PART 5

WOLF-RAYET STARS

THE WOLF-RAYET STARS

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Abstract. This review discusses the spectral classification, the absolute magnitude, the position in stellar systems, the physical properties and the evolution of WR stars.

1. Introduction

The study of WR stars is interesting from the point of view of the physics of stellar atmospheres as well as of evolutionary theory. To this type belong the stars with very broad, bright lines ('bands') of He I, He II, also nitrogen, carbon and oxygen at different stages of ionization. A number of nuclei of planetary nebulae show similar spectra, however, these objects differ from the 'classical' WR stars by low luminosities and small masses. There are about 130 WR stars known in the Galaxy (Smith, 1968a), about 60 in the LMC (Westerlund and Smith, 1964), 2 in the SMC and 25 in M33 (Wray and Corso, 1972).

2. Spectral Classification

Beals (1938) has distinguished two approximately parallel sequences in the excitation levels of the spectra: the carbon sequence (WC 6 to 8) with predominance of carbon and oxygen ion emission, and the nitrogen one (WN 5 to 8) in which the nitrogen lines prevail. Common to both is the emission of He I and He II (in WC stars lines of He II are much weaker). The hydrogen spectrum is weak; the Balmer lines are blended by He II. The division is not at all sharp: in the spectra of WN stars there are carbon lines (strong C IV $\lambda\lambda$5801, 5812), in the spectra of WC stars, weak nitrogen lines are present. There are also spectra of intermediate type, WN-C (see Smith, 1968a). A few lines show violet-shifted absorption components (such as triplets of He I, occasionally Pickering lines of He II, lines of N IV-V and C III-IV). A high-temperature absorption spectrum superposed on the emission in some WR stars belongs to early-type components (O5-B2). No forbidden lines occur in WR stars.

There is evidence that 'nitrogen' and 'carbon' anomalies exist also in Of stars (Swings and Struve, 1941; Kumajgorodskaya, 1964) and in some O-B stars (Walborn, 1970, 1971, 1972).

In the case of WC stars, the band width decreases with spectral subclass; there is no such dependence in WN stars.

Modern classifications are more detailed. Unlike WC stars which lie entirely within one sequence of subclasses, Hiltner and Schild (1966) have found two parallel nitrogen sequences: WN-A stars have comparatively narrow lines, a strengthened continuum and often a superposed early absorption spectrum; in WN-B stars, bright lines are wider and stronger. The interval of ionization potentials in the spectrum of WN-B is larger than that in the analogous spectrum of WN-A, however the number of sub-

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classes is smaller (i.e. for the sequence as a whole this interval is narrower). Among WN-A stars, there are many spectroscopic binaries (however, there are also such stars which do not show evidence of duplicity). Among WN-B spectroscopic binaries also occur, but very seldom. It is believed that the branching of the nitrogen sequence is due to more profound reasons than the simple fact of the existence of binary and single WN stars.

3. Absolute Magnitudes

At the present time there are individual determinations of absolute magnitudes, M_v, for 24 galactic WR stars (Table I; see Rublev, 1963, 1965a, 1970a; Rublev and Cherepashchuk, 1974). For the LMC, there are M_v determinations for about 40 stars (Westerlund and Smith, 1964; Smith, 1968b); mean values are given in Table II.

The luminosities of the WN stars (Figure 1) correlate with spectral classes, increasing in the interval WN4 to WN7-8 on the average from $-3\overset{m}{.}4$ to $-6\overset{m}{.}2$ (Galaxy) and from $-3\overset{m}{.}9$ to $-6\overset{m}{.}3$ (LMC); the dispersion is large, $\sim \pm 0\overset{m}{.}7$ to $1\overset{m}{.}0$. There are no systematic luminosity differences between nitrogen '*B*' ('single') and '*A*' ('binary') stars.

Luminosities of WC stars do not correlate with spectral classes; here binaries in each subclass are on the average brighter than single stars. For a 'mixture' of both of them, $M_v = -5.8$ (Galaxy) and $-5\overset{m}{.}4$ (LMC) with a dispersion of $\sim \pm 1\overset{m}{.}0$.

TABLE I

Magnitudes M_v of galactic WR stars

HD	Sp	M_v	HD	Sp	M_v	d (kps)
4004	WN5-B	−3.4	190918	WN5.5-A	−6.8	2.85
9974	WN4-A	−3.9	190918	WR	−5.2:	
16523	WC6	−4.6	191765	WN6-B	−5.5	3.10
50896	WN5-B	−4.9	192103	WC8	−5.6	3.10
68273	WC8(+08)	−6.2	192163	WN6-B	−5.7	2.65
68273	WR	−4.4	192641	WC7(+B2)	−4.8	1.95
92740(*)	WN7-A	−6.7	192641	WR	−4.6:	
92740(*)	WR	−6.3:	193077	WN6-A	−5.0	1.85
93131	WN7-A	−6.3	193576	WN6-A	−5.3	1.70
93131(*)	WR	−5.6:	193576	WR	−3.5	
93162(*)	WN7-A	−6.1	193793	WC6(+05)	−6.9	2.40
93162(*)	WR	−5.2	193793	WR	−6.0	
151932(*)	WN7-A	−6.4	193928	WN6-B	−4.6	1.70
152270(*)	WC(+08)	−5.7	211853	WN6.5-A	−5.8	
152270(*)	WR	−4.8:	211853	WR	−4.3:	
165763	WC6	−6.5	214419	WN-7A	−5.1	
186943	WN5-A	−4.4	214419	WR	−4.3	
187282	WN5-A	−3.8				

