Part IX

MATHEMATICAL AND PHYSICAL OBJECTS

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Between Mathematics and Physics[1]

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The distinction between mathematical and physical objects has probably played a greater role shaping the philosophy of mathematics than the distinction between observable and theoretical entities has had in defining the philosophy of science. All the major movements in the philosophy of mathematics may be seen as attempts to free mathematics of an abstract ontology or to come to terms with it. The reasons are epistemic. Most philosophers of mathematics believe that the abstractness of mathematical objects introduces special difficulties in accounting for our ability to know them, to refer to them and even to entertain beliefs about them. These difficulties—supposedly absent even in the case of the most theoretical physical objects—make mathematical objects especially problematic and philosophically unattractive.

Few have questioned this epistemic thesis or the ontic distinction it presupposes. LaVerne Shelton (1980) challenged the abstract-concrete distinction some years ago in an unpublished APA address. Susan Hale continued this line in her dissertation and related articles (Hale 1988a, 1988b), and I raised doubts about Hartry Field's taking spacetime points and regions as a concrete, physical ontological foundation for his nominalization of physics (Field, Resnik 1985). Here I want to question the clarity of the mathematical-physical (and corresponding abstract-concrete) distinction by focussing on the ontology of theoretical physics.

According to most philosophers of mathematics, mathematical objects differ from physical objects in being causally inert and totally disconnected from spacetime. I will focus on these two ways of distinguishing mathematical and physical objects, for coupling either with the popular causal or informational theories of knowledge, reference and *de re* belief yields the dogma of the special epistemic inaccessibility of mathematical entities.

Of course, the inability of causal and informational epistemologies to account for our knowledge of mathematical objects counts as much against those epistemologies as it does against the belief in abstract, mathematical objects. But I do not plan to use this defense of mathematical objects here. Instead I will try to undermine the major epistemic critiques of mathematical objects by arguing that modern physics blurs the mathematical/physical distinction as drawn in spatiotemporal or causal-informational terms

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(section 1). In section 2, I will apply my conclusions concerning physics in questioning certain non-interference assumptions—claims to the effect that the physical world is metaphysically independent of mathematical existence and truth—that underlie recent attempts to free physical science from its dependence on mathematical objects.

1. Physics Blurs The Mathematical/Physical Distinction

Anti-realists in the philosophy of mathematics are invariably realists about physical objects. Indeed, pursuing an anti-realist program in the philosophy of mathematics seems to have little point if one is an instrumentalist in physics. An instrumentalist need only remark that the mathematical component of a physical theory is as much an instrument as the rest of it, and leave it at that. In contrast, we find mathematical anti-realists starting with a distinction between physical and abstract or mathematical entities and attempting either to combine an anti-realist interpretation of mathematics with scientific realism or to show that science can make do without mathematical objects. Thus it is appropriate to appeal to contemporary physics and cosmology to undercut attempts by mathematical anti-realists to distinguish between mathematical and physical objects.

1.1. Quantum Particles

Let me start then by considering some physical objects that upon further examination appear as mathematical as they do physical. I have in mind quantum particles. Due to the difficulties in interpreting to quantum theory, one might think quantum particles unsuitable candidates for physical objects. Relative to debates in the philosophy of physics this may be so; but from the standpoint of today's nominalist philosophers of mathematics, some of whom take spacetime points as physical objects, quantum particles count as both real and clearly physical.

The term "particle" brings to mind the image of a tiny object located in spacetime. But, on what seems to be the consensus view of the puzzling entities of quantum physics, this image will not do. Most quantum particles do not have definite locations, masses, velocities, spin or other physical properties most of the time. Quantum mechanics allows us to calculate the probability that a particle of a give type has a given "observable" property. But that does not even imply that if we, say, detect a photon in a given region of spacetime then the photon occupied that position prior to our attempts to detect it or that the photon would have been in that region even if we had not attempted to detect it. Prior to its detection a photon is typically in a state that is a superposition of definite (or pure) states, and quantum theory contains no explanation of how a photon or any other quantum system goes from a superposition into a definite state. Recent mathematical critiques of hidden variable theories indicate that this mysterious feature of quantum mechanics is virtually unavoidable.(Shimony) In the face of this, one might still say that quantum particles are tiny bits of matter with very weird properties—ones that are only partially analogous to classical physical properties. But there is a further problem.

