CORRESPONDENCE.

HIGHER TRIGONOMETRY FOR SCHOOLS.

To the Editor of the *Mathematical Gazette.*

DEAR SIR,—Professor Carslaw's review of Siddons and Hughes' *Trigonometry*, Parts III and IV, shows such a lack of understanding of the problem of school teaching that I am moved to write to you and to urge that the needs of school teaching should get a little more help and sympathy from the *Gazette.*

A man who is used to dealing with university students will naturally treat analysis with extreme rigour. A man who has to deal with boys knows that such a treatment for beginners in the study of analysis would produce no results at all except with boys of very exceptional mathematical ability, and that with such boys it is nearly always better to point out the difficulties as they occur, but to pass them by for the moment.

Just as it is absurd with beginners in geometry to approach the subject with all the rigour which men like Hilbert, Peano, Whitehead and Russell have reached, so most teachers of boys will agree that their first introduction to infinite series and functions of complex numbers should not aim at the standard of rigour of books such as Bromwich's *Infinite Series.* The history of the subject is some guide to the teaching of it; the rigour of to-day is a modern growth and is not suitable for a boy who is beginning the subject—he cannot appreciate it because to him it is meaningless until he has handled some infinite series and functions of complex numbers.

A reviewer of a book should consider for whom the book is written and should not judge a book written for schoolboys by the standard he expects from those same boys a few years later after a university course.

We have tried to make it clear in this book that we did not attempt to produce a book that would satisfy the tests of modern analysis, but we set out to give the boy a preliminary canter round part of the course, pointing out its difficulties and pitfalls, and to prepare him for the hard work he would have to do when he comes to cover it under university conditions, or even, in the case of exceptional boys, later in his school career. Any teacher of boys knows the value of a preliminary survey of a difficult subject. To attempt strict rigour with beginners would only choke them off altogether. Professor Carslaw seems to question the need or wisdom of including such a survey. But, apart from the fact that questions on these subjects are set in Entrance Scholarship Examinations, I am convinced that such a course is desirable; students at the university who have gone through such a course at schools say how much it has helped them and also say that, without it, the lectures at the university would have been beyond them.

For many years schoolmasters have wanted a book on Higher Trigonometry that paved the way for boys to go on to the more exact treatises and the lectures that they would meet at the universities. One essential of such a book seemed to be that it should be honest and point out that there were difficulties that would need to be investigated at a later stage.

Many books are written which do not even point out where they are making assumptions that need justification. On page 303 we say, " In this book we do not propose to deal thoroughly with the question of convergence . . . because it seems best for a student to study the subject *after* he has had some acquaintance with infinite series; on the other hand we shall try to point out places at which we make assumptions about convergence and at which difficulties arise." Lower down on the page we advise that the student should go on after reading this book to study the subject in books that deal with it rigorously. Also we frequently refer to the dangers and pitfalls in the use of complex numbers (see pp. 289, 300).

Professor Carslaw is quite right in saying that our statements about differentiation and integration are careless in detail. They do not apply even to power series, and it should have been said that no criterion applicable to other series could be so much as enunciated within the range of our treatment of convergence. But, on the general principle of acquiring some familiarity with a process before discussing its justification, I am entirely unrepentant.

I contend that a boy who goes to the university able to differentiate or integrate the terms of a series and also the sum of the series, and with the knowledge that the two results may be equated *in some cases, but with a clear understanding that the equating of the two results is a matter that needs investigation,* is better prepared to undertake that investigation than one who has been taught to equate the two results without any warning, or than one who has never even thought of the possibility of using differentiation or integration in connection with series. Is Professor Carslaw fair to us when he says, "Bromwich in § 60 pointed out that the 'proof' which starts with the assumption that $\sin x$ and $\cos x$ can be expressed as Power Series is not logically complete. Siddons and Hughes, on the other hand, consider the 'Proof' quite sound"? In the book under review we state quite clearly, "We shall assume that $\sin \theta$ can be expressed as a series of powers of θ" (page 252). That is an honest statement, and any impartial judge must acknowledge that we have made it clear that our 'proof' depends on that assumption.

Professor Carslaw suggests that we should have used Tannery's theorem to obtain the infinite products for $\sin x$, etc., and his interesting article on all this work appears in the *Mathematical Gazette* (p. 71); but does he imagine that it is suitable for a first treatment for a boy who has had no introductory course?

