

ON NONBLOCKING MULTICONNECTION NETWORKS COMPOSED OF DIGITAL SWITCHING MATRICES

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Abstract

In the paper we study the conditions under which multiconnection networks are nonblocking. A multiconnection network deals with the connections of pairs $\{(T_1, T_2)\}$ where T_1 is a subset of the input terminals and T_2 is a subset of the output terminals. We investigate networks composed of digital switching matrices. Such networks can be treated as a very general case encompassing many kinds of networks used in practice as well as studied theoretically.

We present four routing strategies and then develop conditions under which multiconnection networks are nonblocking when each of these strategies is used. We also show that the obtained conditions reduce to known results for some values of network parameters.

1. Introduction

A multiconnection network deals with the connections of pairs $\{(T_1, T_2)\}$ where T_1 is a subset of the input terminals and T_2 is a subset of the output terminals [5]. Multiconnections are often used in practice, for example, for conference connections, for loop connection tests, or for connections of several subscribers to a single service circuit, such as a signal sender. A special case of multiconnection networks are broadcast networks, in which $|T_1| = 1$. Recently the special interest in multiconnection networks is caused by their application in broadband systems used for the transmission and switching of high bit-rate signals, including *TV* and high speed data [14], [15].

Space-division multiconnection networks have been dealt with by several authors. Hwang and Jajszczyk have given the conditions under which such networks are nonblocking [4], [5], [6]. The results obtained for space-division

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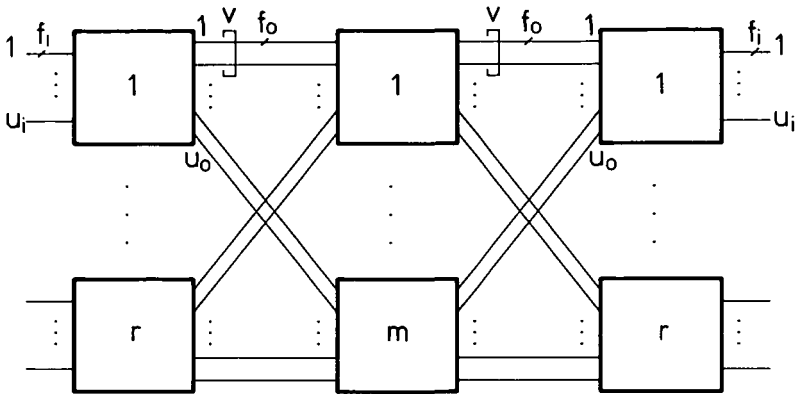


FIGURE 1. Three-stage network; u : number of links, v : number of links between the same switches, f : number of channels in a link.

networks are applicable to time-division networks having separate time and space stages. However, they cannot be applied to networks composed of digital switching matrices, the introduction of which is changing design concepts for digital switching networks.

Digital switching matrix (DSM) mix in the same chip, or in the same printed circuit board, time and space switching [1]. In the matrix information can be transferred from any channel of any incoming PCM link to any channel of any outgoing PCM link. The structures and some features of various switching networks composed of such elements have been discussed in several papers [2], [7], [9], [13], [16].

In this paper the conditions under which multiconnection networks composed of DSMs are nonblocking will be formulated and proven. To illustrate the principle we shall consider the three-stage network shown in Fig. 1. The structure is similar to that proposed by Clos [3]. The main difference is the multiple linkage connection between pairs of switches in successive stages (this multiple linkage connection will be referred to as a “bundle of links”) and that each link can carry more than one channel. The analysis of such a network resolves itself into the analysis of a multiple linkage space-division network.

Let I denote the set of input terminals (input channels) and O the set of output terminals (output channels). A connecting pair in a multiconnection network is defined to be a pair $(T_1, T_2): T_1 \subseteq I, T_2 \subseteq O$ such that each terminal in T_1 is to be connected to every terminal in T_2 . In a (q_1, q_2) -multiconnection network all connecting pairs must satisfy the restrictions $|T_1| \leq q_1$ and $|T_2| \leq q_2$. A multiconnection network is nonblocking if, regardless of what state the network is currently in, a connecting pair involving only idle terminals can

always be connected by a subgraph of the network which is link-disjoint from all subgraphs connecting previous pairs.

Throughout this paper it is assumed that each rectangular switch has the fan-in, fan-out property, i.e., any subset of inlets can be connected simultaneously to any subset of outlets. The above assumption is, of course, valid for space-division switches. However, it cannot be used for some kinds of digital switching matrices. For example, in the DSM described by Belforte et al. the address memory controls the output of the speech memory, and therefore we can send a speech sample from any channel of any incoming PCM link to a multiplicity of outgoing channels, but not vice versa [1]. Some structures of digital switches having both sides of speech memory controlled have been reported [12]. For the sake of generality, we shall use the “fan-in, fan-out” assumption. Therefore, we can restrict ourselves to consider pairs (T_1, T_2) consisting of at most one terminal from each input switch and at most one terminal from each output switch, because if we can connect one input (output) terminal to T_2 (T_1), then all terminals in the same input (output) switch can be connected to T_2 (T_1). Hence, we may assume $r \geq q_1$ and $r \geq q_2$ without a loss of generality. We also assume that overlapping of two or more connecting pairs is not allowed.

Throughout the paper by $\lfloor x \rfloor$ we denote the greatest integer less than or equal to x , and by $\lceil x \rceil$ we denote the smallest integer greater than or equal to x .

2. Routing strategies

We shall modify the four routing strategies proposed by Hwang in order to use them in the discussed networks. The strategies can be formulated as follows.

STRATEGY 1. A connecting path for each pair (t_1, t_2) , $t_1 \in T_1$, $t_2 \in T_2$ is set up independently of a path for another pair (t_1, t_2) . By “independence” we shall mean that any channel is not used by more than one pair (t_1, t_2) .

