ON THE ORIGIN OF THE ALGOL SYSTEMS

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1. Properties of the Algol Systems

The 4th edition of the Finding List for Observers of Eclipsing Variables (Koch *et al.*, 1963) contains 145 sufficiently well-observed eclipsing binaries brighter than $8{\cdot}5^{\mathrm{m}}$ at maximum light. Among them, 59 binaries, or 41%, are systems with both components on the main sequence. The second largest group, 52 binaries or 36% of all systems, are systems similar to Algol. These can be characterized as follows:

(1) The primary (more massive) components are main-sequence stars, fitting well into the mass-luminosity relation defined by visual binaries and by eclipsing binaries with both components on the main sequence (detached systems).

(2) The secondary components are of later spectral type than the primaries, and can be best characterized as subgiants. They are overluminous for their mass as well as for their spectral class.

(3) As a rule, the secondary components fill their respective critical Roche lobes (innermost Lagrangian surfaces).

These properties can be demonstrated on the accompanying Figures 1–3, based on our re-discussion of 46 Algol systems with sufficiently reliable data. Full circles refer to binaries in which both spectra are measurable for radial velocities; hence, the determination of their absolute dimensions is free from any assumptions. However, the data are usually not particularly accurate, and an attempt was made in our discussion (Plavec, 1967*b*) to estimate the uncertainty; therefore, in the figures, circles connected by a straight line refer to the same star. Open circles represent binaries in which only the spectrum of the primary was measured. In order to solve the mass function for the masses, an additional assumption is required. In our discussion, (Plavec, 1968*b*), an effort was made to apply two independent assumptions and to compare the results; again, the uncertainty is marked in the figures.

Figure 1 is a H–R diagram for the primary (more massive) components. The broken heavy line marks the locus of single stars of luminosity class V. A few primaries may be already somewhat evolved off the main sequence (as W UMi or SZ Psc), and the primary of RZ Sct is apparently a supergiant, but otherwise the primaries scatter along the luminosity class-V line.

Figure 2 is similarly a H–R diagram for the secondary components. Although a few of them are giants of luminosity class III, and others show practically no overlumi-

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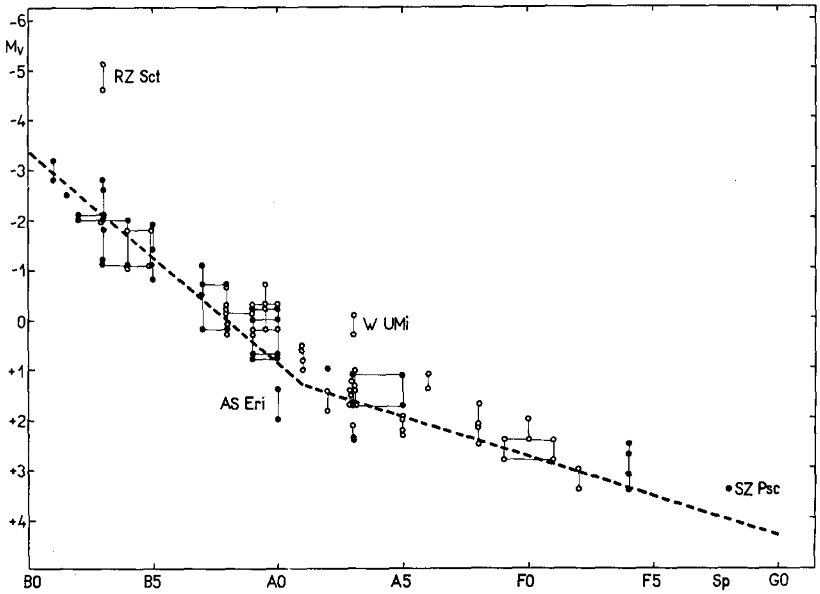