Notes:

1. Sp – from Hiltner and Schild (1966).
2. M_v – from the dependence of apparent distance moduli of OB stars and interstellar line intensities; (*)-estimated from the data of Graham (1965) and Smith (1968b).
3. d for stars in Cygnus is from color excesses following from the dependences between colour excess and apparent distance moduli of near-by OB stars.

TABLE II

M_v of WR stars in LMC (bracketed is the number of objects) (Westerlund and Smith, 1964)

Stars	Field	In OB associations	In 30 Dor complex
WN	−4.4(21)	−5.2(9)	−6.4(7)
WC	−5.2(3)	−5.5(5)	–

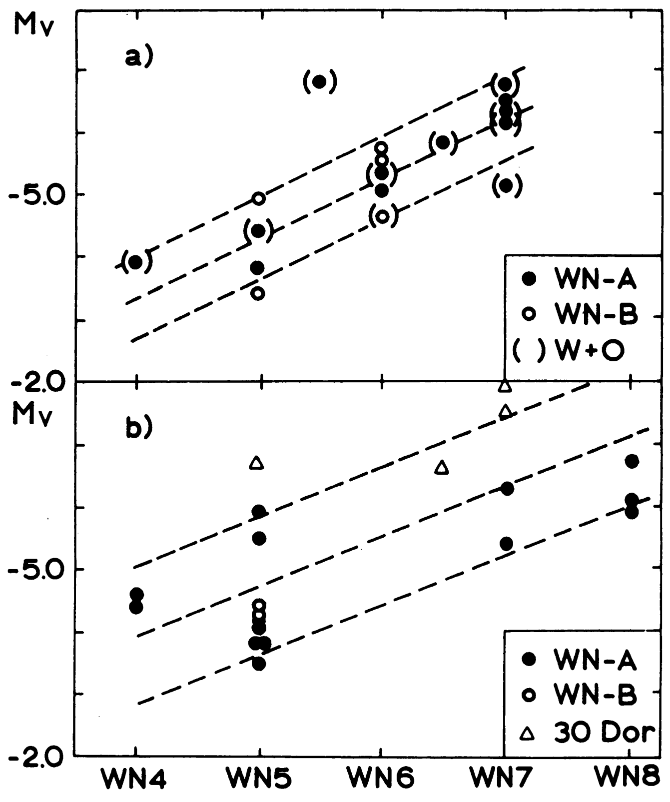

Fig. 1. Relation 'Sp-M_v' for WN stars of Galaxy (a) and LMC (b). For LMC narrow-band M_v magnitudes (Smith) are given; triangles – WN in 30 Dor complex.

For the 'field' stars in the LMC, luminosities are on the average lower than for the members of OB associations; the difference is especially great for WN stars.

WR stars in the Galaxy and in the LMC differ slightly in absolute magnitudes (galactic WN are possibly somewhat weaker, and WC stars are slightly brighter); WR stars in M33 possess similar luminosities (Wray and Corso, 1972).

The LMC and the Galaxy differ in the content of WR stars. The portion of carbon stars in the Galaxy is 1.5–2 times larger; WN6 and WC6-8, which are rather numerous in the Galaxy, are absent in LMC (among carbon stars there are only WC5 stars).

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Smith (1968b, c) has made a list of luminosities and distances of about a hundred galactic WR stars. For these estimations (from spectral class) *mean* values $\bar{M}_v$(WR) and mean intrinsic color indices of single LMC-member stars have been used. Luminosities of galactic binaries (WR+OB) have been synthesized from mean luminosities. In view of a large dispersion of absolute magnitudes of WR stars in each subclass and a problematic character of identification of a number of binaries from indirect features, one may not ascribe to these data the weight of individual determinations. Comparison with the direct determinations (Table I) reveals significant differences (luminosities obtained by Smith are often exaggerated – see Rublev, 1970b). The same has been inferred by Crampton (1971a, b) who studied the relation of WR stars with H II regions (his corrections make M_v values by Smith appreciably closer to the data of Table I).

4. WR Stars in Stellar Systems

WR stars are representatives of a very young population I, which is revealed in the analysis of their distribution in the Galaxy (Mikulašek, 1969).

The distribution with galactic longitude has maxima along spiral arms which is characteristic of OB stars as well. Unlike the latter, WR possess a wide ($\Delta l \simeq 80°$) 'zone of avoidance' towards the anticenter of the Galaxy (Vorontsov-Velyaminov, 1948; Roberts, 1958, 1962; Sim, 1968). One may think that star formation in this region containing very young objects (T Tauri-stars, groups of stars aged $\sim 5 \times 10^5$ yr) occurred only recently, so that even the most massive stars have not left the main sequence (Mikulašek, *loc. cit.*)

The concentration in spiral arms can be observed in figures by Smith (1968c, 1973a) who has constructed the distribution of WR stars in the galactic equatorial plane; the relationship with H II regions and OB associations can be seen.