When we have a system of several particles of the same kind, two photons, for example, there are no quantum mechanical means for tagging them at the beginning of an interaction and re-identifying them at the end of it. Suppose, for example, we run an experiment in which we detect two photons one above the other at one location and later detect two photons (intuitively, the same photons) again one above the other at a location to the right of the first. One might expect that there is a fact as to which photon is on top at the end of the experiment. But quantum theory recognizes no such fact; for such a fact must be described by reference to the trajectories the particles take in travelling between the two locations. Since their initial and final positions in spacetime are fixed, they have definite trajectories only if they have definite velocities, violating the uncertainty principle. The situation is even worse when we combine quantum theory with special relativity, for then there is no fact as to whether we have the same two photons. It might be, for example, that one photon splits into an electron-positron pair, whose members in turn annihilate one another producing the photon we detect. Moreover, between our two detections such processes might occur indefinitely many times. Actually, the same is true of experiments "tracking" a single photon. The one with which we start might be destroyed before we see the one with which we end.

Thus it is better to think of particles as features of spacetime—more like fields—rather than as bodies travelling through spacetime. This view is seconded in the following passage from a recent account of particle physics directed at other scientists

> In the most sophisticated form of quantum theory, all entities are described by fields. Just as the photon is most obviously a manifestation of the electromagnetic field, so too is an electron taken to be a manifestation of an electron field. ... Any one individual electron wavefront may be thought of as a particular frequency excitation of the field and may be localized to a greater or lesser extent dependent upon its interaction. (Dodd, p.27; See also Teller)

Now by thinking of the behavior of iron filings around a magnet we can get an intuitive grasp of what a magnetic field is and its intensity or direction at a given point. However, quantum fields are not distributions of physical entities, rather they are roughly distributions of probabilities. As the electron field varies its intensity over spacetime so does the probability of an interaction involving electrons.(Cf.Teller, pp.612-613.) And remember we cannot think of the electrons as definitely present prior to any electron interaction we observe.

How, then, are we to think of quantum particles and fields? My proposal is that we take the mathematics as descriptive rather than as "merely representational". Fields and particles are functions from spacetime points to probabilities. If that construes fields as mathematical or quasi-mathematical entities, so be it. One advantage to this approach is that it allows us to recognize the individual particles of a given type of particle field as mathematical components of that field. One disadvantage is that it rules out the possibility of explaining the connection between a quantum field and its observed manifestations in familiar physical terms. But quantum theory all but rules this out anyway.

Of course, this proposal blurs the distinction between mathematical and physical objects. Either physical objects become mathematical or mathematical objects acquire physical attributes. I have no difficulties with this. For adherents of the traditional view of physical objects, on the other hand, the challenge is to reduce quantum particles to an uncontroversially physical basis.

How might they respond? One approach would be to identify probabilities with propensities and quantum fields with distributions of propensities, and then argue that distributions and propensities, themselves, are physical entities. But it is unclear to me how one could make a compelling case that this is a genuine *physical* alternative to an openly mathematical account in terms of probabilities and functions.

An analog to Hartry Field's (1980) formulation of Newtonian gravitation would be much more convincing, if one could make it work. The idea would be to treat quantum particles and fields as structural features of spacetime specified in non-mathematical terms. Just as Field was able to avoid referring to quantities, such as temperature, by using physical predicates, such as 'warmer than', one might avoid referring to particles and fields by describing regions of spacetime as 'electronic', 'photonic' or 'electromagnetic'. I do not know whether it is possible to carry through this idea in any reasonably convincing way, and those far more expert than I have already expressed their doubts. (See, e.g., Hellman, p.116.) But let me add a worry of my own.

Suppose that we manage to define 'photonic', 'electronic', etc. in rich enough physical terms to characterize the structure of spacetime regions occupied by photons, electrons, etc. Now suppose that we want to describe the two photon case I discussed earlier. At the beginning and end of the experiment we have two photonic regions of spacetime. But there is no quantum mechanical fact as to whether the spacetime region connecting these events is just the sum of two photonic regions, or of photonic, electronic and positronic regions, and so on. Instead the region has a property corresponding to the superposition of infinitely many pure states. We can make sense of such superpositions using variables ranging over mathematically characterized pure states. In a Hartry Field style formulation, however, variables are restricted to spacetime points and regions. Thus we might be able to define spacetime predicates that allow us to reformulate talk of finitely many specific pure states and superpositions of them as talk of regions. But it is unclear to me how we can recover infinite superpositions without introducing questionable devices such as infinite disjunctions. Even then it may be that we cannot recover the full use of quantification over particles that physics requires. Unfortunately, I am too ignorant of quantum field theories to know how serious a problem this is.

