ON THE ESTABLISHMENT OF TERRESTRIAL COORDINATE SYSTEM BY CLASSICAL TECHNIQUES

Ya. S. Yatskiv

Main Astronomical Observatory,
Ukrainian Academy of Sciences,
Kiev, U.S.S.R.

ABSTRACT

The definition and practical realization of the astronomical system of terrestrial coordinates are discussed. In particular, the separation of secular polar motion and nonpolar drift of observatories are touched upon as far as those phenomena are crucially intertwined with the realization of a terrestrial coordinate system.

1. INTRODUCTION

Many different definitions and practical realizations of the terrestrial coordinate systems were discussed in the First Colloquium "On Reference Coordinate System for Earth Dynamics" held in Torun in 1974. To avoid repetition, this paper will be limited to a brief summary of the current state of establishing the astronomical system of terrestrial coordinates by classical techniques. The term "classical techniques" is used to refer to optical observations of stars by means of zenith telescopes, photographic zenith tubes, etc. The separation of the secular polar motion and the nonpolar drift of observatories; and the method for maintaining the conventional coordinate system, will also be discussed.

2. ON THE DEFINITION OF THE ASTRONOMICAL SYSTEM OF TERRESTRIAL COORDINATES

All classical techniques measure the orientation of reference directions on the Earth, i.e., plumb lines of observatories or directions to visible reference marks, with respect to the directions from the observer to stars, assuming the latter are known in a nonrotating celestial coordinate system, $(X) = (X_1, X_2, X_3)^T$.

E. M. Gaposchkin and B. Kołaczek (eds.), Reference Coordinate Systems for Earth Dynamics, 155–164.

https://doi.org/10.1017/S0252921100081203 Published online by Cambridge University Press

Since distances are not measured these techniques enable only the directions of the axes of a terrestrial coordinate system to be determined.

For geometrical representation of the results by classical techniques one takes the auxiliary unit sphere and draws from an arbitrary centre the unit vectors, e_i, parallel to the plumb lines of observatories. A vector e_i defines the position of the zenith, Z_i, on the sphere. The usual procedure for expressing position on the sphere is a specification of the directions e_i in terms of arcs on the sphere that measure angular distances from selected cardinal directions or circles. The astronomical latitude ϕ fixes the position of a point in the local meridian plane, which passes through the direction e_i and is parallel to the axis of the Earth's rotation. ϕ is reckoned from the astronomical equator. The position of the plane itself is specified by the angle that it makes with the meridian plane through an arbitrarily chosen reference point on the sphere. According to a resolution of the Washington Conference in 1884 the meridian plane through the Airy Transit Circle of the Greenwich Observatory was for many years adopted as the initial reference plane.

This formal geometrical definition of the astronomical system of terrestrial coordinates does not depend upon any interpretation of directly observed phenomena which determine the positions of the cardinal point and circle on the sphere. Really, the basis of any system of terrestrial coordinates, $(x) = (x_1,x_2,x_3)^T$, is essentially empirical. The system (x) is attached to the observed vectors e_i, or to corresponding zenith positions, z_i. The minimum number of constraints sufficient to define the system uniquely is three. Therefore, if we have more than two vectors e_i, this attachment cannot be arbitrarily realized. To tie down the three rotational degrees of freedom of the system, a certain condition must be imposed, e.g., the minimization of the squares of the zenith displacements from their initial positions in the system (x), (Fedorov et al., 1972). Such a system is called the conventional terrestrial coordinate system (Fedorov, 1979). Where high precision of latitude and time determinations is required it is necessary to specify explicitly the directions of the third axis of the system (x), and the reference point for reckoning longitude. According to the resolutions of the 13th General Assembly of the IAU and the 14th General Assembly of the IUGG the x_3-axis points to the CIO, which is defined by the conventional values of latitudes, ϕ_{oi}, of five ILS stations. The longitude is now referred to the so-called "mean observatory" as defined by the BIH. It was defined at an initial epoch as the point where the meridian through Greenwich and the CIO cuts the equator of the CIO. There was no authoritative approval of such a fiducial point for longitude.

