CORRECTION TO MY PAPER "ON TITCHMARSH-KODAIRA'S FORMULA CONCERNING WEYL-STONE'S EIGENFUNCTION EXPANSION" IN NAGOYA MATHEMATICAL JOURNAL, VOL. 1 (1950), 49-58.

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Recently Mr. Seizô Itô kindly called the author's attention to the fact that the derivation of (4.15) in the above referred paper is insufficient since the paper does not contain the proof of the compactness condition:

$$(1)\quad \lim_{T\uparrow\infty}\left\{\int_{-\infty}^{-T}+\int_{T}^{\infty}\right\}\left|d_u\left[\sum_{j,k=1}^{2}\int_0^u (g(x),\ y_j(x,\ u))\,dp_{jk}^{(a,b)}(u)(f(s),\ y_k(s,\ u))\right]\right|=0$$

uniformly in a and b

for C^2 functions $g(x)$ and $f(x)$ vanishing outside the interval $(a',\ b')$, $-\infty<a<a'<b'<b<\infty$. The purpose of the present note is to give a proof to (1) as follows.

We have, by definition (page 49),

$$L_x y_j(x,\ u)=\left(q(x)-\frac{d^2}{dx^2}\right)y_j(x,\ u)=uy_j(x,\ u).$$

Thus, since $g(x)\in C^2$ vanishes outside the interval $(a',\ b')$, we have, by partial integration,

$$(g(x),\ y_j(x,\ u))=(g(x),\ u^{-1}L_x y_j(x,\ u))=u^{-1}(L_x g(x),\ y_j(x,\ u)).$$

And similarly for $f(x)$. We have also the completeness relation

$$(g(x),\ f(x))=\int_{-\infty}^{\infty}d_u\left[\sum_{j,k=1}^{2}\int_0^u (g(x),\ y_j(x,\ u))\,dp_{jk}^{(a,b)}(u)(f(s),\ y_k(s,\ u))\right]$$

for continuous functions $g(x)$ and $f(x)$ vanishing outside the interval (a',b'). The proof was given on page 57 for C^2 functions $g(x)$ and $f(x)$. The extension to continuous functions may be obtained by customary limiting process. Hence, by the positive definiteness of the density matrix $(dp_{jk}^{(a,b)}(u))$, we have, for C^2 function $g(x)$ vanishing outside the interval $(a',\ b')$,

$$\left\{\int_{-\infty}^{-T}+\int_{T}^{\infty}\right\}d_u\left[\sum_{j,k=1}^{2}\int_0^u (g(x),\ y_j(x,\ u))\,dp_{jk}^{(a,b)}(u)(g(s),\ y_k(s,\ u))\right]$$

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$$\leqq T^{-2}\left\{\int_{-\infty}^{-T}+\int_{T}^{\infty}\right\}du\left[\sum_{j,k=1}^{2}\int_{0}^{u}(L_x g(x),\ y_j(x,\ u)dp_{jk}^{(a,b)}(u)(L_s g(s),\ y_j(s,\ u))\right]$$

$$\leqq T^{-2}\int_{-\infty}^{\infty}du[\quad]=T^{-2}(L_x g(x),\ L_x g(x)).$$

We have also the similar inequality for C^2 function $f(x)$ vanishing outside (a', b'). Therefore we have proved (1) by the Schwarz's inequality

$$\left|\left\{\int_{-\infty}^{-T}+\int_{T}^{\infty}\right\}\right|du\left[\sum_{j,k=1}^{2}\int_{0}^{u}(g(x),\ y_j(x,\ u))\,dp_{jk}^{(a,b)}(u)(f(s),\ y_k(s,\ u))\right]\Big|^2$$

$$\leqq\left\{\int_{-\infty}^{-T}+\int_{T}^{\infty}\right\}du\left[\sum_{j,k=1}^{2}\int_{0}^{u}(g(x),\ y_j(x,\ u))\,dp_{jk}^{(a,b)}(u)(g(s),\ y_k(s,\ u))\right]$$

$$\times\left\{\int_{-\infty}^{-T}+\int_{T}^{\infty}\right\}du\left[\sum_{j,k=1}^{2}\int_{0}^{u}(f(x),\ y_j(x,\ u))\,dp_{j,k}^{(a,b)}(u)(f(s),\ y_k(s,\ u))\right].$$

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