In M33, WR stars also emphasize its spiral structure; the distribution across an arm is approximately uniform. About a quarter of all WR stars are associated here with H II regions; some are at the borders of large OB associations. Following the spiral arms, WR stars are also found in the region of the galactic nucleus (Wray and Corso, 1972). The galactic subsystem of WR stars, being very flat ($\bar{Z}=85$ pc), lies between the subsystems of O associations ($\bar{Z}=65$ pc) and open clusters ($\bar{Z}=110$ pc). Their association with young (O-type) clusters is especially strong for 'late' WN stars (WN7-8) and essentially weaker for WC stars. Binary stars, WR+O with spectroscopically resolved components, show the greatest association of such a kind. Mikulašek estimates the mean ages of WN and WC stars to be $\sim 3.2 \times 10^6$ and $\sim 1.1 \times 10^7$ yr respectively.

Binary stars of WR type occur more often than single ones projected onto H II regions and associations. The cases are interesting when one succeeds in identifying WR stars with close OB groups not only with respect to direction but distance as well. Take, for example, two such identifications in Cygnus (Rublev, 1963).

(a) A group Cyg ID (designation from Kopylov, 1958) includes, besides 15 OB stars, 3 WR stars: HD 190918 (WN5.5-A, a binary), HD 192103 (WC8, a single star), and HD 191765 (WN6-B, single; there is a suspicion that it possesses a ring-shaped gaseous envelope, see Crampton, 1971b).

(b) A complex in the region of 'stellar ring' No. 274 (Isserstedt, 1970) includes, besides P Cyg, 4 WR stars: HD 192641 (WC7+B), HD 193077 (WN6-A, a binary), V444 Cyg (WN6-A, a binary), and HD 193928 (WN6-B, a binary).

Thus, a close stellar complex can contain WR stars of different types (evolutionary age and chemical composition).

There are marked differences in the distribution of WR stars in the Galaxy and in the LMC. In both cases, less luminous WR 'field' stars are more numerous than members of associations. Although some types of WC in the LMC are absent altogether (see Section 3), the distribution of WC stars differs less from that of WN stars here than in the Galaxy. According to Smith (1968c, 1973a), WC9-7 and WN6 stars in the Galaxy are concentrated in the regions close to the nucleus (zone 4-9 Kpc). WR stars of other types are distributed more uniformly, and 'early' WN stars tend to the periphery. The composition of the population of the LMC is roughly similar to the outlying regions of the Galaxy and differs from its interior regions. This preliminary picture has to be confirmed. It is also of importance to investigate the distribution of different types of WR stars in M33.

Many WR stars are members of binary systems. For types WN and WC, the frequency of occurrence in binaries identified from direct criteria is approximately the same, $\sim 37\%$ (Roberts, 1962; Underhill, 1963). For some time past, one often appealed to indirect criteria (the principal one is a strengthened continuum, i.e. the relative weakness of bright lines). It turns out that among WR stars brighter than 10^m binary stars make up 58%. According to Kuhi (1973), this fraction may increase up to $\sim 73\%$ (the statistics are based on 15 northern stars brighter than 10^m). This point is very interesting since one may readily account for the origin of WR stars by the duplicity only.

Smith (1973b) has found that binary WN and WC stars differ in separations between components. For WN stars they are no greater than $\sim 65\ R_\odot$, while for WC stars, they are markedly larger. In Smith's opinion, the large separation of components is a universal property of binary stars of type WC and is to be connected with peculiarities in their evolution. Note that the statistics here are very poor and unreliable (7 pairs of WN and 3 of WC; in both cases there are exceptions; WN stars HD 190918 and HD 193928, for example, are the wide pairs).

Indirect evidence for the existence of single WR stars is in the presence of gaseous 'ring' envelopes of nebular type around some of them (Johnson and Hogg, 1965; Smith, 1965, 1967; Schmidt-Kaler, 1970; Johnson, 1971). They are observed only around some WN 5, 6, 8 stars of type B (far from all of them), and are seen as regions of high ionization ($T_e \sim 10000$–$20000°$, $\bar{n}_e \sim 10^{-3}$ cm^2) emitting thermal radio emission and expanding at a rate of tens to a hundred of km per second (Lozinskaya, 1970; Georgelin and Monner, 1970). The masses of these nebulae are very large (~ 3 to hundreds of solar masses) and seem to decrease with the increase of Z (Johnson, 1971). Such envelopes may be formed from the interstellar matter swept up by gas streaming from the central stars (Johnson and Hogg, 1965; Pikelner and Shcheglov, 1968; Avedisova, 1972). Such a scheme explains the absence of envelopes in binary stars (the greater part of the ejected material may be transmitted to the companion).