The rotation of the Earth is represented by the motion of the system (x) with respect to the nonrotating celestial system (X). This relative motion is not exactly predictable because of irregularities in the Earth's rotation due to unknown exitations. For observational purposes it is convenient to introduce an intermediate system $(\xi) = (\xi_1,\xi_2,\xi_3)^T$ whose rotation approximates as closely as possible that of the system (x) and at the same time is precisely predictable. Such a system is called the ephemeris terrestrial system (Fedorov, 1979). The selection of the ξ_3-axis of the ephemeris terrestrial system has been discussed by the IAU Working Group on Nutation in connection with the specification of the new set of nutational coefficients (Seidelmann et al., 1979). The proposed ξ_3-axis points to the "Celestial Ephemeris Pole", i.e., to the point that has no nearly diurnal motion with respect to either the nonrotating, (X), or terrestrial, (x), coordinate system. Two other axes rotate about the ξ_3-axis with "ephemeris angular velocity" specified by conventional formulae for the angle between the ξ_3-axis and the first axis of the nonrotating coordinate system, (X). In current practice, the latter is directed to the mean equinox. Recently, Guinot (1979) has proposed a new choice of fiducial point on the equator, the so-called Non-Rotating Reference Origin. The difference between these two reference points is given by the precession in right ascension. Classical astronomical latitude and time observations enable the directions of the axes of the system (ξ) to be determined with respect to the system (x).

3. REALIZATION OF ASTRONOMICAL SYSTEM OF TERRESTRIAL COORDINATES

The astronomical system of terrestrial coordinates should be adequate for representation of the positions of zeniths on the auxiliary sphere and rotation of the Earth as a whole. The realization of any reference system consists of two stages:

1) Determination of initial longitudes and latitudes of a number of observatories based on observations over some convenient time interval;

2) Choice of the procedure for maintaining the system.

Different systems of terrestrial coordinates of particular interest in the study of the Earth's rotation have been realized. Some of them are in widespread current use in astronomy and geodesy:

MPO - The mean pole origin defined by the mean latitudes of observatories at any moment. For determining the mean latitude different methods may be used, such as the well-known Orlov's method.

CIO - The Conventional International Origin.

https://doi.org/10.1017/S0252921100081203 Published online by Cambridge University Press

BIH - The reference system adopted by the BIH and referred to the epoch of 1968.

BIH 1979 - The same as BIH 1968 except for conventional corrections added to the coordinates of the pole and to UT1.

In addition to above astronomical systems of terrestrial coordinates the following ones are discussed:

ST(SU) 1965 - The reference system adopted by Gubanov and Yagudin (1979) for redetermining the UT scale of the USSR for 1955-1974.

UT(SU) 1975 - The new reference system adopted by the USSR Time Service in 1975 (Belotserkovskij and Kaufman, 1979).

Some information concerning the observational data and the primary reference system used for realization of the above reference systems are given in Table 1.

The BIH 1968 system was realized by requiring coincidence of the BIH and CIO poles in 1968 as well as continuity of UT1 with previous values. After the initial adjustment of 1968, the BIH system was kept independent of the ILS results. On the contrary, ST(SU) 1965 system and the UT(SU) 1975 system have been adjusted to that of the BIH only in the average the time intervals of 1962.0 - 1975.0 and 1957.0 - 1971.0, respectively. The merit of any reference system depends not only upon the accuracy of classical techniques, but also upon precise specification of fundamental constants, computational procedures, etc. The comparison of polar coordinates and UT1 obtained in different systems can yield information on the accuracy of the systems under consideration.

Table 1. Astronomical systems of terrestrial coordinates

Name of System	Number of Observatories	Number of Instruments		Interval of Observations	Primary Reference System
		Time	Latitude		
MPO	all	-	all	more than 1.6 yr	instrumental
CIO	5	-	5	1900.0-1906.0	instrumental
BIH 1968	51	48	39	1964.0-1967.0 1967.0-1968.0	CIO
ST(SU) 1965	15	29	-	1955.0-1975.0	BIH 1968
UT(SU) 1975	5	8	-	1957.0-1971.0 1974.0-1975.0	BIH 1968

https://doi.org/10.1017/S0252921100081203 Published online by Cambridge University Press

