differential operators (Springer-Verlag, Berlin 1983–85) [ALPDO] and *An introduction to the complex analysis in several variables* (North-Holland, Amsterdam 1990) [CASV]. The book under review overlaps slightly with both of these books and depends in places on proofs in [ALPDO], though the full definitions and many examples are given here.

Convexity in the title of this book is to be interpreted in a wide sense, particularly pertaining to differential operators. This viewpoint starts by observing that affine functions f from $\mathbb{R}^n$ to $\mathbb{R}$ are the solutions of $\frac{d^2f}{dx^2}=0$ and that harmonic functions F from $\mathbb{R}^n$ to $\mathbb{R}$ are solutions of $\sum \frac{\partial^2 F}{\partial x_i^2}=0$. A convex function on $\mathbb{R}$ is a function whose graph lies below the chord between any two points. This compares a convex function with the affine functions on any interval that dominate the function on the boundary of the interval. A function F from $\mathbb{R}^n$ to $\mathbb{R}$ is subharmonic if for each compact subset K and each harmonic function h on K with $h \geq F$ on the boundary ∂K the inequality $h \geq F$ in K holds. Of course there are continuity and domain conditions omitted from the simplified description above.

The basic results and examples of convex functions and convex sets are thoroughly developed for finite-dimensional spaces with examples and some applications. For example, the inequality of Fenchel and Alexandrov on mixed volumes of convex subsets of $\mathbb{R}^n$ is proved and used to deduce the Brunn–Minkowksi inequality. Similar care and detail are used in the discussion of subharmonic functions and plurisubharmonic functions from an open set X contained in $\mathbb{R}^n$ into $[-\infty, \infty)$. Representations, inequalities and exceptional sets are covered. In the fifth chapter he discusses G-subharmonic functions, where G is a subgroup of $GL_n(\mathbb{R})$, and shows that convex, subharmonic and plurisubharmonic functions correspond to the full, the orthogonal and complex linear groups respectively.

The book is well written with good examples and frequent discussion of the case $n=2$ to help the reader. For the full discussion on subharmonic and plurisubharmonic functions it is essential to have [ALPDO] and [CASV] at hand. This book should be in every mathematics library.

A. M. SINCLAIR

MORTON, K. W. and MAYERS, D. F. *Numerical solution of partial differential equations* (Cambridge University Press, Cambridge 1994), 227 pp., hardcover: 0 521 41855 0, £35.00, paperback: 0 5214 2922 6, £13.95.

Partial differential equations (PDES) are the most widely used models for many scientific, engineering and economic problems. Unfortunately most PDES of any practical interest cannot easily be solved analytically due to nonlinearities and complex geometric domains. The only recourse to solve these problems is by numerical methods.

This book, which can be seen as an introduction and a complement to the classical text of Richtmyer and Morton (1967), is a self-contained account of the theory and applications of mainly finite difference methods to the numerical solution of PDES. The book is aimed at final year undergraduate and first year postgraduate mathematics students but could easily be used with engineering and computer science students with a rudimentary knowledge of PDES.

Parabolic equations in one dimension are introduced first and through the use of a discrete maximum principle a simple error analysis is given. The extensions of these methods to two and three-dimensional and nonlinear problems are discussed. Hyperbolic problems are treated extensively and here discrete Fourier analysis is used not only for deducing necessary stability conditions but also to analyse phase and amplitude errors. At this point the authors formalise the notions of consistency, convergence and stability for linear initial boundary value problems. A readable account of what is often a confusing area for students is given, including Lax's equivalence theorem uniting the concepts of consistency, stability and convergence. The main difficulty with establishing convergence is a proof of stability. Necessary conditions can be obtained using Fourier analysis and sufficient conditions using energy methods. The use of both methods is illustrated by a number of examples.

Finally, the discretisation of elliptic equations is considered along with a discussion on the treatment of curved boundaries. Here a discrete maximum principle technique is introduced to establish numerical stability and then convergence. It is with elliptic equations that the finite element method holds sway and here we find a brief account of this important method. No discussion of elliptic and implicit methods would be complete without mention of the structure and solution techniques of the linear algebraic systems which arise in the solution process. The final chapter deals with iterative techniques for the solution of these equations including the Jacobi, Gauss–Seidel and SOR methods. Fourier analysis again is the main analytical tool, used this time to establish convergence rates and optimal relaxation parameters.

Overall the book is very readable through the use of numerical examples and numerous illustrations. Exercises of various degrees of difficulty are given at the end of each chapter along with directions to further references.

The book would be of use to engineers and scientists requiring an introduction to numerical methods. Realistic applications to complex domains would however require a discussion of the use of generalised coordinates or finite volume methods. The strength of the book however is the rigorous analysis of many of the methods which would be taught to final honours year and MSc level mathematicians.

J. A. MACKENZIE

ROMAN, S. *Field theory* (Graduate Texts in Mathematics, Vol. 158, Springer-Verlag Berlin–Heidelberg–New York–London–Paris–Tokyo–Hong Kong 1995), 272 pp., softcover 0 387 94408 7, £21.00

This is a carefully judged and beautifully written account of what is of course a beautiful subject, pitched at first year graduate level. The exposition is succinct and expects the appropriate level of mathematical maturity, yet always gives all that could be desired in the way of motivational commentary, illustrative examples and the drawing of fine distinctions where necessary. There is a very full range of useful exercises for each chapter. Altogether, the author is to be congratulated on an excellent piece of work: this book will act as a model for beginning research students as to what they should aim for in the way of elegance of argument and exposition; it is also an effective teaching tool and a useful reference.

As to tone and contents the book goes considerably further than the usual undergraduate course in Galois theory and, where standard undergraduate topics are covered, the exposition is more sophisticated and demanding; thus the book acts as a useful revision of any topics that are already known and indicates the new level of maturity needed for graduate study. On the other hand it does not go as far as graduate texts such as D. Winter's *Structure of fields* (Graduate Texts in Mathematics, Vol 16, Springer-Verlag, 1974) or G. Karpilovsky's *Field theory: classical foundations and multiplicative groups* (Marcel Dekker, 1988) in that it contains no cohomology or valuation theory, nor does it treat algebraic function fields; it does however have two very useful chapters on finite fields, going beyond what these older books contain, perhaps reflecting the new importance of this topic in the light of links with coding and signal theory. Certainly it provides an excellent background from which to proceed to more advanced topics such as those already mentioned.

After opening chapters on preliminaries and basic theory the first part of the book discusses field extensions, algebraic independence and separability. Part Two, on Galois theory, uses so-called 'indexed Galois Correspondences' as an organisation tool, treats Linear Disjointness and the Krull Topology and calculates in detail the Galois groups of some small degree polynomials; norms and traces are also considered as well as normal bases. Next, finite fields are considered in some detail; in particular normal bases are exhibited, Steinitz numbers are used to describe