STRATEGY 2. A connecting path for each pair (t_1, T_2) , $t_1 \in T_1$ is set up through a single channel leading from the input stage to a middle-stage switch. Then it is branched out in this switch and leads to all $t_2 \in T_2$.

STRATEGY 3. A connecting path for each pair (T_1, t_2) , $t_2 \in T_2$ is set up through a single channel connecting the output and the middle stages. Then it is branched out in the middle-stage switch and leads to all $t_1 \in T_1$.

STRATEGY 4. A connecting path for each pair (T_1, T_2) is set up through a single middle-stage switch; all elements t_1 and t_2 , $t_1 \in T_1$, $t_2 \in T_2$ are connected to this switch through single channels.

It can be easily seen that for $v f_o = 1$ the presented strategies reduce to the strategies described in [5].

3. Conditions for nonblocking

Consider the pair (T_1, T_2) . Let $t_1 \in T_1$, $t_2 \in T_2$ and let S_i be the set of switches accommodating T_i where $i = 1, 2$. Denote $y_i = |T_i|$. Because of the fan-in, fan-out properties, we may assume that $|T_i| = |S_i| = y_i$. Let s_i be the switch accommodating t_i .

We can formulate the following four theorems.

THEOREM 1. *A two-sided three-stage network composed of DSMs is nonblocking as a (q_1, q_2) -multiconnection network under Strategy 1 if and only if*

$$m \geq 1 + \min\{[(u_i f_i q_1 - 1)/(v f_o)] + [(u_i f_i q_2 - 1)/(v f_o)], [(u_i f_i (r + \min\{q_1, q_2\} - 1) - 1)/(v f_o)]\}. \quad (1)$$

PROOF. Sufficiency. Consider the connection of the pair (T_1, T_2) . Let g_1 be the maximum number of connections between the following sets of inputs and outputs $s_1 - t_1$ and $O - \{s_2 \cup T_2\}$. Let g_2 be the maximum number of connections between the sets $s_2 - t_2$ and $I - \{s_1 \cup T_1\}$. Let g_3 be the maximum number of connections between the sets $s_1 - t_1$ and $s_2 - t_2$. Let g_4 be the number of connections between t_1 and $T_2 - t_2$. Let g_5 be the number of connections between t_2 and $T_1 - t_1$. Let g_6 be the number of connections between t_1 and t_2 .

Similarly as in the proof of Theorem 1 in [5], the numbers g_1 to g_6 , for the worst state of a network, are as follows:

$$\begin{aligned} g_1 &= \min\{(u_i f_i - 1) \min\{q_2, r - 1\}, (r - 1)u_i f_i - y_2 + 1\}, \\ g_2 &= \min\{(u_i f_i - 1) \min\{q_1, r - 1\}, (r - 1)u_i f_i - y_1 + 1\}, \\ g_3 &= \min\{u_i f_i - 1, (u_i f_i - 1)q_2 - g_1, (u_i f_i - 1)q_1 - g_2\}, \\ g_4 &= y_2 - 1, \\ g_5 &= y_1 - 1, \\ g_6 &= 1. \end{aligned}$$

The total number of connections is then $g = g_1 + g_2 + g_3 + g_4 + g_5 + g_6$. Since g depends on y_1 and y_2 linearly, it is easily seen that g achieves its

maximum at $y_1 = q_1$ and $y_2 = q_2$. Hence,

$$g_1 = (u_i f_i - 1) \min\{q_2, r - 1\}, \quad g_2 = (u_i f_i - 1) \min\{q_1, r - 1\}.$$

Define $g^0 = g(q_1, q_2)$. We consider the following four cases.

CASE 1. $q_1 < r, q_2 < r$. Then

$$\begin{aligned} g_1^0 &= (u_i f_i - 1)q_2, & g_2^0 &= (u_i f_i - 1)q_1, & g_3^0 &= 0, \\ g_4^0 &= q_2 - 1, & g_5^0 &= q_1 - 1, & g_6^0 &= 1. \end{aligned}$$

The connections $s_1 - t_1$ to $O - \{s_2 \cup T_2\}$ and t_1 to $T_2 - t_2$ fully occupy $\lfloor (g_1^0 + g_4^0)/(v f_o) \rfloor$ links between the first and the middle stages. Similarly, the connections $s_2 - t_2$ to $I = \{s_1 \cup T_1\}$ and t_2 to $T_1 - t_1$ fully occupy $\lfloor (g_2^0 + g_5^0)/(v f_o) \rfloor$ links between the middle and the third stages. Thus, in the worst state, the number of required middle-stage switches is

$$h = \lfloor (u_i f_i q_1 - 1)/(v f_o) \rfloor + \lfloor (u_i f_i q_2 - 1)/(v f_o) \rfloor + 1. \tag{2}$$

CASE 2. $q_1 = r, q_2 < r$. Then

(1) Between sets $s_1 - t_1$ and $O - \{s_2 \cup T_2\}$, and the sets t_1 and $T_2 - t_2$ can exist $u_i f_i q_2 - 1$ connections (see Case 1); $v f_o \lfloor (u_i f_i q_2 - 1)/(v f_o) \rfloor$ of these connections fully occupy bundles of links coming to $\lfloor (u_i f_i q_2 - 1)/(v f_o) \rfloor$ middle-stage switches.

(2) The number of connections which can be established between the sets t_2 and $T_1 - t_1$ is equal to $r - 1$. The number of elements of the set $I - \{s_1 \cup T_1\}$ limits g_2 (i.e. the maximum number of connections between $s_2 - t_2$ and $I - \{s_1 \cup T_1\}$ to $(r - 1)(u_i f_i - 1)$. Because in (1) only $v f_o \lfloor (u_i f_i q_2 - 1)/(v f_o) \rfloor$ connections of the set $s_1 - t_1$ are used, the following number of connections can be established between the sets $s_2 - t_2$ and $s_1 - t_1$: $\min\{u_i f_i - 1, R\}$, where $R = (u_i f_i q_2 - 1) \bmod (v f_o)$. Thus, connections discussed here can fully occupy bundles of links coming to $\lfloor ((r - 1)u_i f_i + \min\{u_i f_i - 1, R\})/(v f_o) \rfloor$ middle-stage switches.