WC stars do not possess 'ring' nebulae. From the IR-photometry data (Allen *et al.*, 1972), a number of them apparently have dust envelopes.

IR excesses are observed in all WR stars; in the case of WN-A, they are smaller than in the case of WN-B; in WN stars, on the whole, smaller than in WC stars; in WC9 class the excesses are extremely large. For WN stars, the free-free emission from hot gas may be the explanation. In the case of WC9 stars, the best explanation is the thermal radiation by circumstellar dust which may condense from the ejected material and include carbon-rich (graphite?) particles (see Allen *et al.*, 1972).

5. On the Nature of WR Stars

A number of ideas have been suggested to explain the spectra of WR type; the greatest support is enjoyed by two alternative conceptions: a nebular model of Beals (1929, 1944) and a chromospheric-coronal conception of Thomas (1949, 1968, 1973; Underhill, 1973).

According to Beals, the atmosphere of a WR-type star is formed from the material outflowing at a great velocity (~ 1000 km s^{-1}) from the hot star. Main features of WR spectra (differences in the forms and line widths, violet-shifted absorptions) are explained by the Doppler effect. The basic physical process in the atmosphere is a fluorescence caused by the short-wave radiation from the hot 'nucleus'. The common mechanism of line excitation is radiative; correspondingly, the atmosphere belongs to the type of 'heated' ones (the electron temperature T_e is smaller than the temperature T_r of the ionizing radiation from the nucleus). Following the idea of Beals, such an atmosphere is similar to a planetary nebula with enlarged material density and rate of expansion. Thomas' scheme suggests the existence of a comparatively cool photosphere ($T_r \sim 20000$–$50000°$) which turns into an extended, rather dense and very hot envelope ($T_e \sim 100000$–$500000°$) where the bright lines arise. The intermediate layer is likely to be thin; the temperature here may be very high (up to $\sim 10^7$ °). The source of heating is uncertain; originally it was supposed that the heating might be due to the converted mechanical energy coming from the photosphere. Bright lines are excited by collisions; the atmosphere is a 'cooling' type ($T_e > T_r$). In the outlying regions, there is a 'solar-wind'-type outward flow of material. According to Thomas, in WR stars and in the Sun, the extreme cases of nonclassical atmospheres are realized.

Beals' conception used to be severely criticized; however, in the past few years, a lot of evidence for its reality has been obtained. This model naturally co-ordinates and explains practically all the observational facts. From the phenomenological point of view, it seems to be the best.

Sometimes a 'joint' conception is used: accepting the whole structural and kinematic part of Beals' model, and instead of the fluorescence excitation mechanism, one considers mainly collisions ($T_e \gg T_r$). This approach may hardly be considered correct.

6. Chemical Composition of Atmospheres

In connection with the problem of the evolutionary stage of WR stars, the estimates of atmospheric abundance of H, He, C, N and O are important. The first determinations for the number of atoms gave H/He$\sim$0.4–0.55 (Ambartsumian, 1933; Sobolev, 1952). A summary of the later results is presented in Table III. Until recently, practically no estimates of heavy element abundances have been made.

TABLE III

H/He-values (by the number of atoms) in the atmospheres of WR stars

Star, HD	191765 WN6-B	192163 WN6-B	193077 WN6-A	192641 WC7	192103 WC8
Slyusarev, (1955)	–	0.20	–	–	0.083
Rublev (1972a)	0.13–0.14	0.13–0.17	0.09–0.16	0.14–0.17	0.07–0.11
Nugis (1973c, d)	0.06–0.09	0.14–0.23	$\lesssim$0.1	0.06–0.08	$\lesssim$0.024
Mean	0.11	0.18	0.1	0.11	0.07
Smith (1973a)	$\overline{\text{WN4-6}}$		$\sim$0.3		–
OB stars				10.0$\pm$0.1	

TABLE IV

N/He- and C/He-values (by the number of atoms) in the atmospheres of WR stars

HD	19176-5 WN-B	192163 WN-B	193077 WN6-A	192641 WC7	192103 WC8	OB
N/He	0.051	0.075	0.1	–	–	0.0011
C/He	0.0035 Zanstra and Weenen (1950)	0.0018	0.008	1.0 $\overline{\text{WC}}$, $\sim$0.4	0.65	0.0015

In Table IV are presented the results of Nugis and Ilmas (1973c, d) and Nugis and Feklistova (1973); the estimates include an uncertainty factor of $\sim$2–3.

Thus, in comparison with OB stars:

(a) The hydrogen content in WR atmospheres is 50–150 times lower; in the later subclasses of WC it is even less abundant.

(b) In WN atmospheres, the nitrogen content is a factor of 50–100 larger.

(c) In WC atmospheres, the carbon content is possibly 400–700 times the normal.

Thus, WR stars are helium stars with hydrogen-helium envelopes enriched with nitrogen and carbon.

7. Temperatures

Temperatures of WR stars obtained by diverse methods differ greatly; this can be readily explained within the framework of a nebular model.