4. SECULAR POLAR MOTION AND NONPOLAR DRIFT OF OBSERVATORIES

The separation of secular polar motion and displacements of individual observatories are crucially intertwined with the realization of a terrestrial coordinate system. The existence of the secular and long-period variations of the mean longitudes and latitudes of observatories has been proved by many authors. On the other hand, many determinations of the motion of mean pole of the epoch of observation with respect to the CIO, the so-called secular polar motion, have been carried out using the data of the five ILS stations. The extensive discussion on this matter has led to the conclusion, that at present there is no way to resolve the problem of how much is secular polar motion and how much is due to other secular effects providing that the number of stations is limited, for example, to five. However, different analyses were exersised to show qualitatively that the secular trend of the CIO relative to the mean pole, CIO-MPO, was, in the most part, due to the local nonpolar effects of the ILS stations. If the secular term of polar motion does not exist, we should clear up whether there is some long-period motion of the pole. Yatskiv et al. (1979) showed that the long-period components in polar motion which Markowitz (1970) has claimed to reveal are due to local, nonpolar variations of the latitudes of some stations, in particular, the Ukiah station. If the number of participating stations is large enough, as compared with the ILS, the effect of nonpolar drift of some stations on the relative motion of the reference systems would be balanced out. As an example, consider the variation of differences between the yearly mean values of coordinates of pole given by the ILS and the BIH for the period 1962-1979 (BIH Annual Report for 1979). These differences are compared with the variation of the mean latitude of Ukiah ($\bar{\psi}^x_u = -0.2583\psi_u$, $\bar{\psi}^y_u = +0.2559\psi_u$) as well as with the variation of the angle (S_{UM}) between the verticals of Ukiah and mean observatory (Pulkovo, Kazań, Poltava, Kitab). The values of -0.2583 and +0.2559 are the coefficients of the formulae used by the IPMS for calculating the coordinates of pole, x and y, from results of the five ILS stations.

There is a significant similarity between these three curves. As the variations of S_{UM} do not depend on polar motion, one can conclude that the long-period motion of the CIO with respect to the BIH 1968 system is caused mainly by the local nonpolar effect of the Ukiah station. These considerations lead to the conclusion that either the x_3-axis of the conventional terrestrial system, (x), should be directed toward the mean pole of the epoch of observation instead of the CIO, or a kinematic model of the nonpolar drifts of stations, the coordinates of which define the system, (x), should be introduced.

https://doi.org/10.1017/S0252921100081203 Published online by Cambridge University Press

Comparison of the polar coordinates obtained in different systems is capable of giving information on the relative motion of the third axis of these systems, and remains the only means for estimating the accuracy of the adopted conventional terrestrial coordinate system. Figure 1 shows the difference, BIH-DMA, between the BIH 1968 system and the reference system adopted by the DMTC for the reduction of the Doppler observations of artificial satellites. The trends of these systems with respect to the MPO are also depicted. There is significant correlation between the values, DMA-MPO, and the values, BIH-MPO, that gives some indication on the existence of the secular motion of the pole. However, one has to take into acount that the differences between the BIH 1968 system and the DMA system are of the same order as linear trends of BIH-MPO and DMA-MPO. Under the supposition, that there is no correlation between reference systems, BIH, DMA and ILS, the estimation of the standard errors of the yearly mean values of the polar coordinates have been determined:

x - coordinate: $\sigma DMA = \pm 0''.004$, $\sigma BIH = \pm 0''.003$, $\sigma ILS = \pm 0''.011$
y - coordinate: $\sigma DMA = \pm 0''.006$, $\sigma BIH = \pm 0''.004$, $\sigma ILS = \pm 0''.008$

When calculating these estimates, the value of ILS-BIH for 1976 have been rejected as outlier.

5. ON THE PROCEDURE FOR MAINTAINING THE CONVENTIONAL TERRESTRIAL COORDINATE SYSTEM

In current practice a minimum displacement principle, in least-squares sense, is used to fix the reference system to a set of observatories distributed over the Earth's crust (Fedorov, 1975b). Let ρi be the displacement of the zenith Z_i in the conventional terrestrial coordinate system, (x). According to a minimum displacement principle,

$$\sum_{i=4}^{n} \rho i^2 = \min, \tag{1}$$

where n is number of observatories. Taking the displacement along the parallel and meridian, the condition (1) becomes

$$\sum_{i=1}^{n} \{(\ell_i - \ell_{oi})^2 \cos^2 b_{oi} + (b_i - b_{oi})^2\} = \min, \tag{2}$$

where ℓ_i, b_i are the longitude and latitude of the i-th observatory in the system, (x); ℓ_{oi}, b_{oi} are the values of ℓ_i and b_i for the initial epoch T_o.