FIG. 1. *H-R diagram for the more massive components.*

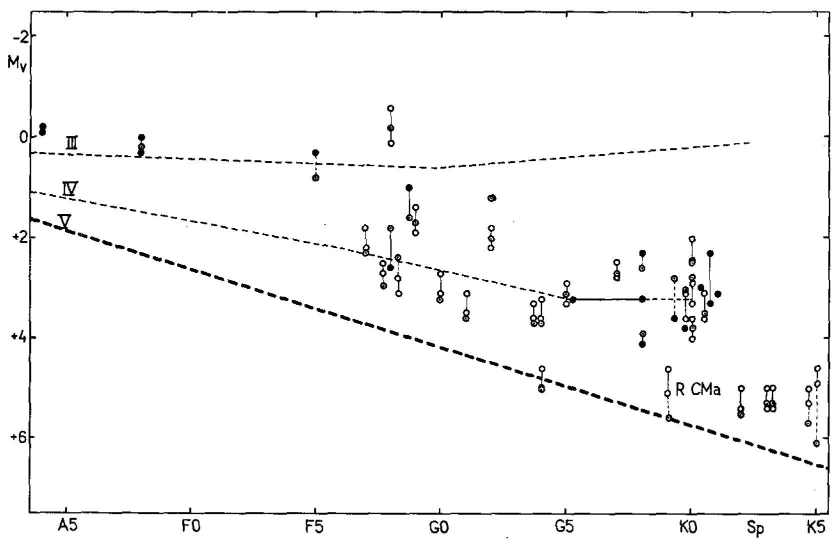

FIG. 2. *H-R diagram for the less massive components.*

nosity for their spectral types (but they may be strongly overluminous for their mass – the secondary of R CMa is an example), the typical secondary is a subgiant, greatly overluminous for its spectral type and mass as well.

Figure 3 shows the relation of the secondary components to the Roche limit. The

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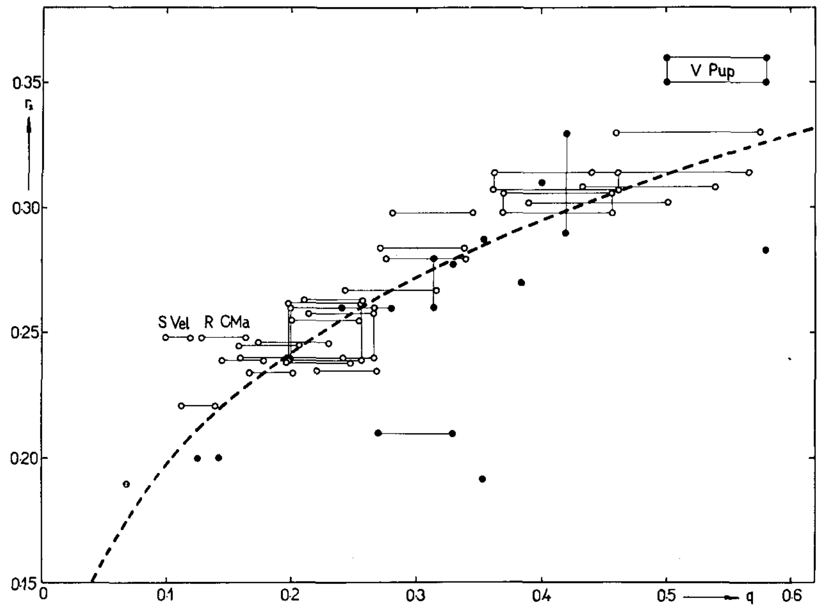

FIG. 3. *Relation of the secondary components to the Roche limit.*

fractional radius of the secondary star, r_2, is plotted against the mass ratio $q = m_2/m_1$. The broken heavy line represents the Roche limit as function of q (Kopal, 1954; Plavec and Kratochvíl, 1964). No star should lie to the left of this line. Actually there are several stars in this forbidden region, which poses an interesting problem discussed elsewhere (e.g., Plavec, 1968*b*). In general, however, most of the secondaries cluster around the Roche limit within the limits of observational errors. A few secondaries are definitely smaller than their respective critical Roche lobes; following Kopal, we speak of systems with undersize subgiants, while most of the binaries considered here are Kopal's semi-detached systems.

Now the important point is that the existence of these Algol-like binaries cannot be explained by the theory of evolution of single stars. According to it, the more massive star is bound to expand first and to develop into a giant. To be exact, the application of the theory of evolution of single stars does provide a possible explanation, but we shall see immediately that it can apply to a few systems only.

2. The Pre-Main-Sequence Contraction

I have in mind the cases where the secondary star is still in the phase of the pre-main-sequence contraction. In very young binaries, the evolutionary lag of the less massive component behind its mate means that its contraction is slower. Therefore, if the difference of the masses is large, the secondary may still be contracting when the primary is already on or near the main sequence. Therefore the radius of the secondary is relatively large, and under favourable conditions it may be comparable with that of the primary.

