action for that claim. The question is, does he not require usurious interest of B?

"Now, calculation on Mr. Milne's hypothesis gives that B in the subsequent agreement pays A, at the expiration of the first half year, £102. 10*s*., and that A then lends to C £102. 10*s*., at £2. 10*s*. per cent. interest for the half year, which is required to complete the whole year: for were he only to lend him £100, A would leave £2. 10*s*. lying idle; and consequently, at the expiration of the year from the original term, C will have to pay A £105. 1*s*. 3*d*., being 1*s*. 3*d*. beyond the legal interest for which the action is brought against B. But on D'Alembert's hypothesis, B will have had only to pay at the expiration of the first half year £$100 \times \overline{1{\cdot}05}^{\frac{1}{2}}$; and C afterwards, for the continuance of the loan to him, and for which B is bound, $\overline{100} \times \overline{1{\cdot}05}^{\frac{1}{2}} \times \overline{1{\cdot}5}^{\frac{1}{2}} =$ £105.

"Should these remarks tend to lessen the difficulty attaching to the case, I should be glad; but if I should be required to reply to objections which may be made to them, I feel that I should be obliged to decline doing so, and will hope that I should not be thought discourteous on that account.

"Yours truly,

"Benj. Gompertz.

"152, King's Road, Brighton,
"3 Nov., 1853."

CALCULATION OF THE ODDS OF THROWING ANY SPECIFIED NUMBER WITH TWO, THREE, FOUR, OR MORE DICE.

To the Editor of the Assurance Magazine.

Sir,—Some persons have supposed that the doctrine of probability rather fosters than discourages habits of gambling. No doubt the error of such a supposition arises from the known facility with which its principles can be applied to games at cards and dice. It has, however, been employed to expose the nefarious practices of many, who have developed very alluring though dishonest and fatal schemes for realizing money; and through the authority and influence of your *Journal*, the science of probability might be turned to some account in exposing those pernicious practices that are of nightly occurrence in many establishments in London, especially at the West End. The uninitiated and unwary, who seek amusement in these dens of infamy, might at all events be put on their guard, by having in their possession the *true odds* in every case where betting is resorted to on games of chance; and at the same time the usefulness and importance of your *Magazine* would be considerably augmented. With this view I have made the following calculations; and at a future time, I may direct my attention to other forms and shapes under which this insidious and dangerous practice presents itself.

When the throwing is with two dice, that are homogeneous and dynamically accurate (which is *never* the case in gambling houses), the probability of throwing either of the numbers

3 or 11 is $\frac{2}{36}$; the odds against are 17 to 1

4 or 10 is $\frac{3}{36}$ „ „ 11 to 1

5 or 9 is	$\frac{4}{36}$;	the odds against are	8 to 1
6 or 8 is	$\frac{5}{36}$	,, ,,	31 to 5
7 is	$\frac{6}{36}$	,, ,,	5 to 1

When the throwing is with three dice, the probability of throwing either of the numbers

4 or 17 is	$\frac{3}{216}$;	the odds against are	71 to 1
5 or 16 is	$\frac{6}{216}$	,, ,,	35 to 1
6 or 15 is	$\frac{10}{216}$	,, ,,	103 to 5
7 or 14 is	$\frac{15}{216}$	,, ,,	67 to 5
8 or 13 is	$\frac{21}{216}$	,, ,,	65 to 7
9 or 12 is	$\frac{25}{216}$	,, ,,	191 to 25
10 or 11 is	$\frac{27}{216}$	,, ,,	7 to 1

When the throwing is with four dice, the probability of throwing either of the numbers

5 or 23 is	$\frac{4}{1296}$;	the odds against are	323 to 1
6 or 22 is	$\frac{10}{1296}$	,, ,,	643 to 5
7 or 21 is	$\frac{20}{1296}$	,, ,,	319 to 5
8 or 20 is	$\frac{35}{1296}$	,, ,,	1261 to 35
9 or 19 is	$\frac{56}{1296}$	,, ,,	155 to 7
10 or 18 is	$\frac{80}{1296}$	,, ,,	76 to 5
11 or 17 is	$\frac{104}{1296}$	,, ,,	149 to 13
12 or 16 is	$\frac{125}{1296}$	,, ,,	1171 to 125
13 or 15 is	$\frac{140}{1296}$	,, ,,	289 to 35
14 is	$\frac{146}{1296}$	,, ,,	575 to 73

In the preceding calculations, I have used the following well known process:—

If p denote the number of dice, n any particular number to be thrown, then 6^p=the whole number of combinations; and the different ways in which n can be thrown is the number of combinations in which $a+b+c+$, &c. p terms can be made equal to n; the several numbers from 1 to 6 being successively substituted for a, b, c, &c. This will be the same as if we raise $(x+x^2+x^3+x^4+x^5+x^6)$ to the pth power, and determine the coefficient of x^n, which may readily be done as follows:—

$$(x+x^2+\ldots\ldots x^6)^p=x^p\left(\frac{1-x^6}{1-x}\right)^p=x^p(1-x^6)^p(1-x)^{-p}$$

giving to p any of the values 2, 3, 4, &c., and expanding and performing the multiplication. Any coefficient in the resulting product, divided by 6^p, will denote the probability of throwing the number which is the index of x, the term to which the coefficient belongs.

Your obedient Servant,

GEO. SCOTT, A.I.A.

Fortescue House, Twickenham,
Dec. 16*th*, 1853.

DETERMINATION OF SURPLUS.

To the Editor of the Assurance Magazine.

Sir,—To ascertain the sum which a Society may safely appropriate as a bonus being one of the most momentous problems that can fall within the scope of an actuary's duties, I may perhaps be permitted to offer a few observations on the subject.

I will suppose, then, that a mutual Society has been in existence five years, and that the amount of pure divisible surplus is sought, with a view to the declaration of a bonus. After payment of the preliminary expenses, or those attendant upon the formation of the Society, the cost of management, and the claims on account of deaths, the sum s remains to credit of the Company. The present value of the future *gross* and *net* premiums on the existing policies $=$V and v respectively, that of the policies themselves being v'. The working expenses hitherto average e per annum; and n policies on an equality have been issued yearly. Now $V-v'+s$ cannot be called actual surplus, since no allowance is made for future expenses, which must necessarily be incurred before the profits on the future premiums (of which $V-v$ is the present value) can be realized. To estimate this important deduction, we can but proceed upon the experience of the past; if therefore A be the *net* premiums receivable annually on the policies issued, $\frac{v}{A}$ will denote the average number of years these policies have to run, and the working expenses during such period $=\frac{ve}{A}$. Now we may reasonably assume, if nothing be known to the contrary, that in this time $\frac{nv}{A}$ new policies will be granted, so that the fair proportion of the sum $\frac{ve}{A}$ which

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