embeds into the Turing degrees. We will show that in a certain extension of ZF (which is incompatible with ZFC), this holds for all partial orders of height two, but *not* for all partial orders of height three. Our proof also yields an analogous result for Borel partial orders and Borel embeddings in ZF, which shows that the Borel version of Sacks' question has a negative answer.

We will end the thesis with a list of open questions related to Martin's conjecture, which we hope will stimulate further research.

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JUSTIN MILLER, ***Intrinsic density, asymptotic computability, and stochasticity***, University of Notre Dame, Notre Dame, IN, USA, 2021. Supervised by Peter Cholak. MSC: Primary 03D32, 03D30. Keywords: Intrinsic density, asymptotic computation, stochasticity, randomness.

Abstract

There are many computational problems which are generally "easy" to solve but have certain rare examples which are much more difficult to solve. One approach to studying these problems is to ignore the difficult edge cases. Asymptotic computability is one of the formal tools that uses this approach to study these problems. Asymptotically computable sets can be thought of as almost computable sets, however every set is computationally equivalent to an almost computable set. Intrinsic density was introduced as a way to get around this unsettling fact, and which will be our main focus.

Of particular interest for the first half of this dissertation are the intrinsically small sets, the sets of intrinsic density 0. While the bulk of the existing work concerning intrinsic density was focused on these sets, there were still many questions left unanswered. The first half of this dissertation answers some of these questions. We proved some useful closure properties for the intrinsically small sets and applied them to prove separations for the intrinsic variants of asymptotic computability. We also completely separated hyperimmunity and intrinsic smallness in the Turing degrees and resolved some open questions regarding the relativization of intrinsic density.

For the second half of this dissertation, we turned our attention to the study of intermediate intrinsic density. We developed a calculus using noncomputable coding operations to construct examples of sets with intermediate intrinsic density. For almost all $r \in (0,1)$, this construction yielded the first known example of a set with intrinsic density r which cannot compute a set random with respect to the r-Bernoulli measure. Motivated by the fact that intrinsic density coincides with the notion of injection stochasticity, we applied these techniques to study the structure of the more well-known notion of MWC-stochasticity.

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CHENG PENG, ***On Transfinite Levels of the Ershov Hierarchy***, National University of Singapore, Singapore, 2018. Supervised by Yue Yang. MSC: 03D28, 03D55. Keywords: Turing degree, Ershov hierarchy.

Abstract

In this thesis, we study Turing degrees in the context of classical recursion theory. What we are interested in is the partially ordered structures $\mathcal{D}_\alpha$ for ordinals $\alpha < \omega^2$ and $\mathcal{D}_a$ for notations $a \in \mathcal{O}$ with $|a|_o \geq \omega^2$.

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