Such a binary has a relatively very high probability of being discovered as an eclipsing variable, since the fractional dimensions of the stars are large and comparable with each other, and the eclipses tend to produce deep minima. However, the duration of this phase is quite short. We shall see in Section 4 that the theory of large-scale mass exchange predicts another 'Algol' phase, photometrically almost equally favourable, but with a duration about 50 times longer. Our calculations indicate that the observed number of the young binaries to these transformed systems should be about 1:30.

Nevertheless it is worth while to look for the binaries with young contracting subgiants. However, the most probable combination is not a genuine semi-detached system with a contact subgiant, but a binary in which the secondary is already significantly smaller than its Roche limit – this is because towards its end the pre-main-sequence contraction is slower. Roxburgh (1966) suggested that KO Aquilae might be such a binary. However, the absolute dimensions of this system are very poorly known. I think that young contracting subgiants could be best identified spectroscopically.

3. Phases of Secular Expansion

It is evident that the existence of a great number of the semi-detached binaries similar to Algol can be explained only if we admit that the evolution of each component of the binary is greatly affected by the presence of the other star. In the theory I am presenting here it is assumed that the dominant role is played by the Roche limit,

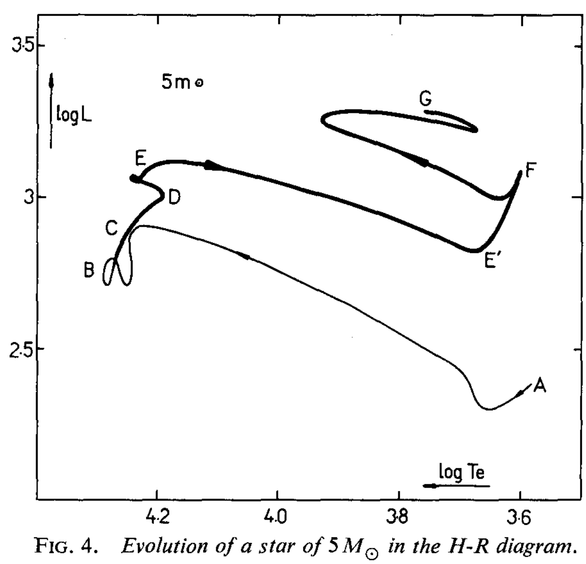

FIG. 4. *Evolution of a star of* $5 M_\odot$ *in the H-R diagram.*

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which represents the upper boundary to the permitted size of each component. Each component is expected to follow its normal course of evolution as long as it remains smaller than its Roche limit. When, however, its expansion due to the internal forces brings its surface to the Roche limit, the star begins to lose mass. Because the more massive stars develop faster, it is always the primary component that reaches the Roche limit first.

Figure 4 shows the evolution of a star of 5 $M_{\odot}$, according to Iben (1965), and Figure 5 shows the corresponding secular change of its radius. We recognize two phases of