Color temperatures are rather low in the photographic interval: $\sim 18000°$ on the average (allowing for the interstellar reddening), corresponding to B8-9 stars (Vorontsov-Velyaminov, 1945; Rublev, 1963, 1972b). They decrease very much with wavelength (Andrillat, 1957; Kuhi, 1966) in single stars $\bar{T}_c \simeq 27000°$ near $\lambda 3500$, and $\sim 8500°$ near $\lambda 9500$. The continuum is 'smooth' (no marked 'jumps').

The analysis of narrow-band light curves of the eclipsing binary V 444 Cyg gave the distribution of spectrophotometric temperature over the stellar disk of a WR component (Cherepashchuk and Khaliullin, 1972). It increases from the peripheral regions ($T(\lambda 5000) \simeq 6000$–$15000°$) towards the center ($T(\lambda 5000) \simeq 50000$–$100000°$!); the outlying zones are the major contributors to the visual luminosity in the continuum.

The continuous radiation from the envelope arises in recombinations and free-free transitions; the distribution of energy here is, roughly speaking, exponential; thus, the color (i.e. Planck) temperature always turns out to be low and decreases with λ (Sobolev, 1947). The observed distribution is the result of superposing the continua of the 'nucleus' and the envelope. This circumstance may be used to estimate the temperatures of the envelope, i.e. to find a relationship between the temperatures T_e and T_* (Planck temperature of the nucleus) which would represent the observed continuum of the star (Rublev, 1972b). Results for the photographic wavelength region are presented in Figure 2. It is seen that the mean envelope temperatures are lower than those

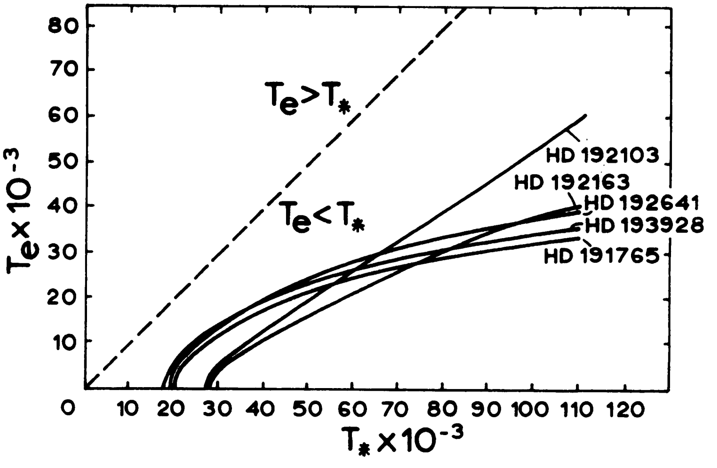

Fig. 2. Relations between temperatures T_* of WR nuclei and mean temperatures T_e of their envelopes (from unreddened continua in the interval $\lambda\lambda 0.385$–0.5μ; see the text).

of the nuclei ($\bar{T}_e \simeq 0.4\, T_*$ on the average). In a number of stars, the values of $\bar{T}_e$ are limited ($\lesssim 50000°$). The temperatures of the nuclei have the limitation ($T_* \gtrsim 3000$–$40000°$). If we adopt the Zanstra temperatures T_* (see below), we obtain $25000° \lesssim \lesssim \bar{T}_e \lesssim 45000°$.

From the energy balance condition of free electrons in a He III region, it turns out that $T_e(\text{He III}) \lesssim \frac{2}{3}\, T_*$ approximately (Nugis, 1973a, b). The minimum estimate (T_e

(He III) $\sim$ 20000–30000°) can be made from the Pickering He II lines (a 'limit-decrement' method – see Rublev, 1964a, 1972c: in the high-temperature region it is very sensitive to errors in intensities and yields results probably close to the lower limit of T_e). Cherepashchuk and Khaliullin (1972) have found that in WR atmospheres, at large optical depths, the absorption coefficient in the red increases with λ due to the contribution to the electron scattering by the free-free transitions. A rough estimate of the amount of this contribution gave $T_e \lesssim 50000°$.

The intensity analysis of UV lines of C III in γ^2 Velorum (Castor and Nussbaumer, 1972) has given for the outer layers of the WC8-atmosphere $T_e \simeq 20000$–$25000°$. All this shows that the electron temperatures are far lower than those required by the chromospheric-coronal model ($T_e \sim 10^5$ deg). A probable value of T_e in the deep layers of the envelope is $\sim$ 50000°; in the outer layers, it is 2–2.5 times smaller; the mean value is $\sim$ 30000–40000°.

Let us dwell on the estimate of Planck temperatures of the nuclei. In our opinion, they are quite close to the effective temperatures; that is they correspond to the entire radiation flux.