In the worst state the sets of middle-stage switches described in (1) and (2) are disjoint. In order to connect the pair (t_1, t_2) an additional switch is required. Therefore, the total number of required middle-stage switches is

$$h \lfloor (u_i f_i q_2 - 1)/(v f_o) \rfloor + \lfloor ((r - 1)u_i f_i + \min\{u_i f_i - 1, R\})/(v f_o) \rfloor + 1. \tag{3}$$

Because for positive integers a, b, c, d holds

$$d \lfloor a/b \rfloor + \lfloor (c + d((a \bmod b))) / b \rfloor = \lfloor (ad + c)/b \rfloor \tag{4}$$

and $(r - 1)u_i f_i + u_i f_i - 1 = u_i f_i q_1 - 1$ (in Case 2 we have $q_1 = r$), expression (3) can be rewritten as follows

$$h = \min \{ \lfloor (u_i f_i q_1 - 1)/(v f_o) \rfloor + \lfloor (u_i f_i q_2 - 1)/(v f_o) \rfloor, \lfloor (u_i f_i (r + q_2 - 1) - 1)/(v f_o) \rfloor \} + 1. \tag{5}$$

CASE 3. $q_1 < r, q_2 = r$. By symmetry to Case 2 we have

$$h = \min \{ [(u_i f_i q_1 - 1)/(v f_o)] + [(u_i f_i q_2 - 1)/(v f_o)], [(u_i f_i (r + q_1 - 1) - 1)/(v f_o)] \} + 1. \quad (6)$$

CASE 4. $q_1 = q_2 = r$. Then

$$g_1^0 = g_2^0 = (u_i f_i - 1)(r - 1), \quad g_3^0 = u_i f_i - 1.$$

Because $(u_i f_i - 1)(r - 1) + u_i f_i - 1 = q_j(u_i f_i - 1)$, for $j = 1, 2$, Case 4 is identical to Cases 2 and 3.

Summarising Cases 1–4 we obtain Theorem 1. Necessity can be proved with ease by constructing sequences of calls which lead to the occupancy of such a number of switches which is given by formula (1).

Theorem 1 is illustrated by the following simple example.

EXAMPLE 1. Let us consider a network having the following parameters: $q_1 = q_2 = r = 2, u_i f_i = 5, v f_o = 2$. By Theorem 1 the network is nonblocking for $m \geq 8$. The worst state of the network is presented in Figure 2.

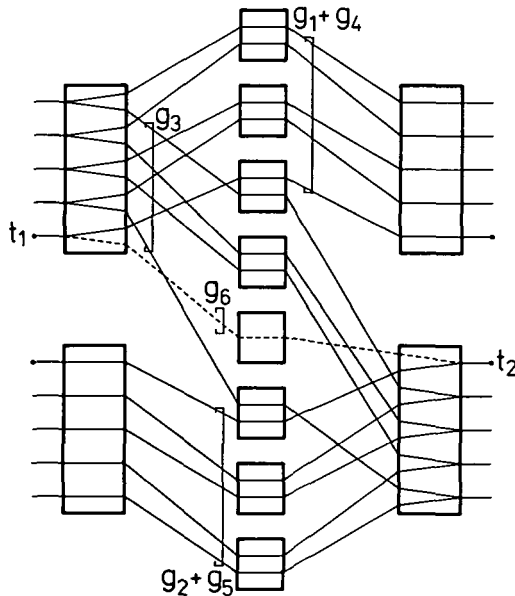


FIGURE 2. The worst state in the (2,2)-multiconnection network controlled by using Strategy 1.

THEOREM 2. *A two-sided three-stage network composed of DSMs is nonblocking as a (q_1, q_2) -multiconnection network under Strategy 2 if and only if*

$$m \geq \max_{1 \leq y_2 \leq q_2} \{ \min \{ \lfloor (u_i f_i - 1) / (v f_o) \rfloor + y_2 \lfloor (q_1 (u_i f_i - 1)) / (v f_o) \rfloor + \min \{ d_1, d_2 \}, \lfloor (r u_i f_i - q_1) / (v f_o) \rfloor + d_1, \lfloor ((r - y_2) u_i f_i + q_1 y_2 (u_i f_i - 1)) / (v f_o) \rfloor + d_2 \} \}, \tag{7}$$

where

$$d_1 = \begin{cases} \lfloor (q_1 - 1) / (v f_o - R_1) \rfloor + 1, & \text{for } q_1 \leq \lceil R_2 / R_1 \rceil v f_o - R_2, R_1 \neq 0, \text{ and} \\ & y_2 \lfloor (q_1 (u_i f_i - 1)) / (v f_o) \rfloor \leq \lfloor (u_i f_i (r - 1) - q_1 + R + 1) / (v f_o) \rfloor; \\ \lfloor (q_1 + R_2 - 1) / (v f_o) \rfloor + 1 & \text{for } q_1 > \lceil R_2 / R_1 \rceil v f_o - R_2, R_1 \neq 0 \\ \text{or } y_2 \lfloor (q_1 (u_i f_i - 1)) / (v f_o) \rfloor > \lfloor (u_i f_i (r - 1) - q_1 + R + 1) / (v f_o) \rfloor \\ \text{or } R_1 = 0; \end{cases} \tag{8}$$

$$d_2 = \begin{cases} \lfloor (q_1 - 1) (v f_o - R_1) \rfloor + 1, & \text{for } q_1 \leq \lceil R_3 / R_1 \rceil v f_o - R_3, R_1 \neq 0; \\ \lfloor (q_1 + R_3 - 1) / (v f_o) \rfloor + 1, & \text{for } q_1 > \lceil R_3 / R_1 \rceil v f_o - R_3, R_1 \neq 0; \\ \lfloor (q_1 - 1) (v f_o) \rfloor + 1, & \text{for } R_1 = 0; \end{cases} \tag{9}$$