This statistical approach could be realized in the following manner. At the start of perhaps a number of Chandlerian periods the average values, $(\lambda_{oi}, \phi_{oi})$, of the observed astronomical longitudes and latitudes, (λ_i, ϕ_i), referred to the ephemeris

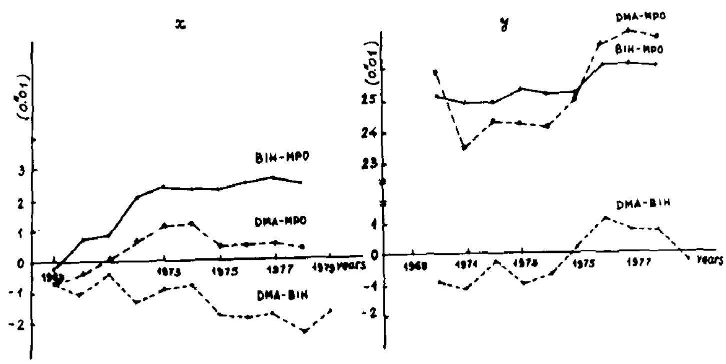

Figure 1. Variations of the yearly mean differences: BIH-MPO (solid line), DMA-BIH (dashed line with solid circles) and DMA-MPO (dashed line with open circles).

system, (ξ), is used for calculating UT1 and polar coordinates, (x,y), from the system of equation (3):

$$\begin{aligned} &\phi_i - \phi_{oi} - (x \cos \lambda_{oi} + y \sin \lambda_{oi} + z) = v_i \\ &UT0_i - UTC - (-x \tan \phi_{oi} \sin \lambda_{oi} + y \tan \phi_{oi} \cos \lambda_{oi} + t) = v_i', \end{aligned} \tag{3}$$

where $UT0_i$ is the Greenwich mean time derived from observations of star transits at the i-th observatory using the initial longitude λ_{oi}; UTC is the universal coordinated time broadcast by means of time signals; t is the value of UT1-UTC; z is nonpolar latitude variation, which is common for all participating observatories; v_i, v_i' are residual which is assumed to be independent random variables. Then the process is repeated for calculating the more accurate estimates of mean longitude and latitude under the condition that UT1 and x, y are given. As a result, the values λ_{oi}, ϕ_{oi} are obtained, which one can adopt as conventional coordinates of the i-th observatory for the initial epoch T_o, i.e., ℓ_{oi}, b_{oi}. At some convenient time interval like a one or two-year period, the estimates of the systematic errors of individual instruments along with UT1 and polar motion are determined from the solution of (3). The former as well as the information on the observation accuracy are used to weight the equations (3) when solving for subsequent time interval.

The difficulty of maintaining the conventional system, (x), is caused, first of all, by the fact that poor network configuration prevents the independent estimates of the unknowns x, y, z, t

https://doi.org/10.1017/S0252921100081203 Published online by Cambridge University Press

from the solution of equations (3). For example, the covariance matrix calculated for the worldwide net of about eighty observatories, the BIH network, is given in Table 2. The figures placed to the right and from above a diagonal of Table 2 correspond to the solution of the equations (3) with the weights adopted by the BIH. The other ones correspond to the solution of the equations (3) without weights. According to Korsuń and Emetz (1980) existence of correlation between x, t and z could result in the seasonal effect of polar motion and UT1 which have been recently corrected by the BIH (so-called BIH 1979 system).

Table 2. Normalized covariance matrix of the unknowns of equations (3).

	x	y	z	t
x		-0.14	-0.26	0.07
y	-0.05		0.04	0.04
z	-0.53	-0.11		0.02
t	-0.14	0.22	0.10	

The difficulty of maintaining the conventional coordinate system is increased by the fact that the observatories are located on different crustal plates that are in relative motion. Therefore, the above assumption on the nature of residuals, v_i, v_i', is not valid and one should use some statistical model of these residuals to solve the equations (3). Let us suppose

$$v_i = v_{s,i} + v_{r,i} , \qquad (4)$$

where $v_{s,i}$ is nonpolar systematic displacement of the i-th observtory; $v_{r,i}$ is independent random variable. For this model, the estimates of the unknown, x, y and UT1 of the equations (3) would be consistent, if the vectorial mean value, $v_{s,i}$, taken over the entire set of observatories, is zero, namely,