In spite of all that Professor Carslaw says there are other points of view, and many practical teachers encourage me to think that the book presents a most difficult subject in a form intelligible to beginners, and honestly and fairly points out assumptions and difficulties, while treating examples where possible with a rigour not approached in any other text-books at present possible for school use.

But to return to my original plea: will the *Mathematical Gazette* help us by giving us more articles on actual school teaching? I can well imagine the editors saying, "Write such articles and we will publish them." But who will venture to write such articles if they are going to be criticised by the standard of modern analysis by men who seem to be entirely out of sympathy with school teaching?

Our parent Association, the A.I.G.T., was founded by schoolmasters for the benefit of mathematical teaching in schools. I have before me a copy of the first report of that Association. That report gives the list of members, 61 in all: of those, 51 were schoolmasters. To-day I suppose the majority of the members of the Mathematical Association are engaged in teaching in schools; they do not begrudge the space given to the many admirable reviews by eminent mathematicians of books of extreme mathematical rigour, but they would like to have all books intended for school purposes reviewed from the school point of view by men who understood what are the needs of the schools, and not treated as though they were intended for university students.

My letter has already run to great length, but I must say that it is not all university professors who are blind to the school problem; I should like to pay my tribute to the great help school mathematics has received from some of the most eminent of pure mathematicians: they have displayed a wonderful understanding of the problem of teaching in schools, they have appreciated the fact that the rigour required of university students is not to be expected of the schoolboy beginning the study of pure mathematics, and that the boy must be led gradually to strict rigour. Such men have given a helping hand to many schoolmasters and so to many boys who have become good mathematicians; but the men who think that the mere schoolboy should be fed from the start with mathematical food of the strictest rigour are doing their best entirely to stop the flow of promising mathematicians to the universities; if schoolmasters followed their advice, the number of boys choosing mathematics as their special subject would soon tend to the limit zero.

Finally, I would suggest that the *Mathematical Gazette* should throw open its pages to a discussion of the following three possible ways of dealing with Higher Trigonometry in schools:

(*a*) A treatment that is rigorous from the start.

(*b*) A first course that does not pretend to be rigorous and makes many assumptions that need justification, but points out the assumptions and dangers and pitfalls to be investigated later.

(*c*) The old-fashioned course which gives " proofs " that make assumptions that are not stated, and slurs over difficulties and pitfalls without any warning (*e.g.* proofs of the power series for $\sin x$ and $\cos x$ that neglect an infinite number of infinitely small quantities in the most light-hearted fashion).

My answer would be

(*a*) The book has yet to be written that will make this possible. And will it ever be possible except with very few boys? And how many schoolmasters are capable of such a treatment? Even if they know all about the work, they so seldom get a boy of that class that they cannot have much experience of teaching it, and it would be better for them to leave the work to the university teacher.

(*b*) is to me the best course. I believe the really larger-minded university teachers will approve of it and see that it paves the way for their work; the boy will come to them with a knowledge of the ground and prepared to face the difficulties.

(*c*) which is the course books have catered for in the past, seems to me thoroughly bad. Boys who have been trained on that line go to the university very ill prepared—they do not even see that there are difficulties to be faced.

A symposium on this would be valuable, and I hope that other schoolmasters will express their views and not be frightened by the fear of " high brow " criticism from men who have not had school experience.—Yours, etc.,

A. W. SIDDONS.

P.S.—I do not wish to imply that criticism from men with university experience is valueless; it will be very helpful, provided it has some sympathy with the conditions that obtain in schools of various types.

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766. " During the three years which I spent at Cambridge, my time was wasted. . . . I attempted mathematics, . . . but I got on very slowly. The work was repugnant to me, chiefly from my not being able to see any meaning in the early steps in algebra. . . . In my last year I worked with some earnestness for my final degree of B.A., and brushed up my Classics, together with a little Algebra and Euclid, which latter gave me much pleasure. . . . The logic of Paley['s *Evidences* and *Natural Theology*] gave me as much delight as did Euclid. . . . I was very intimate with Whitley, who was afterwards Senior Wrangler. . . . I also got into a musical set, I believe by means of my warm-hearted friend, Herbert, who took a high wrangler's degree."—From *Autobiography of Charles Darwin*, pp. 21, 22, 23, 24. [Whitley, Rev. C., afterwards Canon of Durham. Herbert, John Maurice, afterwards County Court Judge of Cardiff and Monmouth Circuit.]