsistent logics. In the 1930s, 1940s, and 1950s, in a somehow latent way, some scholars have debated formal and logical theses of paraconsistent flavour without effectively starting up such a theoretical field. Stanisław Jaśkowski is one of the first to contribute to the paraconsistent field in a very intentional way, with full awareness of the meaning of his contribution to it. In fact, his discussive logic D_2 is a mature propositional paraconsistent logic. In 1950s, in an independent and more radical way, the initial investigations of Newton da Costa will lead him to conceive and introduce his paraconsistent logics $C_n, 1 \leq n \leq \omega$, definitely published in 1963. Such a full-fledged system of paraconsistent logics, whose later investigation has opened up the field of paraconsistency to the worldwide community of scholars, is an important event to the recent history of logic and constitutes the first meaningful contribution of a Brazilian thinker to logic and to the Western philosophy.

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Matthew Harrison-Trainor, ***The Complexity of Countable Structures***, University of California, Berkeley, CA, USA, 2017. Supervised by Antonio Montalbán. MSC: 03D45. Keywords: Computability theory, computable structure theory.

Abstract

We prove various results about the complexity of countable structures, both computable and arbitrary. We will describe some of the more important results.

§1. Scott ranks. We begin by investigating descriptions of countable structures in the infinitary logic $\mathcal{L}_{\omega_1\omega}$. Given a countable structure $\mathcal{A}$, we can find a sentence φ, a Scott sentence for $\mathcal{A}$, which describes $\mathcal{A}$ up to isomorphism in the sense that $\mathcal{A}$ is the unique countable model of φ. We can assign a complexity, the Scott rank of $\mathcal{A}$, to $\mathcal{A}$; this is the quantifier complexity of the simplest Scott sentence.

Given an $\mathcal{L}_{\omega_1\omega}$ sentence φ, which we think of as a theory defining a class of structures, what might the set of Scott ranks of the models of φ be? We call such a set of Scott ranks the Scott spectrum of φ. Under projective determinacy, we get a complete descriptive-set-theoretic characterization of which sets of ordinals are the Scott spectra of a sentence.

Theorem 1.1 (Harrison-Trainor; ZFC + PD). *The sets of ordinals which are the Scott spectra of $\mathcal{L}_{\omega_1\omega}$-sentences are exactly the sets of the following forms, for some $\mathbf{\Sigma}^1_1$ class of linear orders $\mathcal{C}$:*

1. *The well-founded parts of orderings in $\mathcal{C}$,*
2. *The orderings in $\mathcal{C}$ with the non-well-founded part collapsed to a single element, or*
3. *The union of (1) and (2).*

Using the same ideas, we also solve three open questions. We answer a question of Montalbán by showing, for each $\alpha < \omega_1$, that there is a $\Pi^{\mathtt{in}}_2$ theory with no models of Scott rank less than α. We also answer a question of Knight and Calvert by showing that there are computable models of high Scott rank which are not computably approximable by models of low Scott rank. Finally, we answer a question of Sacks and Marker by showing that δ^1_2 is the least ordinal α such that if the models of a computable theory T have Scott rank bounded below ω_1, then their Scott ranks are bounded below α.

We also look at Scott sentences for finitely generated groups. Every finitely generated structure automatically has a $\Sigma^{\mathtt{in}}_3$ Scott sentence, which is relatively simple. It turns out that many groups have a simpler d-$\Sigma^{\mathtt{in}}_2$ Scott sentence—a conjunction of a $\Sigma^{\mathtt{in}}_2$ and a $\Pi^{\mathtt{in}}_2$ sentence—and hence have Scott rank at most 2. This led Knight to conjecture that every finitely generated group has a d-$\Sigma^{\mathtt{in}}_2$ Scott sentence. To resolve this conjecture, we first give a general characterization of the finitely generated structure with d-$\Sigma^{\mathtt{in}}_2$ Scott sentences, and then use this characterization to build a finitely generated group which achieves the maximum possible descriptive complexity.

THEOREM 1.2 (Harrison-Trainor, Ho). *There is a computable finitely-generated group G which has no d-Σ_2^{in} Scott sentence. Its index set is Σ_3^0 m-complete.*

On the other hand, every finitely generated field has a d-Σ_2^{in} Scott sentence.

We also answer a question of Millar and Sacks.

THEOREM 1.3 (Harrison-Trainor, Igusa, Knight). *There is a computable structure of Scott rank ω_1^{CK} whose computable infinitary theory is not $\aleph_0$-categorical.*
Millar and Sacks had shown that there is such a structure $\mathcal{A}$ which is not computable, but which satisfies $\omega_1^{\mathcal{A}} = \omega_1^{CK}$.

§2. Decidably presentable structures. A structure is decidably presentable if it is isomorphic to a decidable structure, i.e., one whose full elementary diagram is computable. Goncharov asked whether there is a classification of the decidably presentable structures. We use an index set result to show that there is no reasonable classification.

THEOREM 2.1 (Harrison-Trainor). *The index set of the decidably presentable structures is Σ_1^1-complete.*

This is the same complexity as the naive definition of being decidably presentable.

§3. Structures on a cone. If $\mathcal{A}$ is a natural structure—by which we mean an informal notion of a structure that might show up in the normal course of mathematics, and which was not constructed explicitly as a computability-theoretic counterexample—then arguments about $\mathcal{A}$ will generally relativize. So we can study natural structures by studying arbitrary structures relativized "on a cone". This gives us a way of making precise statements about the imprecise notion of a natural structure. We begin a classification of the degree spectra of relations on a cone. One of our results here is that (on a cone) any degree spectrum which contains a non-Δ_2^0 degree contains all 2-CEA degrees. With Csima we give a complete classification of the degrees of categoricity on a cone: they are exactly the iterates of the jump.

§4. Functors and interpretations. With Melnikov, Miller, and Montalbán we investigate the deep connections between infinitary interpretations and functors. An interpretation of one structure $\mathcal{A}$ in another structure $\mathcal{B}$ induces a functor which produces copies of $\mathcal{A}$ from copies of $\mathcal{B}$. Moreover, the interpretation induces a homomorphism from the automorphism group of $\mathcal{B}$ to the automorphism group of $\mathcal{A}$. We show that this reverses: given a functor from $\mathcal{B}$ to $\mathcal{A}$, or a homomorphism from the automorphism group of $\mathcal{B}$ to $\mathcal{A}$, we can recover an interpretation.

§5. Computable algebra. We prove various results in computable algebra. Among these, we answer an open question of Downey and Kurtz.

THEOREM 5.1 (Harrison-Trainor). *There is a computable left-orderable group with no computable copy with a computable ordering.*

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