Dr ten Bruggencate:

We started in Göttingen a programme to study the stratification of the solar photosphere. By measuring the profiles of the infra-red O lines at the centre of the disk, we found a distinct asymmetry of the line-profile; the violet wing being stronger than the red wing. This can be understood by using the three streams model of Böhm. The excitation potential of the O lines is so high that the hot rising elements of granulation contribute much more to the line-profile than the cooler elements going downwards. The asymmetry disappears if the lines are observed near the solar limb.

Dr Athay:

Dr de Jager presented a diagram showing a compilation of the measures of isotropic turbulence in the chromosphere that showed turbulent velocities increasing with height. Some of these results, particularly those of Suemoto and Redman (*M.N.* 1955), were based on the assumption of a uniform, spherically symmetric chromosphere. If one includes temperature fluctuations as large as those we have been discussing, the increase of turbulence with height will not be as rapid as indicated by de Jager's diagram. Would Dr de Jager comment on this point and also comment on the extent to which the other results may be influenced by such non-uniformities?

Dr de Jager, in reply to the questions from Dr Athay:

The data derived from the Redman-Suemoto observations have been recomputed by using the temperature distribution given in Fig. 2 of my lecture and the non-uniformity there assumed. An exception has been made for the helium lines: from an unpublished computation made at Utrecht, I have the impression that the helium lines are mainly formed in a layer between 2000 and 3000 km. high. The same follows from the v_{turb} values, derived from the He lines by Redman and Suemoto: at $h<2000$ km. these are equal to the values at $h=2000$ km.; and afterwards they increase with h. I have excluded the helium points for $h<2000$ km. from Fig. 2.

2. EVIDENCE FOR TURBULENT MOTIONS IN STELLAR ATMOSPHERES

By K. O. Wright, *Dominion Astrophysical Observatory*

The application of the principles of hydrodynamics has become an important feature of modern astrophysical investigations, but the first mention of turbulence in stellar atmospheres was made by Rosseland (1) as long ago as 1928 in his paper on 'Viscosity in the Stars'. He showed that if there were any differential motions taking place in these atmospheres, the scale must be so great that turbulence of a 'bewildering complexity' would result. By 1934 the word 'turbulence' appeared frequently in the literature and, for some fifteen years, was often used somewhat indiscriminately to explain any stellar phenomenon relating to stellar spectra not otherwise accounted for. However, about 1949 students of hydrodynamics and astrophysics began to pool their ideas and it is believed that, although many of the problems of astrophysics are exceedingly complex, the implications of turbulent phenomena and their applications to astrophysics are becoming better known.

This paper presents in some detail the spectroscopic evidence for turbulence in the atmospheres of stars other than the Sun as obtained from curves of growth and from line-profiles, and also discusses some observations made during the atmospheric eclipses of the ζ Aurigae-type of stars. Other data related to turbulent phenomena will be discussed only briefly.

Following Rosseland's paper, Unsöld (2) and McCrea (3) discussed turbulence in the solar atmosphere and Menzel (4) noted that the systematic differences in the velocity-curves of different elements observed in the spectra of Cepheid variables might be evidence of this phenomenon. However, Struve and his students at the Yerkes Observatory were the first to discover a new phenomenon in stellar spectra that could be ascribed to 'turbulence'. The 'gradient effect' was first noted by Struve (5) in the spectrum variable 17 Leporis, and was studied in some detail by Elvey (6) in eight stars. They found that, particularly for giant stars, stellar multiplet intensities did not agree with intensities calculated from theoretical consideration of the sum rules, nor those observed in the solar flash spectrum. Struve and Elvey (7) explained the phenomenon in terms of the curve of growth and large turbulent motions in the atmospheres of giant stars.