$$\begin{aligned} R &= (u_i f_i - 1) \bmod (v f_o); \\ R_1 &= (q_1 (u_i f_i - 1)) \bmod (v f_o); \\ R_2 &= u_i f_i (r - 1) - q_1 + 1 + R - \\ &\quad - \min \{ y_2 \lfloor (q_1 (u_i f_i - 1)) / (v f_o) \rfloor, \lfloor (u_i f_i (r - 1) - q_1 + R + 1) / (v f_o) \rfloor \} v f_o; \\ R_3 &= \{ y_2 R_1, (r - y_2) u_i f_i + y_2 R_1 - \\ &\quad - \min \{ \lfloor (u_i f_i - 1) / (v f_o) \rfloor, \lfloor ((r - y_2) u_i f_i + y_2 R_1) / (v f_o) \rfloor \} \} v f_o. \end{aligned}$$

PROOF. Sufficiency. Consider the connection of the pair (T_1, T_2) . Let g_1 be the maximum number of connections between the sets $s_1 - t_1$ and $O - S_2$. Let g_2 be the maximum number of connections between the sets $S_2 - T_2$ and $I - \{s_1 \cup T_1\}$. Let g_3 be the maximum number of connections between the sets $s_1 - t_1$ and $S_2 - T_2$. Let g_4 be the number of connections between the sets T_1 and T_2 . It is easily seen that:

$$\begin{aligned} g_1 &= \min \{ u_i f_i - 1, (r - y_2) u_i f_i \}, \\ g_2 &= \min \{ y_2 (u_i f_i - 1) \min \{ q_1, r - 1 \}, (r - 1) u_i f_i - y_1 + 1 \}, \\ g_3 &= \min \{ (u_i f_i - 1) y_2, y_2 (u_i f_i - 1) q_1 - g_2, u_i f_i - 1 - g_1 \}, \\ g_4 &= y_1. \end{aligned}$$

Because the total number of connections increases in y_1 , so we can take $y_1 = q_1$. Hence, g_2 can be rewritten as follows:

$$g_2 = \min \{ y_2 (u_i f_i - 1) q_1, (r - 1) u_i f_i - q_1 + 1 \}.$$

The total number of connections is then $g = g_1 + g_2 + g_3 + g_4$. Define $g^0 = g(q_1, y_2)$. We consider all possible combinations of the terms of g_1, g_2, g_3 , and g_4 in g^0 . There are 12 such combinations. We express a combination as follows:

$$g_{1,i} + g_{2,j} + g_{3,k} + g_4,$$

where

$$\begin{aligned} g_{1,1} &= u_i f_i - 1, & g_{1,2} &= (r - y_2) u_i f_i; \\ g_{2,1} &= y_2(u_i f_i - 1)q_1, & g_{2,2} &= (r - 1)u_i f_i - q_1 + 1; \\ g_{3,1} &= (u_i f_i - 1)y_2, & g_{3,2} &= u_i f_i - 1 - g_1, \\ g_{3,3} &= y_2(u_i f_i - 1)q_1 - g_2; \\ g_4 &= q_1. \end{aligned}$$

Note that the appearance of $g_{1,1}$ or $g_{2,1}$ implies that $g_3 = 0$. Furthermore, $g_{1,i} + g_{3,2} = g_{1,1}$ and $g_{2,j} + g_{3,3} = g_{2,1}$. Finally, it is easily to verify that $g_2 = g_{2,2}$ implies $g_{3,1} > g_{3,2}$. Thus, there are only three distinct expressions for g , namely

$$\begin{aligned} g_{1,1} + g_{2,1} + g_4, \\ g_{1,1} + g_{2,2} + g_4 = g_{1,2} + g_{2,2} + g_{3,2} + g_4, \\ g_{1,2} + g_{2,1} + g_4 = g_{1,2} + g_{2,2} + g_{3,3} + g_4. \end{aligned}$$

Let h denote the number of required middle-stage switches. We consider the following three cases related to the expressions presented above.

CASE 1. $g_{1,1} + g_{2,1} + g_4$. In this case we can distinguish the following three kinds of connections.

- (1) The connections $s_1 - t_1$ to $O - S_2$ can fully occupy bundles of links coming to $\lfloor (u_i f_i - 1)/(v f_o) \rfloor$ middle-stage switches.
- (2) The connections $S_2 - T_2$ to $I - \{s_1 \cup T_1\}$ can fully occupy bundles of links coming to $y_2 \lfloor q_1(u_i f_i - 1)/(v f_o) \rfloor$ middle-stages switches.
- (3) The remaining q_1 connections are established while the bundles of links connecting middle-stage switches and third-stage switches belonging to S_2 are partially occupied by $y_2 R_1 = y_2(q_1(u_i f_i - 1)) \bmod (v f_o)$ connections $S_2 - T_2$ to $I - \{s_1 \cup T_1\}$. The connections T_1 to T_2 can be established by partially occupied bundles of links if $q_1 \leq y_2(v f_o - R_1)$. These links lead to $\lceil q_1/(v f_o - R_1) \rceil = \lfloor (q_1 - 1)/(v f_o - R_1) \rfloor + 1$ middle-stage switches. If $q_1 > y_2(v f_o - R_1)$ then the connections T_1 to T_2 and the connections partially occupying bundles of links can be established through $\lceil (q_1 + y_2 R_1)/(v f_o) \rceil$ middle-stage switches.