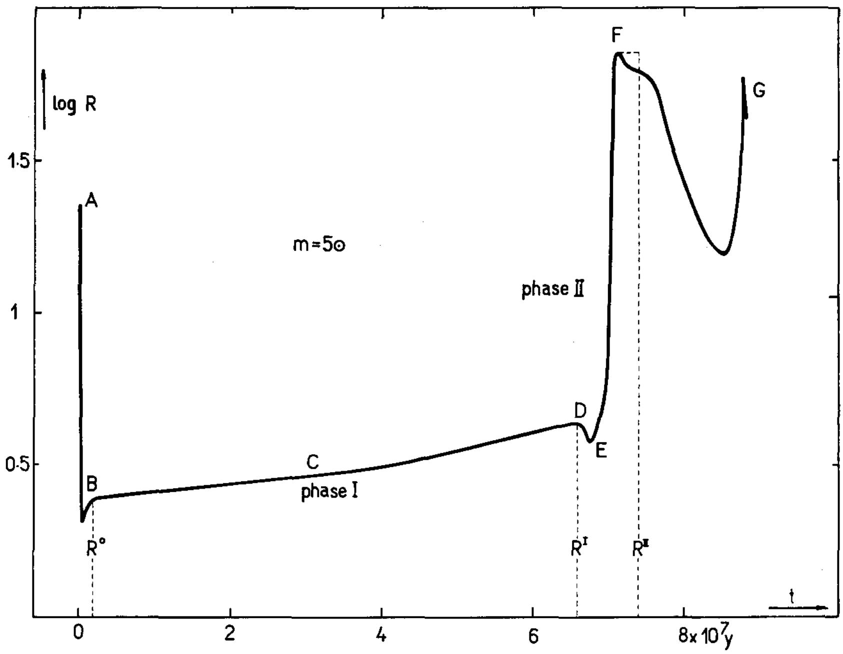

FIG. 5. *Secular change of radius with time for a 5 $M_{\odot}$ star.*

expansion, at which the star can attain the Roche limit: Phase I (or A, as Kippenhahn and Weigert call it) of slow expansion when the star burns hydrogen in its core and its envelope expands simultaneously as the convective core contracts; and Phase II (or B) when hydrogen is burning in the shell surrounding the helium-rich core, and the star expands rapidly, changing into a late-type giant. Certainly there are further phases of expansion in the later evolution of this giant, but these will not be considered here.

Whether and when the primary component fills its Roche limit, depends on the mass and distance of the secondary component. A more detailed discussion has been published elsewhere (Plavec, 1967*a*, 1968*a*). May it suffice here to say that the primary reaches the Roche limit in Phase I if the period of the system is shorter than P^{I}, given by the equation

$$\log P^{\mathrm{I}} = 0{\cdot}73 \log M_1 - 0{\cdot}86 - T(q). \tag{1}$$

Similarly, the primary component attains the Roche limit in Phase II if the period of the system fulfils the inequality

$$P^{\mathrm{I}} < P \leqslant P^{\mathrm{II}}, \tag{2}$$

where

$$\log P^{\mathrm{II}} = 2{\cdot}20 \log M_1 + 0{\cdot}04 - T(q). \tag{3}$$

Here the periods are expressed in days, M_1 is the mass of the primary component, and $T(q)$ is a slowly varying function of the mass ratio q; for the sake of a rough estimate, we can take $T(q) = -0{\cdot}45$.

From (1) it follows that in Phase I, the Roche limit is reached by the primary component in short-period systems only, since P^{I} is about 0·8 day for a primary of 3 $M_\odot$, and about 1·9 day for a primary of 9 $M_\odot$. Nevertheless, just these short-period systems appear to be the progenitors of many of the Algol systems.

4. The Mass Exchange

Crawford (1955) was the first to suggest that the semi-detached binaries are products of a large-scale mass transfer between the components. Rough model calculations by Morton (1960) and by Smak (1962) added a great support to this hypothesis, but I think I am correct to say that the definitive theoretical proof that such a process is physically possible was not afforded until in the model calculations by Kippenhahn and Weigert (1967). During the past year, further calculations illuminating the problem have been performed not only in Göttingen (Giannone *et al.*, 1968) but also in Warsaw (Paczyński, 1966, 1967) and by our group at Ondřejov (Plavec *et al.*, 1968*a*, *b*).

The work of all the three groups is based on the same principles. Rotational and tidal distortion, affecting only the outer layers of the stars, are neglected, and spherical symmetry is assumed as well as circular orbits.

All the material driven by the expanding star beyond the Roche limit is expected to flow rapidly to the other component and settle down on it. It is assumed that the total mass and the total amount of orbital angular momentum remain preserved. (This may not be the best assumption, but we have as yet little observational evidence contrary to this assumption.)

In this 'conservative case', the distance between the two components varies according to the law

$$A = A_0 \left(\frac{M_1^0 \cdot M_2^0}{M_1 \cdot M_2} \right)^2, \tag{4}$$

where the noughts refer to the values at the beginning of the mass exchange.

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The fractional size of the Roche limit has been tabulated (Kopal, 1954; Plavec, 1968*a*) or can be represented by the following approximate formula:

$$r_1^* = 0{\cdot}38 + 0{\cdot}2 \log(M_1/M_2); \tag{5}$$

the absolute mean radius of the Roche limit is then

$$R_1^* = A{\cdot}r_1^*. \tag{6}$$

When the mass exchange begins, both factors in (6) decrease, so that the Roche limit of the mass-losing star shrinks rapidly. However, Equation (4) shows that A reaches a minimum when the two stars are equal in mass, and then begins to increase again. Eventually, this increase more than compensates the monotonous decrease of r_1^*, and the absolute radius of the Roche limit begins to increase again.

