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## Thoughts About Numbers: Ancient, Medieval, and Modern

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If we are to believe the ancient Greek philosopher Aristotle, the early Pythagoreans thought that the principles governing Number are “the principles of all things,” the concept of Number being more basic than *earth, air, fire, or water*, which were according to ancient tradition the four building blocks of matter. To think about Number is to get close to the architecture of “what is.”

So, how far along are we in our thoughts about numbers?



Figure 1.1. René Descartes (1596–1650) © RMN-Grand Palais / Art Resource, NY

The French philosopher and mathematician René Descartes, almost four centuries ago, expressed the hope that there soon would be “almost nothing more to discover in geometry.” Contemporary physicists dream of a final



Figure 1.2. Jean de Bosschere, “Don Quixote and his Dulcinea del Toboso,” from *The History of Don Quixote De La Mancha*, by Miguel De Cervantes. Trans. Thomas Shelton. Constable and Company, New York, 1922

theory.<sup>1</sup> But despite its venerability and its great power and beauty, the pure mathematics of numbers may still be in the infancy of its development, with depths to be explored as endless as the human soul, and *never* a final theory.

Numbers are obstreperous things. Don Quixote encountered this when he requested that the “bachelor” compose a poem to his lady Dulcinea del Toboso, the first letters of each line spelling out her name. The “bachelor” found<sup>2</sup>

“a great difficulty in their composition because the number of letters in her name was 17, and if he made four Castilian stanzas of four octosyllabic lines each, there would be one letter too many, and if he made the stanzas of five octosyllabic lines each, the ones called *décimas* or *redondillas*, there would be three letters too few...”

“It must fit in, however you do it,” pleaded Quixote, not willing to grant the imperviousness of the number 17 to division.

*Seventeen* is indeed a prime number: there is no way of factoring it as the product of smaller numbers, and this accounts – people tell us – for its occurrence in some phenomena of nature, as when the seventeen-year cicadas all emerged to celebrate a “reunion” of some sort in our fields and valleys.

Prime numbers, despite their *primary* position in our modern understanding of numbers, were not specifically doted over in the ancient literature before Euclid, at least not in the literature that has been preserved. Primes are mentioned as a class of numbers in the writings of Philolaus (a predecessor of Plato);

<sup>1</sup> See Weinberg’s book *Dreams of a Final Theory: The Search for the Fundamental Laws of Nature*, by Steven Weinberg (New York: Pantheon Books, 1992).

<sup>2</sup> See Chapter IV of the Second Part of the *Ingenious Gentleman Don Quixote of La Mancha*.



Figure 1.3. Cicadas emerge every 17 years. Photo by Bob Peterson

they are not mentioned specifically in the Platonic dialogues, which is surprising given the intense interest Plato had in mathematical developments; and they make an occasional appearance in the writings of Aristotle, which is not surprising, given Aristotle's emphasis on the distinction between the *composite* and the *incomposite*. "The incomposite is prior to the composite," writes Aristotle in Book 13 of the *Metaphysics*.

Prime numbers do occur, in earnest, in Euclid's *Elements*!

There is an extraordinary wealth of established truths about whole numbers; these truths provoke sheer awe for the beautiful complexity of prime numbers. But each of the important new discoveries we make gives rise to a further richness of questions, educated guesses, heuristics, expectations, and unsolved problems.