In the worst state the sets of switches described in (1)–(3) are disjoint. Hence, the number of required middle-stage switches is

$$h = \lfloor (u_i f_i - 1)/(v f_o) \rfloor + y_2 \lfloor (q_1(u_i f_i - 1))/(v f_o) \rfloor + d, \tag{10}$$

where

$$d = \begin{cases} \lfloor (q_1 - 1)/(v f_o - R_1) \rfloor + 1, & \text{for } q_1 \leq y_2(v f_o - R_1), \\ \lfloor (q_1 + y_2 R_1 - 1)/(v f_o) \rfloor + 1, & \text{for } q_1 > y_2(v f_o - R_1). \end{cases} \quad (11)$$

CASE 2. $g_{1,1} + g_{2,2} + g_4 = g_{1,2} + g_{2,2} + g_{3,2} + g_4$. In this case we can distinguish the following three kinds of connections.

(1) $v f_o \lfloor (u_i f_i - 1)/(v f_o) \rfloor$ connections $s_1 - t_1$ to $O - S_2$ fully occupy bundles of links coming to $\lfloor (u_i f_i - 1)/(v f_o) \rfloor$ middle-stage switches.

(2) The connections $S_2 - T_2$ to $I - \{s_1 \cup T_1\}$ can fully occupy bundles of links coming to at most $y_2 \lfloor q_1 (u_i f_i - 1)/(v f_o) \rfloor$ middle-stage switches. However, the set $I - \{s_1 \cup T_1\}$ limits the number of these connections to $g_{2,2} = u_i f_i (r - 1) - q_1 + 1$. In the worst state, output channels belonging to $S_2 - T_2$ are also connected to $R = (u_i f_i - 1) \bmod (v f_o)$ input channels belonging to $s_1 - t_1$ which have not been used in (1). Hence, the connections described here fully occupy bundles of links coming to the following number of middle-stage switches

$$\min \{y_2 \lfloor (q_1 (u_i f_i - 1))/(v f_o) \rfloor, \lfloor (u_i f_i (r - 1) - q_1 + R + 1)/(v f_o) \rfloor\}.$$

(3) The connections T_1 to T_2 are established while the bundles of links connecting middle-stage switches and third-stage switches belonging to S_2 are partially occupied by the following number of connections described in (2)

$$R_2 = u_i f_i (r - 1) - q_1 + 1 + R$$

$$- \min \{y_2 \lfloor (q_1 (u_i f_i - 1))/(v f_o) \rfloor, \lfloor (u_i f_i (r - 1) - q_1 + R + 1)/(v f_o) \rfloor\} v f_o.$$

If $R_1 = 0$ or

$$\lfloor (u_i f_i (r - 1) - q_1 + R_1 + 1)/(v f_o) \rfloor < y_2 \lfloor (q_1 (u_i f_i - 1))/(v f_o) \rfloor$$

then, in the worst state, these remaining connections are established to a single switch belonging to S_2 . In this case the connections T_1 to T_2 require $\lceil (q_1 + R_2)/(v f_o) \rceil = \lfloor (q_1 + R_2 - 1)/(v f_o) \rfloor + 1$ middle-stage switches.

If $R_1 \neq 0$ and

$$\lfloor (u_i f_i (r - 1) - q_1 - R_1 + 1)/(v f_o) \rfloor \geq y_2 \lfloor (q_1 (u_i f_i - 1))/(v f_o) \rfloor$$

then, in the worst state, these remaining connections are established to $\lceil R_2/R_1 \rceil$ switches belonging to S_2 . In this case, similarly as in Case 1, the connections T_1 and T_2 require $\lfloor (q_1 - 1)/(v f_o - R_1) \rfloor + 1$ middle-stage switches for $q_1 \leq \lceil R_2/R_1 \rceil v f_o - R_2$ and $\lfloor (q_1 + R_2 - 1)/(v f_o) \rfloor + 1$ middle-stage switches for $q_1 > \lceil R_2/R_1 \rceil v f_o - R_2$.

In the worst state sets of switches described in (1)–(3) are disjoint. Hence, the number of required middle-stage switches is

$$h = \lfloor (u_i f_i - 1)/(v f_o) \rfloor + \min \{y_2 \lfloor (q_1 (u_i f_i - 1))/(v f_o) \rfloor, \lfloor (u_i f_i (r - 1) - q_1 + R + 1)/(v f_o) \rfloor\} + d_1, \quad (12)$$

where d_1 is expressed by (8).

By using formula (4) we can rearrange formula (12) to

$$h = \min \{ \lfloor (u_i f_i - 1) / (v f_o) \rfloor + y_2 \lfloor (q_1 (u_i f_i - 1)) / (v f_o) \rfloor, \lfloor (r u_i f_i - q_1) / (v f_o) \rfloor \} + d_1. \quad (13)$$

REMARK 1. We can note that $g_{1,2} < g_{1,1}$ only if $r = y_2$, i.e. $g_{1,2} = 0$. Therefore, for $g_{1,2} + g_{2,2} + g_{3,2} + g_4$, (1) is related to connections $s_1 - t_1$ to $S_2 - T_2$, but not to connections $s_1 - t_1$ to $O - S_2$.

REMARK 2. For $y_2 q_1 (u_i f_i - 1) \leq u_i f_i (r - 1) - q_1 + R + 1$ the number of required middle-stage switches for Case 2 is equal or greater than that for Case 1.

CASE 3. $g_{1,2} + g_{2,1} + g_4 = g_{1,2} + g_{2,2} + g_{3,3} + g_4$. In this case we can distinguish the following three kinds of connections

(1) $v f_o y_2 \lfloor (q_1 (u_i f_i - 1)) / (v f_o) \rfloor$ connections $S_2 - T_2$ to $I - \{s_1 \cup T_1\}$ fully occupy bundles of links coming to $y_2 \lfloor (q_1 (u_i f_i - 1)) / (v f_o) \rfloor$ middle-stage switches.