Note the following:

(a) If one examines the ionization of He II in a hydrogen-helium envelope, one then obtains the 'intrinsic' temperature interval for the temperature of the exciting nucleus from the condition of co-existence of He III and He II zones (radiating in He II and He I-lines). In the case of WR stars, $70000° \lesssim T_* \lesssim 100000°$; the values of the limits are determined here by the high content of helium and the comparatively small size of the nucleus (Rublev, 1974a). The same temperature interval is obtained in calculating the evolution of a close binary which leads to the formation of a WR-type star: $T_* \simeq$ $\simeq 74000$–$82000°$; if one allows for the ejection of material at the WR stage, the upper limit is then shifted to $\sim (10$–$11) \times 10^4$ K (Tutukov and Yungelson, 1973a, b). Similar values follow from the analysis of eclipsing binaries. This has already been mentioned with respect to the color temperature which may reach 100000° in the case of the nucleus of V444 Cyg. From the ratio of surface brightnesses of the WR nucleus and the O7-component in CQ Cephei (Kartashova, 1974), the brightness temperature of $\sim$ 80000° (and $M_{\mathrm{bol}} \simeq -10.2$! – see further) is obtained.

(b) Temperatures of nuclei can be determined by the Zanstra technique; one needs only to allow for the differences between conditions in stellar envelopes and those in nebulae (see Rublev, 1964b, 1972d; Rublev and Cherepashchuk, 1974). The mean Zanstra temperature scale for Of stars obtained in this way from the line He II λ4686 is in good agreement (up to $\pm$5–10%) with the available values of T_{eff}. This characterizes both the reliability of the method and the degree of its applicability to estimates of *effective* temperatures (Rublev, 1974b).

Results of determinations from the line He II λ4686 for 25 WR stars are presented in Figure 3; all these temperatures fall within the interval 70000–110000°, there being no correlation with the spectral class (concealed by unresolved binaries?). It is possible that in each subclass, WC and WN-B stars are hotter on the average than WN-A. Nugis (1973c, e) has estimated Zanstra temperatures of four WR stars from

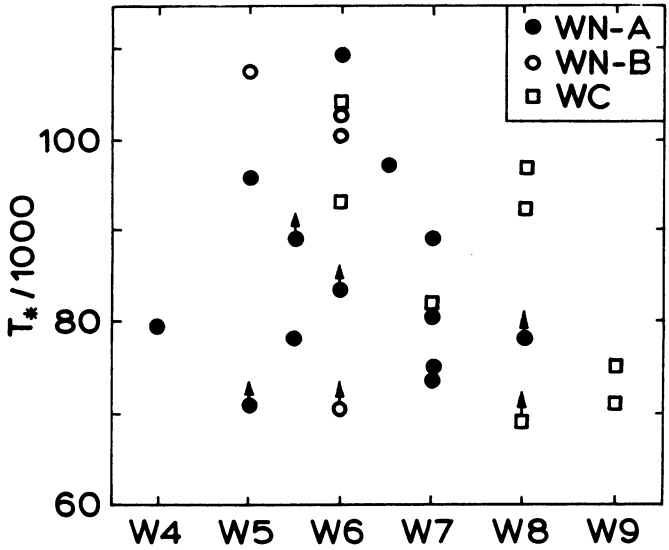

Fig. 3. Zanstra temperatures for nuclei of WR stars from He II lines: unresolved binaries are marked by arrows.

lines of different ions (He II, N IV, N V, C IV, O IV). If one takes into account overlapping of bands of photoelectric absorption, the temperatures turn out to be very similar.

Morton (1970, 1973) has obtained a 'low' range of Zanstra temperatures (25000–54000°) from radio emission of the nebular envelopes of WN stars. The models available cannot be used in this case: the radiation emitted after the transformations in a WR star envelope is 'low-temperature': the UV part of the spectrum is weakened and the visible part is strengthened. Accordingly, such Zanstra (i.e. colour) temperatures prove to be lower than the effective temperatures. In the calculation of bolometric corrections for WR stars, one has to allow for the contribution of the envelope (in lines and continuum) to m_v values (the result of transformation of the UV-spectrum of the nucleus, see Rublev, 1965b; Nevsky and Rublev, 1963). BC-values are in this case far smaller than for a black body and for hot models (in the case of WC stars, there is a 25000–40000° lowering, i.e. a shifting to early-O-star models – see Figure 4).

8. Physical Parameters

Luminosities of WR stars are very high ($-9^m \lesssim M_{bol} \lesssim -11^m.5$). Since they are much higher and the masses of WR stars are ~3 times smaller than in the O-components of binaries, then, in view of the luminosity excess, WR stars surely cannot be in the main sequence evolutionary stage. On the 'temperature-luminosity' diagram (Figure 5), they lie between the main sequence and the sequence of homogeneous helium models; the major energy source is apparently helium burning. The radii of nuclei (found from

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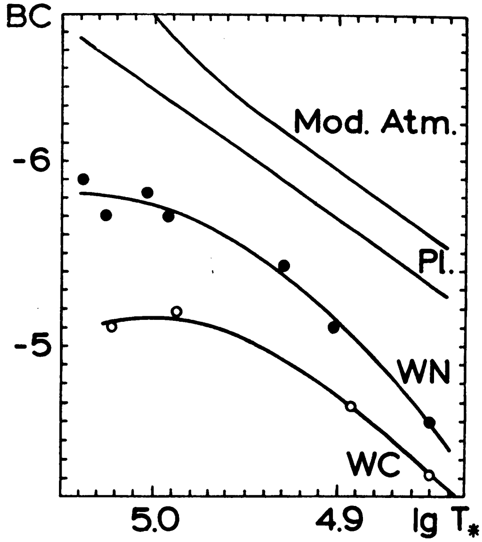

Fig. 4. Bolometric corrections for WR stars (with Planck radiation from the nuclei). For comparison the BC curves of the black body (Pl.) and those of high-temperature model atmospheres are presented.