This has serious consequences for the process of mass loss. Consider the beginning of this process. The expanding star has just reached the Roche limit and has lost a small fraction of its mass. This loss cannot affect perceptibly the deep layers, and the star tends to continue its expansion. If it were free to adjust itself to thermal equilibrium, it would have a larger radius. But the radius is imposed upon it in the form of the

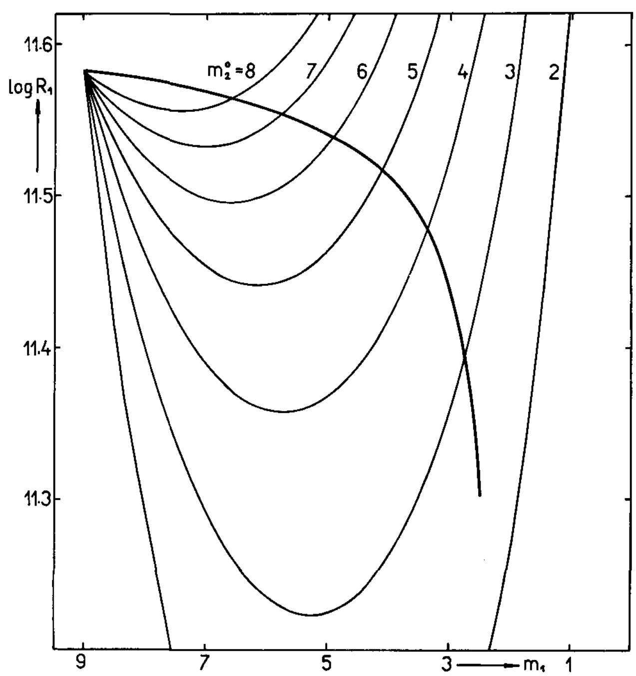

FIG. 6. *Change of the Roche limit during mass loss of a star with $M_1{}^0 = 9\ M_\odot$, for various initial masses of the secondary $M_2{}^0$. Abscissa is the instantaneous mass M_1.*

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Roche limit, and the Roche limit contracts instead of expanding. Therefore the star deviates strongly from thermal equilibrium, and the time-dependent terms in the equation for energetical balance play an important role. As usual under such conditions, the evolution is rapid – in our case it proceeds roughly on the Kelvin time-scale of the outer layers. The mass loss goes on, and is particularly rapid at the early phases, when the star's radius is forced to decrease so rapidly.

It is not difficult to understand that the rapidity of the mass loss depends also on the initial mass ratio. The greater is the initial disparity in masses, the more pronounced is the variation of the Roche radius. This can be seen from Figure 6, where we see how the size of the Roche limit of a star originally of 9 $M_{\odot}$ varies during the mass exchange in dependence on the original mass of the other star.

The actual computation of the evolutionary sequence of stellar models deviating from thermal equilibrium is made by means of Henyey's method for integrating the time-dependent equations of stellar structure (Henyey *et al.*, 1964). In our Ondřejov group, we have investigated several cases of mass exchange in Phase I, using this method modified by S. Kříž for this purpose. The necessity of a modification lies in the fact that we have one more unknown – the time interval Δt during which the star