(2) The set $O - S_2$ limits the number of the connections $s_1 - t_1$ to $O - S_2$ to $u_i f_i (r - y_2)$. In case $g_{1,1} > g_{1,2}$, input channels belonging to $s_1 - t_1$ can be connected to $\min \{u_i f_i - 1, y_2 R_1\}$ output channels belonging to $S_2 - T_2$, because $y_2 R_1$ channels belonging to $S_2 - T_2$ have not been used in (1). Thus, the connections described here can fully occupy bundles of links coming to the following number of middle-stage switches

$$\min \{ \lfloor (u_i f_i - 1) / (v f_o) \rfloor, \lfloor ((r - y_2) u_i f_i + y_2 R_1) / (v f_o) \rfloor \}.$$

(3) The output channels belonging to $S_2 - T_2$ which have not been used in (1) and (2) can use the following number of connections with free channels belonging to $I - \{s_1 \cup T_1\}$

$$R_3 = \min \{ y_2 R_1, (r - y_2) u_i f_i + y_2 R_1 - \min \{ \lfloor (u_i f_i - 1) / (v f_o) \rfloor, \lfloor ((r - y_2) u_i f_i + y_2 R_1) / (v f_o) \rfloor \} v f_o \}.$$

In the worst state these connections are established with $\lceil R_3 / R_1 \rceil$ switches belonging to S_2 , if $R_1 \neq 0$. Similarly, as in Case 1, the connections T_1 to T_2 require $\lfloor (q_1 - 1) / (v f_o - R_1) \rfloor + 1$ middle-stage switches if $q_1 \leq \lceil R_3 / R_1 \rceil v f_o - R_3$ and $\lfloor (q_1 + R_3 - 1) / (v f_o) \rfloor + 1$ middle-stage switches if $q_1 > \lceil R_3 / R_1 \rceil v f_o - R_3$. If $R_1 = 0$, then $R_3 = 0$ and the connections T_1 and T_2 require $\lfloor (q_1 - 1) / (v f_o) \rfloor + 1$ middle-stage switches.

In the worst state the sets of middle-stage switches described in (1)–(3) are disjoint. Hence, the number of required middle-stage switches is

$$h = y_2 \lfloor (q_1 (u_i f_i - 1)) / (v f_o) \rfloor + \min \{ \lfloor (u_i f_i - 1) / (v f_o) \rfloor, \lfloor ((r - y_2) u_i f_i + y_2 R_1) / (v f_o) \rfloor \} + d_2, \quad (14)$$

where d_2 is expressed by (9).

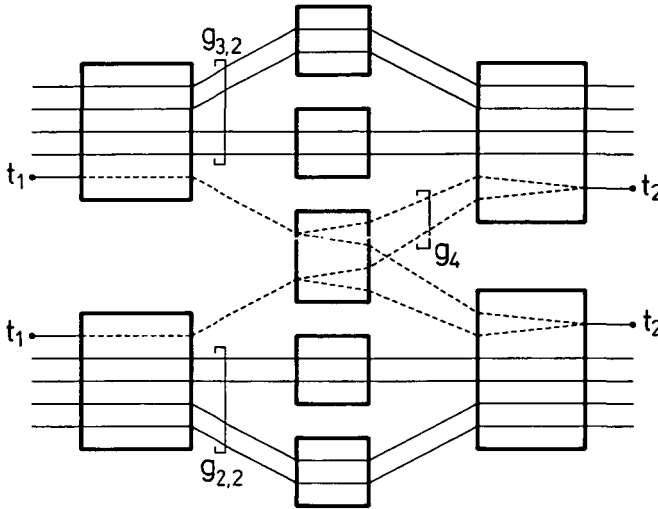


FIGURE 3. The worst state in the (2, 2)-multiconnection network controlled by using Strategy 2.

By using formula (4) we can rewrite (14) obtaining the following expression

$$h = \min \{ \lfloor (u_i f_i - 1) / (v f_o) \rfloor + \lfloor (q_1(u_i f_i - 1)) / (v f_o) \rfloor, \lfloor ((r - y_2)u_i f_i - y_2 q_1(u_i f_i - 1)) / (v f_o) \rfloor \} + d_2. \quad (15)$$

REMARK 3. For $u_i f_i - 1 \leq (r - y_2)u_i f_i$ Case 3 is transformed to Case 1.

Summarising Cases 2 and 3 and choosing the maximum value of $h(q_1, y_2)$ for $1 \leq y_2 \leq q_2$ we obtain Theorem 2. Necessity can be proved with ease by constructing a sequence of calls which leads to the occupancy of the switches of formula (7).

Theorem 2 is illustrated by the following example.

EXAMPLE 2. Let us consider a network identical to that of Example 1. Under Strategy 2 this network is nonblocking for $m \geq 5$. The worst state of the network is illustrated in Figure 3. This state corresponds to Case 2 of the proof, and, in particular, to the combination: $g_{1,2} + g_{2,2} + g_{3,2} + g_4$, where $g_{1,2} = 0$. Note that Case 2 is proved only for the combination $g_{1,1} + g_{2,2} + g_4$ which gives the same m as the combination appearing in our example.

THEOREM 3. A two-sided three-stage network composed of DSMs is nonblocking as a (q_1, q_2) -multiconnection network under Strategy 3 if and only if

$$m \geq \max_{1 \leq y_1 \leq q_1} \{ \min \{ \lfloor (u_i f_i - 1) / (v f_o) \rfloor + y_1 \lfloor (q_2(u_i f_i - 1)) / (v f_o) \rfloor + \min \{ d_3, d_4 \}, \lfloor (r u_i f_i - q_2) / (v f_o) \rfloor + d_3, \lfloor ((r - y_1)u_i f_i + q_2 y_1(u_i f_i - 1)) / (v f_o) \rfloor + d_4 \} \}, \quad (16)$$