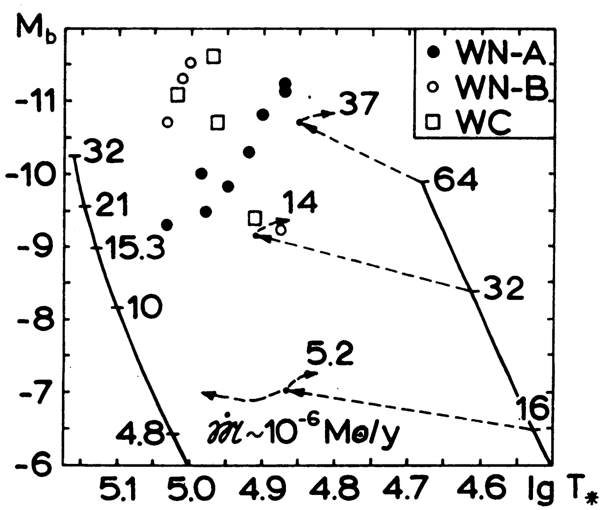

Fig. 5. Positions of WR stars in the 'temperature-luminosity' diagram. Shown are the sequences for homogeneous models of normal chemical composition and purely helium models; the dashed lines represent the general scheme of evolution with the appearance of WR stars in close binaries (Tutukov and Yungelson, 1973a; see the text); figures are for masses in solar units; for $\mathfrak{M} = 16 \odot$ the direction of evolution with mass loss is shown. A developing sequence of nitrogen A-type stars can be seen.

temperatures T_* and visual luminosities allowing for the contribution of the envelope) are shown in Figure 6. They lie in the range ~2–9 $R_\odot$; the mean value is ~4–5 $R_\odot$. One can see the temperature dependence for WN-A stars (single and unresolvable binaries): the radii of cooler nuclei are larger.

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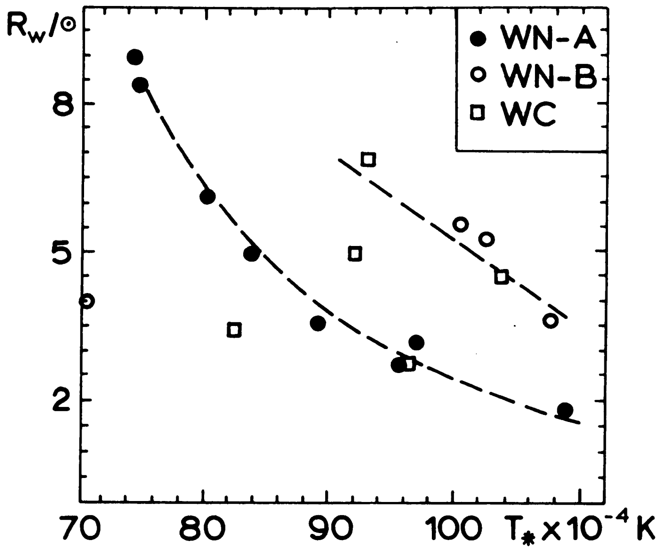

Fig. 6. 'Temperature-radius' diagram for the nuclei of WR stars.

The masses estimated from binary stars are confined within 5–15$\mathfrak{M}_\odot$ (see Kuhi, 1973; Rublev and Cherepashchuk, 1974). This mass range is adopted for single WR stars as well. With such masses, bolometric luminosities are considerably larger than the critical Eddington luminosity: the force of gravity cannot hold WR atmospheres. Outflow of matter caused by radiation pressure upon free electrons (and, to a lesser extent, beyond the He II ionization limit) occurs (Rublev, 1964c; see also Rublev and Cherepashchuk, 1974).

The outflow velocities are of the order of the observed ones (i.e. correspond to line widths); the mass loss is $\simeq(1\text{–}8)\times 10^{-5}\mathfrak{M}_\odot$ per year. The electron density in the deep layers at the boundary of the "nucleus", is $n_e^* \simeq 10^{13}$ cm^{-3}. The depths of the envelopes beyond the He II and He I ionization limits are of the order 10^6 and 0.05, respectively; the one caused by the electron scattering is $\sim$0.5–1.5 (Rublev, 1965c; Rublev and Cherepashchuk, 1974). Independent estimates yield similar values ($n_e^* \sim (0.7\text{–}1)\times$ $\times 10^{13}$ cm^{-3} (Cherepashchuk, 1972) $\dot{\mathfrak{M}} \sim -10^{-5}\ \mathfrak{M}_\odot$ per year (Lozinskaya, 1973; etc.).