1.2. Epistemic Breakdowns

On the traditional view, a major epistemic difference between physical and mathematical objects is that the former can and the latter cannot participate in a causal process that permits us to detect them. It is pertinent, then, that physicists now recognize physical entities suffering from the very epistemic disability ascribed to mathematical objects. Take, for example, the photon-electron-positron-photon transformations we discussed earlier. Quantum field theory posits processes of this type that happen so fast that they are in principle undetectable. Virtual processes, as they are called, need not even conserve energy or momentum so long as the total processes of which they are components do. Black holes in spacetime supply another example. According to physicist Clifford Will, "there is no way for any external observer to determine, for example, the total number of baryons... [inside a black hole] " and "there must exist [such] a singularity of spacetime at which the path or world line of an observer who hits it must terminate, and physics as we know it must break down". (Will, p.28)

I would expect someone to reply at this point that, unlike mathematical objects, black holes, virtual processes and other examples that one might cite are supposed to be part of the causal network and to have physical effects, and that, furthermore, the theories that posit them are empirical theories supported by empirical evidence. Yet doesn't this sort of response beg the question? True, given a mathematical/physical distinction, one can argue that mathematical objects don't participate in the causal network and mathematical theories remain silent concerning it. But in view of our earlier considerations, the point about the causal network is no longer evident. Indeed, we might now argue that mathematical objects, in the form of quantum fields, do participate in the causal network.

Furthermore, as I have argued elsewhere (Resnik 1989), even supposedly purely mathematical theories have observational consequences and are justified in part on the basis of observational evidence. For instance, we might appeal to recursion theory to predict the results of a computer run, or conversely, use the results of computer runs to convince ourselves of properties of a Turing algorithm or to support a step in a proof. Admittedly, these inferences depend upon taking certain auxiliary hypotheses for granted; and given a mathematical-physical distinction, one might argue that the real empirical content should accrue to those hypotheses rather than to the mathematical ones.

In any case, empirically confirmed theories are as much committed to the existence of numbers and other paradigmatic mathematical entities as they are to black holes or virtual processes, because these theories posit numbers and their ilk as the solutions to equations, dimensionless constants, and so on. Undercutting this point would require reformulating physics along the lines of Hartry Field's program. Given the heavy use that program so far made of spacetime regions and points, we would still need a clear mathematical-physical distinction to determine whether Hartry Field style physics succeeds in eliminating mathematical objects.

So far I have argued that 1) because quantum particles are probability fields or something akin to them, they will impede attempts to distinguish between mathematical and physical objects on spatio-temporal or causal grounds; and 2) that undetectable physical processes count against distinguishing mathematical and physical objects on the basis of physical detectability. Although the mathematical-physical distinction is central to the standard approach to the philosophy of mathematics, these claims underscore the need to clarify it and our current inability to do so.

2. Non-Interference Claims.

Hartry Field once argued along the following lines that good mathematics should be conservative though it need not be true: Physical theories are supposed to be non-conservative in the sense of having observational consequences, which do not follow from just our prior observational knowledge. Not so for mathematics; it is true in all possible worlds, a priori, and the realm it treats has no effect upon physical reality. Thus a mathematical theory with empirical consequences would have something wrong with it—it might even be inconsistent. (1980, pp.12-14.)

Field is right, of course, that good physics should have novel observational consequences. That is why physicists formulate their theories in terms that are prone to produce such consequences. For example, if they construct a theory introducing a new particle, they are very likely to give it a mass, spin, charge, etc., which are likely to allow them to derive observable consequences from their theory. To put the matter metaphorically, physicists typically equip their theories with at least girders extending towards the observable world even when they may not know how to build full bridges. Field is also right that mathematicians do not aim for observational consequences. Instead they try to describe structures that may or may not be realized in the physical world. Thus they do not assign "physical" properties to the objects they posit and find it far more important to relate them to other mathematical objects. This sociology may be responsible for the popular intuition that the mathematical realm does not interfere with the physical one, but plainly it is no argument for that intuition.

The intuition also surfaces in an argument of Terrence Horgan's defending a nominalization of science using counter-factual conditionals with mathematical antecedents Here is what he says:

> Since sets are not supposed to be part of the world's spatio-temporal causal nexus, that nexus would be exactly as it is whether or sets existed or not; for sets would not causally influence the concreta in the spatio-temporal causal nexus, and the idea of their non-causally influencing those concreta is just unintelligible.(Horgan)

We find the intuition again in Geoffrey Hellman's recent book (Hellman), which contains an account of applied mathematics similar to Horgan's. Although Hellman translates statements of mathematicized science as second-order, strict implications, he paraphrases them as counterfactuals with antecedents such as "if *X,f* were any omega sequence". He notes that for his approach to work, we must assume that the presence of such omega sequences will not affect the way the world actually is. To see what is at stake here, consider the presumably true statement "The number of stars is finite". Suppose that Hellman rendered this as, roughly, "If *X,f* were any omega sequence, then some member of it would be greater than its proxy for the number of stars". Then an omega sequence of stars would falsify this translation. To avoid such irrelevant counter-examples, Hellman restricts this and similar translations of applied mathematical claims to omega sequences that do not interfere with the way the world actually is. Of course, such claims will be non-vacuously true only if it is logically possible for such non-interfering omega sequences to exist; and, accordingly Hellman recognizes the need for a postulating possibilities of this type.