$$\sum_{i=1}^{n} v_{s,i}^{m} \cos \lambda_{oi} = 0, \quad \sum_{i=1}^{n} v_{s,i}^{m} \sin \lambda_{oi} = 0, \quad \sum_{i=1}^{n} v_{s,i}^{p} = 0 ,$$

where $v_{s,i}^{m}$, $v_{s,i}^{p}$ are the components of nonpolar displacement, $v_{s,i}$, along the meridian and parallel respectively. The deviation from this condition results in the relative motion of the different astronomical systems of terrestrial coordinate. This consideration leads to a conclusion that the conventional terrestrial coordinate system should be attached, in the statistical sense, to the observatories at relatively stable sites having on the average no mutual equatorial displacement of the zeniths and no systematic meridional components of the zenith displacements. The realization of such a reference system could be achieved by means of the comparison of the different systems of terrestrial coordinates with each other and with the MPO in the manner that is similar to the compilation of general catalogues in fundamental astrometry. After realization of such a reference system, a special procedure has to be used to minimize the effects of the changes in the number of participating observatories and the observational programs. This procedure might be similar to that employed by the BIH, though some improvement is possible (Yatskiv et al., 1979)

REFERENCES

Belotserkovskij, D.Yu. and Kaufman, M.B.: 1979, in D.D. McCarthy and J.D.H. Pilkington (eds.) "IAU Symposium No. 82, Time and the Earth's Rotation", D. Reidel, Dordrecht, pp. 23-27.

Fedorov, E.P., Korsuń, A.A., Major, S.P., Panchenko, N.I., Taradij, W.K., Yatskiv, Ya.S.: 1972, "Motion of the Earth's Pole for the Years 1890.0-1969.0, Naukova Dumka, Kiev.

Fedorov, E.P.: 1975a, Astrometrija i Astrofizika, Kiev, No. 27, pp. 3-13.

Fedorov, E.P.: 1975b, in B. Kolaczek and G. Weiffenbach (eds.), "IAU Colloquium No. 26, On Reference Coordinate Systems for Earth Dynamics", Warsaw, pp. 63-77.

Fedorov, E.P.: 1979, in D.D. McCarthy and J.D.H. Pilkington (eds.) "IAU Symposium No. 82, Time and the Earth's Rotation", D. Reidel, Dordrecht, pp. 89-101.

Gubanov, V.S. and Yaguidin, L.I.: 1979, in D.D. McCarthy and J.D.H. Pilkington (eds.), "IAU Symposium No. 82, Time and the Earth's Rotation", D. Reidel, Dordrecht, pp. 47-51.

Guinot, B.: 1979, in D.D. McCarthy and J.D.H. Pilkington (eds.), "IAU Symposium No. 82, Time and the Earth's Rotation", D. Reidel, Dordrecht, pp. 7-18.

Kolaczek, B.: 1979, in D.D. McCarthy and J.D.H. Pilkington (eds.), "IAU Symposium No. 82, Time and the Earth's Rotation", D. Reidel, Dordrecht, pp. 125-128.

Korsuń, A.A. and Emetz, A.I.: 1980, in Proceedings of the 4th International Symposium, "Geodesy and Physics of the Earth", Karl-Marx-Stadt (in press).

Mandelbrot, B.B. and McCamy, K.: 1970, "On the Secular Polar Motion and the Chandler Wobble", Geophys. J. R. Astr. Soc., 21, pp. 217-232.

Markovitz, N.: 1970, in L. Mansihna, D.E. Smylie, A.E. Beck (eds.), "Earthquake Displacement Fields and the Rotation of the Earth", Springer-Verlag, New York, pp. 69-81.

Mironov, N.T.: 1973, Astron. Circular, No. 769, pp. 7-8.

Orlov, A.Ya.: 1941, Bull. Sternberg Institut, Moskva, No. 8.

https://doi.org/10.1017/S0252921100081203 Published online by Cambridge University Press

Seidelmann, P.K. et al.: 1979, Final Report of the IAU Working Group on Nutation, 17th General Assembly of the IAU, Montreal.
Yatskiv, Ya.S., Korsuń, A.A., Mironov, N.T.: 1979, in D.D. McCarthy and J.D.H. Pilkington (eds.), "IAU Symposium No. 82, Time and the Earth's Rotation", D. Reidel, Dordrecht, pp. 29-39.