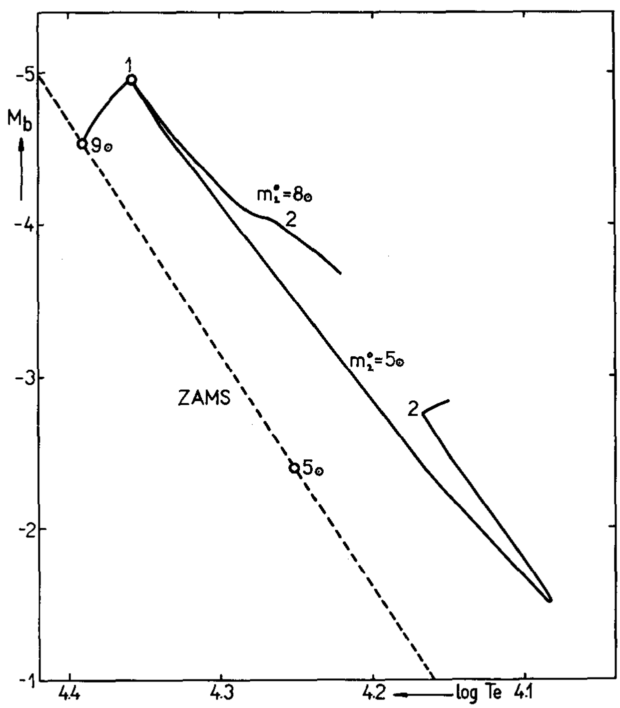

FIG. 7. *Theoretical H-R diagram shows the evolution of a star of* 9 $M_{\odot}$ *during mass loss. Mass exchange begins at the point* '1', *rapid mass loss ends at* '2', *slow mass loss follows.*

loses the mass ΔM – but we have also one additional equation of condition, namely Equation (6) which fixes the radius.

Figure 7 shows the evolution of a primary component of 9 $M_{\odot}$ in a system where the other component has a mass of 5 $M_{\odot}$ in the first case, and 8 $M_{\odot}$ in the second case. The primary was supposed to reach the Roche limit at point '1', where its central hydrogen content has dropped from $X_c = 0{\cdot}602$ to $X_c = 0{\cdot}25$. As soon as the mass loss begins, the star's luminosity decreases, because a part of the luminous flux is absorbed in the expanding outer layers. This process is much more conspicuous if the original masses are very different, because the radius decreases more rapidly.

The phase of rapid mass loss is relatively very short, 10^4–10^5 years. Figure 8 shows

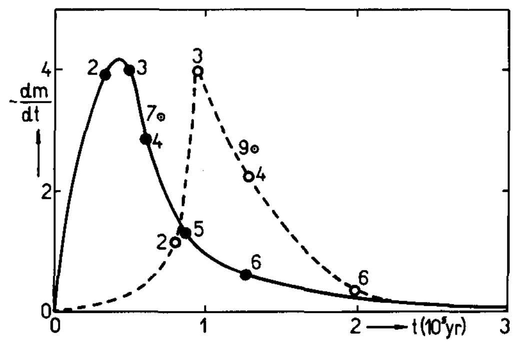

FIG. 8. *The rate of mass loss (in 10^{-5} solar masses per year): Full line, initial masses 7 $M_{\odot}$ + 5 $M_{\odot}$; dashed line, 9 $M_{\odot}$ + 8 $M_{\odot}$.*

the variation with time of the rate of mass loss for two configurations with the two stars initially not much different. The average rate of mass loss is 2×10^{-5} $M_{\odot}$/yr. For the combination 9 $M_{\odot}$ + 5 $M_{\odot}$, the rates are ten times higher.

A star of 9 $M_{\odot}$ loses, during the phase of rapid mass transfer, a total of 2·4 $M_{\odot}$ if the secondary has initially 8 $M_{\odot}$, and 5·2 $M_{\odot}$ if the secondary has 5 $M_{\odot}$. It has been found by our three groups that the final stage of the rapid mass loss can be determined approximately but much more easily by computing a series (not an evolutionary sequence) of stationary models with continuously decreasing mass. In Figure 6, the heavy line is the locus of such models in thermal equilibrium. Its intersection with the appropriate curve describing the change of the radius of the Roche limit indicates approximately the final mass of the mass-losing component at the end of the rapid mass transfer, when the star's thermal equilibrium is very nearly restored.

5. The Algol Stage

The final stage of the rapid mass loss in Phase I is a subgiant, overluminous for its mass and usually also for its spectral type. This subgiant continues to fill the critical

Roche lobe. Its evolution proceeds now again on the nuclear time-scale. With increasing chemical inhomogeneity in its interior, where hydrogen continues to be converted into helium, the core shrinks and the envelope expands slowly. This phase is analogous to the main-sequence slow expansion (Phase I in Figure 5), and its duration is of the same order. The star fills the Roche lobe but no rapid mass loss occurs, since simultaneously with the expanding star the radius of the Roche limit increases, too. This is because the mass-losing star is now the less massive component, and with continuing mass loss the disparity of masses increases and causes the distance A to increase, as follows from (4). Although the rate of mass loss is very low, the total amount of material lost in this stage is not negligible; e.g., according to Kippenhahn and Weigert (1967), in the system consisting initially of stars of 9 $M_\odot$ and 5 $M_\odot$, the new secondary loses another 0·7 $M_\odot$ within $1{\cdot}8 \times 10^7$ yr. This phase ends when the mass-losing star begins to contract and detaches itself from the Roche limit.