At this point we should note an important difference between Hellman, on the one hand, and Horgan and Field, on the other. Hellman is a modal-structuralist. He believes that mathematics is about structures, but he does not posit them outright. Instead, he characterizes individual mathematical structures using modal operators and posits only that it is logically possible for them to be realized rather than that they are actually realized. If we may think of structures for a moment as universals, then Hellman's approach is Aristotelian—structures exist only as realized. By contrast, Field and Horgan think of mathematical objects under what is called the objects-platonist view; they see mathematical objects as full-fledged abstract objects that cannot be identified with anything physical.

Given the Field-Horgan view of mathematical objects, it is almost immediate that they do not interfere with the physical world, and, hence that it is logically possible for them to exist while the actual world remains as it now is. This presupposes, of course, that fields, waves, and particles are not mathematical—in particular, not functions or (impure) sets.

Hellman's position is much more subtle and his attempts to work it out are very interesting. He spends quite a bit of effort exploring ways in which we might formulate non-interference conditions. The first way is simply to take the phrase "does not interfere with the way the world actually is" as acceptable as it stands, and use this phrase in antecedents of counterfactual renderings of applied mathematical statements. This would change the statement about the stars to "if *X,f* were any omega sequence which would not interfere with the way the world actually is, then some member of it would be greater than its proxy for the number of stars". We would also use the phrase to state the following possibility postulate: it is possible that there is an omega sequence which does not interfere with the way the world actually is. Now, as Hellman emphasizes, making sense of these conditionals and this postulate presupposes that the terms in which they are formulated make sense. He takes this to mean that it makes sense to refer to the actual world (or the actual condition of a given physical system) apart from any relativization to a language or theory or conceptual framework, and so on (p.100). Of course, the world Hellman has in mind is supposed to contain no mathematical objects. So at this point he too employs the same presupposition used by Field and Horgan.

As it turns out, Hellman is not happy referring to the world as it actually is, because it presupposes a strong physical realism. Due to this, Hellman moves on to explore ways of formulating non-interference conditions in more precise and metaphysically neutral terms. His general strategy is to introduce non-mathematical predicates R and use them to formulate clauses of the form

x,y,z,..,w stand in R if and only if x,y,z,...,w
actually stand in R, for all *actual* objects
x,y,z,...,w.

Given an appropriate set of predicates, we could use such clauses to state that in a given counterfactual situation all the actual objects have exactly the properties and relationships to each other that they actually have.

Can we find an appropriate set of predicates? Hellman divides this question in two: First, can we find a set of non-mathematical (or synthetic) predicates for picking out the set of actual objects under consideration in a given application? Second, can we be sure that through their extensions these predicates fix a physical world corresponding to our usual mathematical descriptions of the actual world? (Pp.129-139.) For example, it is not enough to have a predicate true of just stars, we must also insure that our clauses only have models in which the number of stars is whatever it actually is. Hellman devotes most of his effort to investigating the second question. In the end, he combines a cautious optimism that it has a positive answer with a lingering fondness for the strong realist approach (p.142).

Hellman's caution concerning the second question contrasts with the cavalier attitude he takes toward the first, ontological one. This, he says, "is relatively unproblematic, for the particular context of application usually involves a given domain of material objects to which a piece of mathematics is to be applied, and it suffices to cover this domain with synthetic predicates" (p.132). Alright, let us suppose that the context of application is quantum electrodynamics, QED. What are the synthetic predicates for picking out the relevant material objects? I guess Hellman would say something like 'electron', 'photon', 'positron', since those are the types of particles involved. But, in the light of our earlier attempt to understand what quantum particles are, it would be appropriate to object that we need a more basic set of synthetic predicates in order to reflect the apparent ontic complexity of electrons, photons and positrons. Since that might require Hellman to carry out more of a Hartry Field style nominalization of quantum physics than he wants, I can imagine him replying: "Look, although we may be unable to state what an electron is without using mathematical terms, electrons are whatever they are independently of our mathematical descriptions. And surely they are physical objects. Thus, since my predicate 'electron' is a primitive predicate true of just them, it is a suitable synthetic predicate for formulating non-interference conditions needed for applying mathematics in QED." Now I see two problems with this sort of reply. First, it takes exactly the strong realist stance towards electrons that at this point Hellman is trying to avoid taking towards the actual world. Second, it simply takes it for granted that electrons are physical objects. It seems unlikely to me that Hellman can carry out his program for applying mathematics without presupposing a problematic mathematical-physical distinction.

