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EDITORIAL ANNOUNCEMENT

Editorial "Symmetries and differential equations"

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This special issue celebrates the remarkable research career of George W. Bluman, following his recent retirement as a Professor from the Mathematics Department at University of British Columbia. In 1967 at the California Institute of Technology, George completed his PhD thesis, "Construction of Solutions to Partial Differential Equations by the Use of Transformation Groups". Since then, this subject and the broader field of symmetry methods applied to differential equations have been his main academic interest. Outside of research and teaching are his activities in university administration, in school mathematics education, and in Jewish Canadian culture and history (his parents were saved by the heroic Japanese consul Chiune Sempo Sugihara during the second world war).

Among George's approximately 100 scientific publications, there are well-recognized and pioneering advances in the theory and applications of Lie symmetry reduction methods, their extensions to non-classical symmetries and potential symmetries, and the construction of associated conservation laws. In addition, George has written four books on these subjects, and each of them has been a must-read item in the education of several generations of early-career researchers, including 20 of his own graduate students, many close collaborators, and many more interested students and researchers around the world.

This special issue called for contributions within these aforementioned areas of interest by his research colleagues (*) and leading researchers.

Symmetries are an intrinsic (coordinate-free) feature of a given differential equation.

They can be computed in an algorithmic way and provide a powerful tool for many purposes:

- finding exact solutions;
- deriving conservation laws by Noether's theorem;
- constructing mappings of nonlinear equations into linear equations;
- detecting integrability properties;
- finding invariants and differential invariants; and
- formulating discretizations with good properties.

By now, Lie symmetries have been applied to a huge variety of differential equations across many different fields in the scientific and mathematics literature.

The paper by Praturi, Plumacher, and Oberlack investigates the Lie point symmetries of the equations governing a statistical description of turbulence in compressible fluid flow, which is a relatively new and promising area of application. These symmetries are found to comprise the well-known Lie point symmetries admitted by the Euler equations of compressible fluid flow, as well as certain statistical symmetries which are related to measures of intermittency and translation-invariance of moments seen in turbulent flows.

Olver's paper studies a symmetry problem arising from image processing. In this application, the differential invariants of the projective group acting on functions defined on the two-dimensional projective plane are central to methods of image recognition. The invariants are constructed and completely classified by the use of the method of equivariant moving frames.

The paper by Koval, Bihlo∗ and Popovych carries out an extended symmetry analysis of a maximally symmetric Fokker–Planck equation – also called the Kolmogorov equation – in three independent variables. They compute all point symmetries, both continuous and discrete, and determine all inequivalent one- and two-dimensional subalgebras together with a classification of Lie symmetry reductions. These results are applied to construct wide families of exact solutions.

In the paper by King, Richardson and Foster, they study the dynamics of interfaces in a family of slow-diffusion equations with strong absorption. For a given equation, self-similar solutions of the equation and of its asymptotic limits play a central role in the analysis of the dynamics. Different types of behaviour include changes in direction of propagation, detaching from an absorbing obstacle, formation of two interfaces by film rupture, and extinction.

Extensions and generalisations of Lie symmetries, in particular the non-classical method, are very fruitful and have been used in many applications.

The paper by Boadbridge, Cherniha and Goard describes a class of non-linear reaction-diffusion equations in two spatial dimensions having a non-classical symmetry which leads to a reduction of each equation in the class to a system of two linear equations. This system is used to construct exact time-dependent solutions. Applications to a predator–prey model of Lotka-Volterra type are discussed.

The paper by Tarayrah, Pitzel and Cheviakov∗ compares two widely used frameworks for defining approximate symmetries. Through two examples of non-linear families of wave equations, they show that the frameworks yield different approximate symmetries and have different stability properties. The application of approximate symmetries to constructing approximately invariant solutions is illustrated for a physical example of waves in a fibre-reinforced hyper-elastic solid.

The problem of finding conservation laws of a differential equation when Noether's theorem is inapplicable has been extensively studied from the viewpoint of multipliers. This has led to a modern generalisation in which multipliers satisfy the adjoint of the conditions defining variational symmetries, without the need or use of a Lagrangian.

Anco's∗ two papers undertake an extensive study of adjoint symmetries (sometimes called cosymmetries) which are a generalisation of conservation law multipliers, in analogy with how symmetries can be viewed as generalising variational symmetries. For any system of differential equations, there is shown to be three different actions by infinitesimal symmetries on the linear space of adjoint symmetries, and these actions are used to construct an associated bilinear Lie bracket for adjoint symmetries as well as a pre-symplectic (Noether) operator that maps infinitesimal symmetries into adjoint-symmetries. Applications to a variety of nonlinear systems of PDEs are presented.

The paper by Hydon revisits the long-studied problem of inverting the total divergence operator, which is essential for obtaining the components of a conservation law from multipliers. A new, efficient method utilising partial Euler operators and partial scalings to obtain a line integral formula is presented.

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