We have here a long stage when the system is photometrically quite prominent, when the subgiant fills the Roche limit, and when it is losing mass at a rate of about 10^{-8}–10^{-7} $M_\odot$/yr. This mass flow is probably sufficient to cause the observed phenomena of period changes and gaseous streams.

This is therefore the stage in which we observe the Algol systems. It is interesting to realize how much effort was devoted to the study of an evolutionary phase which appears to be only a fade-out of a much more violent and conspicuous process.

While this theory is, in my opinion, very promising in its qualitative explanation of the evolution of the Algol systems, I would not dare at this stage to attempt a quantitative comparison with an observed system. First of all, the models are still rather crude; the uncertainty introduced already in the models of the single stars by the assumptions about opacity, nuclear energy rate, composition etc., is augmented here by the simplifications mentioned in Section 4.

But, disregarding this, the number of free parameters is rather large. The final stage is not determined uniquely by the original masses. It is clear enough that the position of the heavy line representing the stationary models in Figure 6 will be different for different internal constitution of the mass-losing star. Since the phase of rapid mass loss is very fast, the chemical composition remains practically the same as at the beginning. Therefore when introducing e.g. X_c (abundance of hydrogen in the centre) as a parameter, we introduce in fact the age of the star. Figure 9 shows how the final product of the rapid mass loss depends on X_c.

The phase of slow mass loss is long and the evolution of the subgiant in it is not negligible – this is another complication. If we admit that part of the material leaves the system, we are introducing two more unknown parameters (Paczyński and Ziólkowski, 1967). But perhaps we shall be forced to make this assumption, since it is difficult to understand how the initial secondary can accommodate the huge amount of material carrying a large amount of angular momentum.

All we can do now is to establish which regions of the H-R and mass-luminosity

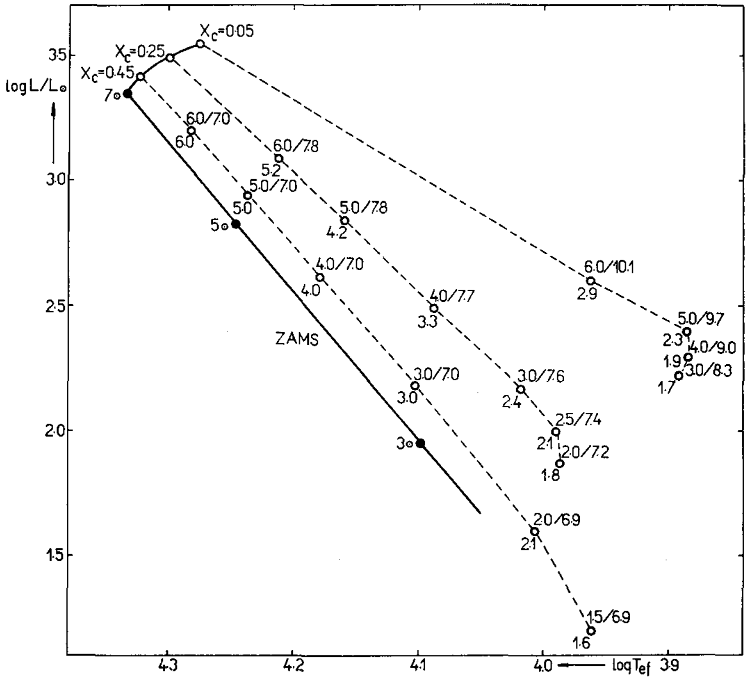

FIG. 9. *H-R diagram of the mass-losing components at the end of rapid mass transfer. Initial mass 7 $M_\odot$. Dashed lines are loci of stars of the same X_c. Lower numbers are final masses of the mass-losing components. Upper numbers are the corresponding initial and final mass of the mass-gaining component, respectively.*

diagrams are covered by the models based on our simple theory. This has been done by Giannone *et al.* (1968), who introduced the 'generalized main sequences', and in a somewhat different way by our Ondřejov group (Plavec *et al.*, 1968*b*).