where

$$d_3 = \begin{cases} [(q_2 - 1)/(vf_o - R_1)] + 1, & \text{for } q_2 \leq [R_2/R_1]vf_o - R_2, R_1 \neq 0, \text{ and} \\ & y_1[(q_2(u_i f_i - 1))/(vf_o)] \leq [(u_i f_i(r - 1) - q_2 + R + 1)/(vf_o)]; \\ [(q_2 + R_2 - 1)/(vf_o)] + 1, & \text{for } q_2 > [R_2/R_1]vf_o - R_2, R_1 \neq 0 \\ & \text{or } y_1[(q_2(u_i f_i - 1))/(vf_o)] > [(u_i f_i(r - 1) - q_2 + R + 1)/(vf_o)]; \\ & \text{or } R_1 = 0; \end{cases} \tag{17}$$

$$d_4 = \begin{cases} [(q_2 - 1)/(vf_o - R_1)] + 1, & \text{for } q_2 \leq [R_3/R_1]vf_o - R_3, R_1 \neq 0; \\ [(q_2 + R_3 - 1)/(vf_o)] + 1, & \text{for } q_2 > [R_3/R_1]vf_o - R_3, R_1 \neq 0; \\ [(q_2 - 1)/(vf_o)] + 1, & \text{for } R_1 = 0; \end{cases} \tag{18}$$

$$\begin{aligned} R &= (u_i f_i - 1) \bmod(vf_o); \\ R_1 &= (q_2(u_i f_i - 1)) \bmod(vf_o); \\ R_2 &= u_i f_i(r - 1) - q_2 + 1 + R - \\ &\quad - \min\{y_1[(q_2(u_i f_i - 1))/(vf_o)], [(u_i f_i(r - 1) - q_2 + R + 1)/(vf_o)]\}vf_o; \\ R_3 &= \min\{y_1 R_1, (r - y_1)u_i f_i + y_1 R_1 - \\ &\quad - \min\{[(u_i f_i - 1)/(vf_o)], [(r - y_1)u_i f_i + y_1 R_1]/(vf_o)\}\}vf_o. \end{aligned}$$

PROOF. Analogous to the proof of Theorem 2

THEOREM 4. A two-sided three-stage network composed of DSMs is nonblocking as a (q_1, q_2) -multiconnection network under Strategy 4 if and only if

$$m \geq 1 + \max_{\substack{1 \leq y_1 \leq q_1 \\ 1 \leq y_2 \leq q_2}} \{ \min\{ (y_1 + y_2)[(u_i f_i - 1)/(vf_o)], [(ru_i f_i - \max\{y_1, y_2\})/(vf_o)] \} \}. \tag{19}$$

PROOF. Sufficiency. Consider the connection of the pair (T_1, T_2) . Let g_1 be the maximum number of connections between the sets $S_1 - T_1$ and $O - S_2$. Let g_2 be the maximum number of connections between $S_2 - T_2$ and $I - S_1$. Let g_3 be the maximum number of connections between $S_1 - T_1$ and $S_2 - T_2$. Let g_4 be the number of connections between T_1 and T_2 .

The numbers g_1 to g_4 , for the worst state of a network, are as follows:

$$\begin{aligned} g_1 &= \min\{y_1(u_i f_i - 1), (r - y_2)u_i f_i\}, \\ g_2 &= \min\{y_2(u_i f_i - 1), (r - y_1)u_i f_i\}, \\ g_3 &= \min\{y_1(u_i f_i - 1) - g_1, y_2(u_i f_i - 1) - g_2\}, \\ g_4 &= 1. \end{aligned}$$

The total number of connections is then $g = g_1 + g_2 + g_3 + g_4$.

Similarly as in the proof of Theorem 2 we consider all possible combinations of the terms $g_1, g_2, g_3,$ and g_4 . Write

$$\begin{aligned} g_{1,1} &= y_1(u_i f_i - 1), & g_{1,2} &= (r - y_2)u_i f_i; \\ g_{2,1} &= y_2(u_i f_i - 1), & g_{2,2} &= (r - y_1)u_i f_i; \\ g_{3,1} &= y_1(u_i f_i - 1) - g_1, & g_{3,2} &= y_2(u_i f_i - 1) - g_2. \end{aligned}$$

Note that the appearance of $g_{1,1}$ or $g_{2,1}$ implies that $g_3 = 0$.

Furthermore, $g_{1,i} + g_{3,1} = g_{1,1}$ and $g_{2,j} + g_{3,2} = g_{2,1}$. Thus, it is sufficient to consider the three combinations

$$\begin{aligned} g_{1,1} + g_{2,1} + g_4, \\ g_{1,1} + g_{2,2} + g_4 = g_{1,2} + g_{2,2} + g_{3,1} + g_4, \\ g_{1,2} + g_{2,1} + g_4 = g_{1,2} + g_{2,2} + g_{3,2} + g_4. \end{aligned}$$

Let h denote the number of required middle-stage switches. We consider the following three cases related to the expressions presented above.

CASE 1. $g_{1,1} + g_{2,1} + g_4$. In this case we can distinguish the following two kinds of connections.

(1) Between the sets $S_1 - T_1$ and $O - S_2 y_1 (u_i f_i - 1)$ connections are established. These connections can fully occupy bundles of links coming to $y_1 \lfloor (u_i f_i - 1) / (v f_o) \rfloor$ middle-stage switches.

(2) Between the sets $S_2 - T_2$ and $I - S_1 y_2 (u_i f_i - 1)$ connections are established. These connections can fully occupy bundles of links coming to $y_2 \lfloor (u_i f_i - 1) / (v f_o) \rfloor$ middle-stage switches.

In the worst state the sets of switches described in (1) and (2) are disjoint. Because in order to establish the connection T_1 to T_2 an additional switch is required, the total number of middle-stage switches is

$$h = (y_1 + y_2) \lfloor (u_i f_i - 1) / (v f_o) \rfloor. \tag{20}$$

CASE 2. $g_{1,1} + g_{2,2} + g_4 = g_{1,2} + g_{2,2} + g_{3,1} + g_4$. In this case we can distinguish the following two kinds of connections

(1) The set $S_1 - T_1$ can realise $y_1(u_i f_i - 1)$ connections; $v f_o y_1 \lfloor (u_i f_i - 1) / (v f_o) \rfloor$ of them can fully occupy bundles of links coming to $y_1 \lfloor (u_i f_i - 1) / (v f_o) \rfloor$ middle-stage switches.