Setting this issue aside, let us ask how sure we can be that the physical world will permit Hellman's non-interference conditions to be met. Hellman's case is different from the Field-Horgan case, because, as far as I can tell, for Hellman, to add an omega sequence or a continuum or an iterative hierarchy to the world is not to add something abstract but rather something concrete. Adding new concreta to the world might change the behavior of the things already there. Of course, that depends upon what we mean by the actual world. Are we to count just its population and their properties and relations taken in extension? Or should we also include its laws? Adding a new planet to our solar system will not change the behavior of the other planets so long as the physics of the new world is appropriately different. Thus if we do not include its physics in our description of the actual world, then it is logically possible to add more stuff to it and leave the original stuff as it is. I do not see any place for the laws of physics in Hellman's sketches of how we use synthetic predicates to spell out non-interference conditions; so it may be that he would be willing to take the actual world *sine* physics approach to non-interference. But once we free our world of its physics, its unclear how and whether Hellman's position differs significantly from the Horgan-Field position, according to which the additional entities are abstracta.

In any case, other passages (pp.100-101) suggest that Hellman has the actual world *cum* physics in mind when he speaks of non-interference. So long as we restrict ourselves to applications of mathematics to small collections of physical objects viewed as "isolated systems", the possibility of there being additional physical objects outside such systems that realize various mathematical structures seems relatively unproblematic. But suppose we consider instead applications of mathematics to cosmology. As I understand it, one consequence of the Big Bang theory is that the amount of matter in the universe is finite. Now let us suppose that the Big Bang theory is part of a description of how the world actually is. Then it would not be logically possible for there to be an infinite amount of matter and for our world to remain as it actually is.

Here I am probably appealing to the world *cum* its physics and its initial conditions. We can avoid using "initial conditions" in this example, by going to even more speculative theories; for some cosmologists are trying to develop theories of the origins of the universe that dispense with initial conditions. Thus under the usual understanding of "physically possible" the actual universe would be the only physically possible one.

There is a way for Hellman to defuse these objections. By counting spacetime points or regions as already part of the physical world, the conditions for applying Hellman-style mathematics can often be met without invoking additional physical objects—perhaps even in the cosmology case. However, if we take the less controversial stand of not counting points and regions as part of the physical world, then even applying number theory forces one to confront non-interference problems. For excluding spacetime points and regions from the set of actual physical objects leaves a (presumably) finite set.

3. Conclusion

I will conclude with some observations from my own structuralist perspective. (See Resnik, 1981, 1988.) On this view, numbers, functions and sets (at least) are positions in structures, and their identity is determined by their relationships to other positions in the structure to which they belong. One consequence of this is that usually there is a fact as to whether the positions of one structure are identical to those of another only if the two structures are part of a containing structure. Taking spacetime and the natural number sequence, for instance, as separate structures deprives us of a context in which we can speak of there being a fact as to whether the natural numbers are identical to any omega sequence of spacetime points. In particular, it deprives us of a context in which it makes sense to say that while have admitted spacetime points we have admitted no numbers. From this structuralist point of view, nominalists should aim to avoid committing themselves to the *structures* exhibited by so-called mathematical objects and not merely to avoid committing themselves to mathematical objects *per se*. For there simply may be no fact as to whether they have met the latter aim. To illustrate this point, consider the next passage from Hellman.

> ...When it comes to mathematics, however, we need not regards its abstract structures as literally part of the actual world.... [it] is convenient (perhaps essential) in describing [a material] configuration in detail, but the configuration is "already there" prior to the mathematical description, and independently of any ... [mathematical structure] that may be invoked in such a description (pp.127-128).

But if the configurations (structures) are there, then, from the structural point of view, it is simply unclear as to what it is for the corresponding mathematical objects to fail to be there as well.

Returning to the theme with which I began this paper: rather than worrying about epistemic differences between mathematical and physical objects, it would be better to worry about epistemic differences between various structures—between, for example, infinite and finite ones, or between continuous and discrete ones. While we may find interesting epistemic differences between such structures, I see no reason for interesting epistemic differences to arise between the structures studied in physics and those studied in the traditional (non-global, non-foundational) branches of mathematics.

Note

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