A fact of great importance is that in each single case, the mass ratio is more than reversed, i.e. the roles of the components are always interchanged and the discrepancy in masses increased. As a consequence, the final period is always longer than the original one. Starting from this fact, and from our computations of the type shown in Figure 9, we made rough estimates of the initial parameters of the 46 Algol systems for which we have reliable data. These systems can be explained as products of mass exchange in Phase I, except perhaps for binaries with low masses; however, Kippenhahn *et al.* (1967) have shown that semi-detached systems with subgiants are produced also by primaries of low mass expanding in Phase II.

6. Observational Tests

Although a direct comparison of the theoretical models with observed binaries is evidently difficult, I believe that the theory can be tested by observation.

We have seen (cf. Figure 9) that the final mass of the subgiant is the smaller the smaller was the initial mass ratio. Now in the homogeneous model of a star of 7 $M_{\odot}$, the convective core contains a mass of 1·7 $M_{\odot}$. Within this mass, therefore, hydrogen is deficient and also other abundances are affected by the nuclear reactions. Now only this part of the original star is left at the end of the rapid mass exchange in binaries with a final mass ratio about 0·2; this result obtains roughly also for other initial masses of the primaries. This means that in the observed Algol systems with $q \leqslant 0{\cdot}2$, we should expect anomalous abundances (in particular, deficiency of hydrogen and overabundance of helium) in the atmospheres of both components. Such systems are e.g. AS Eri, DN Ori, AW Peg, XZ Sgr, R CMa, S Equ, and S Vel. The present primary components are naturally much easier to study, but in their atmospheres the effect may be obliterated by strong superficial currents which may develop when the material is falling on this star. It is our plan at Ondřejov to study this problem with our new 79″ telescope.

Another possibility is to catch a binary at the stage of rapid mass loss. This phase is short, but may be spectroscopically rather conspicuous, since the rate of mass loss in favourable cases is as high as 10^{-4} $M_{\odot}$/yr, which is only one order of magnitude smaller than a nova outburst.

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DISCUSSION

Koch: Have any of the listed binaries with small mass ratio, e.g. S Equulei, been studied for abundance anomalies?

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Plavec: Not so far I know. It is a difficult task, because for reliable abundance determinations you need spectra of high dispersion, and our stars are relatively faint.

Masevič: Could you indicate the regions in the H-R diagram where you expect the very fast mass loss to occur?

Plavec: I think I can refer to my Figure 5, which shows that the mass-losing components lie above the main sequence in the general region of the subgiants or giants.

Abhyankar: Have you considered the evolution of the component which is gaining mass?

Plavec: Not yet, but certainly this is a very important problem. We have postulated tacitly that the mass-gaining component is capable of accumulating a huge amount of material, often equal to several solar masses, within about the same time as the primary loses it. It is questionable whether such a 'conservative case' of mass transfer is possible. We must bear in mind that the flowing material carries with it a great amount of angular momentum, which could make the accommodation difficult, since the mass-gaining star may get on the verge of rotational break-up. Personally I consider this problem the weakest point in the whole theory of mass-exchange, and its solution should not be delayed. The result may easily be that we must admit a certain mass loss from the whole system.

Weigert: Dr. Paczyński has really calculated several cases where a mass loss of the whole system was assumed. When we normally do not take into account a mass loss from the system, we do not think that this is the normal case in reality. We only neglect it since we would have to introduce unknown parameters which can change the result to either side. We only wish to see which systems can be explained with the most simple assumptions.

Paczyński: The Warsaw group (B. Paczyński and J. Ziółkowski, *Acta astr.*, **17**, 1967, 7) computed a number of evolutionary tracks for close binaries with arbitrarily chosen rates for the mass- and angular momentum losses. In all these cases the final mass ratio was closer to unity because not all the mass lost by one component was captured by the other star. I think this is in better agreement with the observed mass ratios.

Mammano: The new Bamberg variable BV 412 may be an example of systems in which the more massive component is in contact with its Roche lobe, while the companion is well detached. However, the only anomaly we can see is that the K line is unusually strong for the A0 spectral type, and its intensity varies with phase (A. Mammano, R. Margoni, R. Stagni, *Mem. S.A. It.*, **38**, 1967, in press).