For some time, the reality of the existence of stars with luminosities higher than the critical luminosity, $L_c = 4\pi c G\mathfrak{M}/\kappa$, has been discussed. Analytical studies and calculations support the possibility of constructing self-consistent stellar models with a hydrodynamically outflowing envelope and a core in hydrostatic equilibrium (Bisnovatyi-Kogan and Zeldovich, 1968; Bisnovatyi-Kogan and Nadyorhin, 1972; Bisnovatyi-Kogan, 1973). On the other hand, it has been found that, from the point of view of the present theory on the inner constitution of the stars, such objects cannot exist (Paczyński, 1973, Żytkov, 1973). Note that WR stars with supercritical luminosities really exist, and *this follows immediately from the observational material.* Among the components of binaries (where there are reliable masses), M_v reach -6^m; the masses do not exceed 20 $\mathfrak{M}_\odot$; radiation temperatures are ~80000° and higher (this is obtained directly from the analysis of binaries). For a minimum BC (allowing for the effect of a 'cool' envelope), $L/L_c \simeq 2$. In order to reduce the luminosity here to a critical value, it would be necessary to reduce the bolometric correction (low as it is) by approximately one stellar magnitude.

9. Evolutionary Aspects

The appearance of WR stars in binary systems can be explained by the 'mass exchange' between the components in their evolution (Paczyński, 1967; Kippenhahn and Weigert, 1967; Snezhko, 1967). A more massive star evolves more rapidly. Expanding after exhaustion of hydrogen in the core, it fills the Roche lobe and a rapid outflow of matter towards the companion takes place. When the hydogen envelope is depleted, there remains a hot helium star with a little hydrogen in the outermost layers. Being yet in the main-sequence (hydrogen burning) stage, all carbon and oxygen is transformed into nitrogen. After the commencement of helium burning, the nitrogen undergoes rapid transformations back to oxygen and carbon until it completely disappears from the core. The hydrogen-helium atmosphere of the 'remnant' of the primary may be enriched with either nitrogen or carbon and oxygen (in the case of mixing, see Paczyński, 1973).

The calculations by Tutukov and Yungelson (1973a, b) have shown that objects of the WR-star type develop in this way in ~(3.8–7.5) × 10^6 yr from stars with masses of 16–32 $\mathfrak{M}_\odot$ and fall within a temperature interval of 74000–82000°. The time of such

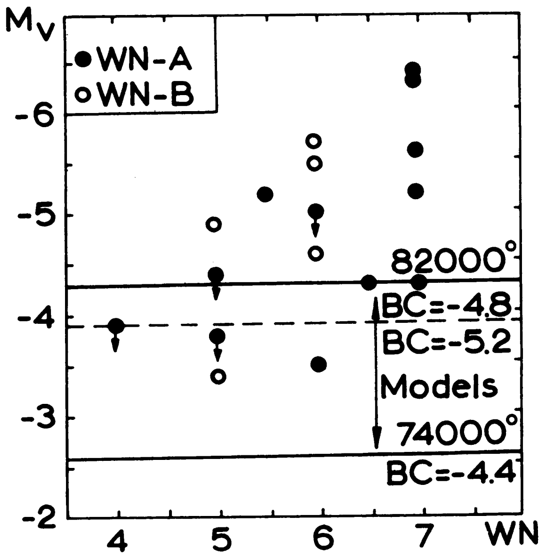

Fig. 7. Comparison of the observed absolute magnitudes M_v of WN stars with optimum results derived from evolutionary models (see the text); dashed line corresponds to the model with $T_* = 82000°$ allowing for BC from the WN's curve of Figure 4.

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evolution agrees with estimates from the age of clusters (Section 4); the temperatures agree with estimates using the Zanstra method (Section 8). However, the luminosities of these models prove to be low. Figure 7 provides a comparison between the observed absolute magnitudes M_v for WN stars and for the models (found from their luminosities and temperatures; the lowest possible bolometric corrections were used i.e. those corresponding to an extreme contribution from the envelope). It can be seen that no more than 1/3 of the stars can fall within the 'permitted' model limits; many WR components of binary stars turn out to be much brighter (see also Figure 5). We should remember that the luminosities of real WR stars exceed the critical; for equilibrium models, they are lower than the critical luminosity.

A possible evolutionary track for single stars has been outlined by Bisnovatyi-Kogan and Nadyozhin (1972). In stars with masses of $\sim$20–30 $\mathfrak{M}_\odot$ after the transition to the red supergiant region (at the core helium-burning stage) an inverse density gradient arises which is related to the supercritical luminosity. It results in a powerful and rapid outflow of matter (up to 0.5 $\mathfrak{M}_\odot$ per year). Having reached the levels close to the hydrogen-burning shell, it diminishes. After these changes in the star's structure, there remains a very hot helium (with a little hydrogen) remnant with a mass of $\sim 10\mathfrak{M}_\odot$, similar to a single WR star. In such a scheme, the WR stage is preceded by an IR-star (corresponding to the stage when the hydrogen-rich envelope is lost). On exhaustion of the nuclear energy sources, WR stars, having still considerable masses, should collapse. After the implosion a relativistic object (a neutron star or a 'black hole') would probably remain.

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