(2) Theoretically, the connections realised by set $S_2 - T_2$ can fully occupy bundles of links coming to at most $y_2 \lfloor (u_i f_i - 1) / (v f_o) \rfloor$ middle-stage switches. However, the set $I - S_1$ limits the number of connections between $S_2 - T_2$ and $I - S_1$ to $(r - y_1)u_i f_i$. The output channels belonging to $S_2 - T_2$ can be additionally connected to $y_1 R = y_1(u_i f_i - 1) \bmod (v f_o)$ input channels belonging to $S_1 - T_1$, which have not been used in (1). Therefore, the

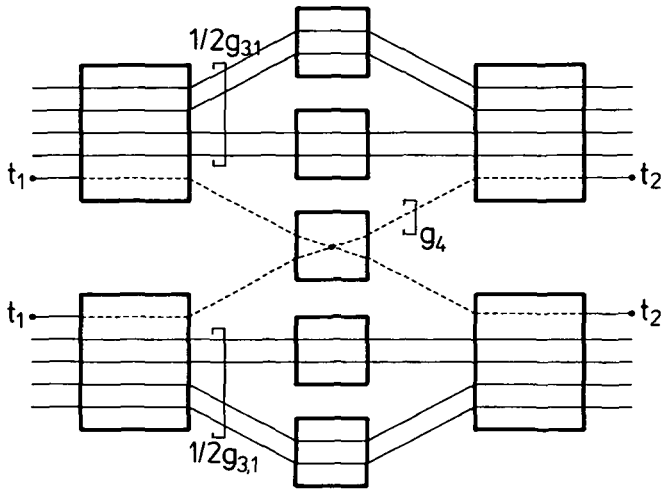


FIGURE 4. The worst state in the (2, 2)-multiconnection network controlled by using Strategy 4.

connections described in (2) can fully occupy bundles of links coming to $\min\{[(r - y_1)u_i f_i + y_1 R]/(v f_o), y_2[(u_i f_i - 1)/(v f_o)]\}$ middle-stage switches.

In the worst state the sets of switches described in (1) and (2) are disjoint. Because in order to establish the connection T_1 to T_2 an additional switch is required, the total number of middle-stage switches is

$$h = y_1[(u_i f_i - 1)/(v f_o)] + \min\{y_2[(u_i f_i - 1)/(v f_o)], [(r - y_1)u_i f_i + y_1 R]/(v f_o)\} + 1. \quad (21)$$

By using formula (4) we can rewrite (21) obtaining the following expression

$$h = 1 + \min\{(y_1 + y_2)[(u_i f_i - 1)/(v f_o)], [(ru_i f_i - y_1)/(v f_o)]\}. \quad (22)$$

CASE 3. $g_{1,2} + g_{2,1} + g_4 = g_{1,2} + g_{2,2} + g_{3,2} + g_4$. By symmetry to Case 2 we have

$$h = 1 + \min\{(y_1 + y_2)[(u_i f_i - 1)/(v f_o)], [(ru_i f_i - y_2)/(v f_o)]\}. \quad (23)$$

Summarising Cases 1 to 3 and choosing the maximum value of $h(y_1, y_2)$ for $1 \leq y_1 \leq q_1$ and $1 \leq y_2 \leq q_2$ we obtain Theorem 4. Necessity can be proved by constructing a sequence of calls which leads to the occupancy of the switches of formula (19).

EXAMPLE 3. Let us consider a network identical to that of Example 1. Under Strategy 4 this network is nonblocking for $m \geq 5$. The worst state of the network is illustrated in Figure 4. This state corresponds to the combination: $g_{1,2} + g_{2,2} + g_{3,1} + g_4$, where $g_{1,2} = g_{2,2} = 0$, $g_{3,1} = 8$, and $g_4 = 1$.

We note that for $f_i = f_o = v = 1$ Theorems 1, 2, 3, and 4 all reduce to the theorems presented in [5], for the symmetrical Clos network. For $q_1 = q_2 = 1$, for all theorems, we obtain the known result: $m \geq 2[(u_i f_i - 1)/(v f_o)] + 1$ [9]. For $r > q_1 q_2$ Theorems 1 to 4 reduce to the theorems presented in [10]. For $f_i = f_o = v = q_1 = q_2 = 1$ we obtain the well-known Clos theorem [3].

Conclusion

In the paper the conditions under which three-stage symmetrical networks composed of digital switching matrices are nonblocking as (q_1, q_2) -multiconnection networks were studied. We can note that although our results have been developed for time-division networks, they are valid also for multiple linkage space-division networks, i.e. networks in which each pair of switches belonging to adjacent stages is connected by more than one link. The obtained results can be easily extended also for non-symmetrical networks, as well as for networks containing more than three-stages.

Since nonblocking multiconnection networks require expansion in the first stage and concentration in the last one we can form various expansion and concentration modules made of square switches (available DSMs are usually square) by using the method proposed in [8]. In some practical applications other approaches to multiconnections may be justified [11], [15].

Throughout the paper we assumed that each rectangular switch has the fan-in and fan-out properties. However, in most cases digital time-division switches have only the fan-out property, i.e. any input channel can be connected to a multiplicity of output channels but not vice versa. In this case Strategies 1 and 3 and Strategies 2 and 4 are in pairs equivalent and, therefore, Theorem 1 is identical to Theorem 3, and Theorem 2 is identical to Theorem 4. The fan-out property does not limit the construction of broadcast networks, i.e. $(1, q)$ -multiconnection networks, which are of practical interest, especially in broadband applications.

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