Since the curve of growth has been studied quite extensively and the results have been related to turbulence phenomena, a brief discussion of its properties seems desirable. For nearly all atoms of astrophysical importance except hydrogen and helium, the principal agents effective in broadening a spectral absorption line are Doppler motions, which produce the observed profiles of weak lines and the cores of strong lines, and damping effects, including those of radiation and collisions, which affect the wings of the lines. For weak absorption lines the intensity, measured as the equivalent width or total absorption, is controlled almost entirely by the Doppler motions of the atoms and is directly proportional to Nf, the number of active atoms producing the line. As more atoms become effective in producing the line, it becomes deeper until the centre becomes saturated and then, for a time, the increase in equivalent width becomes small until the damping effects become important, when the intensity increases again, this time at a rate proportional to the square root of the number of effective atoms. These effects were considered by Voigt (8) and by van der Held (9) and were first applied to solar data by Minnaert and Mulders (10) and by Minnaert and Slob (11). Struve and Elvey (7) calculated curves of growth assuming purely exponential absorption; Menzel (12) and Unsöld (13) made calculations for a Schuster-Schwarzschild atmosphere for pure scattering; and Strömgren (14), Pannekoek and van Albada (15), and Greenstein (16) calculated curves of growth for a Milne-Eddington model atmosphere. Recently, Wrubel (17, 18) made more elaborate calculations for these models using Chandrasekhar's (19) solution of the equation of transfer. Although the results of these calculations give curves which vary appreciably in shape (and in interpretation), their general form is the same and the uncertainty of most stellar observations makes discrimination between them almost impossible.

In addition to the uncertainties of measurement in the ordinates ($\log W/\lambda$) of stellar curves of growth, there are also uncertainties in the abscissae ($\log gf \times \lambda$) in many cases. When using stellar data, we can only rarely study the increase in intensity of a single line by increasing the path length and therefore must employ the theoretical intensity relations calculated from the structure of the atom or, preferably, well-determined oscillator strengths (f-values) such as those determined at Mount Wilson Observatory by King and his co-workers (20, 21). These relative f-values for the atoms, Fe I, Ti I, Ti II, V I, Ni I, Cr I and Co I, seem to be sufficiently accurate for most astrophysical needs, though more absolute data are required for determining stellar abundances. The many uncertainties involved in using stellar data require that as many lines as possible be used in constructing stellar curves of growth. Wright (22) and Bell (23) have derived relative solar oscillator strengths ($\log X_f$-values) which have been used quite frequently, but their use cannot be recommended for accurate work in spite of the abundance of the solar data because the lines observed in the solar spectrum do not all arise from the same effective optical depth, and therefore should not fit the same curve of growth. Other observers have used Greenstein's f-values (16) derived from lines in the spectra of τ Ursae Majoris or of the hydrogen-poor star, υ Sagittarii, or Buscombe's data (24) for α Cygni, but the same remarks apply and, in addition, Greenstein's data are based on solar X_f-values. Further laboratory work is urgently needed for all spectra of astrophysical importance, including high-excitation lines of the elements studied by King, and especially for lines occurring in the spectra of ionized atoms.

In spite of the uncertainties noted above in the determination of stellar curves of growth, a considerable body of data has been accumulated over the past twenty years and permits a few, fairly well-established conclusions to be drawn concerning turbulence in these stellar atmospheres. For most stars, the best-determined part of the curve of growth is the flat transition region since it occurs, in most cases, at the intensity level where the lines are frequently numerous, fairly strong and easily measured. The observed

Doppler motions, derived from the curve of growth, are shown from theory to be a function of the ordinate which for the observed data is log W/λ and for the theoretical curve is log $(W/\lambda \times c/v)$, where v is the Doppler velocity and c is the velocity of light. These motions are therefore determined from the displacement of the ordinates when the observed data are fitted to the theoretical curve. The effective temperature, T, is known sufficiently well from the spectral type of the star. Therefore if v_0 is the observed velocity as determined above, and v_T the turbulent velocity, the following relation applies:

$$v_0^2 = v_T^2 + 2RT/\mu$$

where R is the gas constant and μ the atomic weight of the atom being studied.

In Fig. 1 the available data on turbulent velocities for a number of stars have been plotted against absolute magnitude and spectral type; the size and shape of the symbol is a measure of the turbulent velocity, which is usually determined from lines of neutral and ionized atoms of low excitation potential. The data are given in Table 1, where references are given. Special thanks are due to Dr J. L. Greenstein who generously made available his unpublished data for G-type and metallic-line stars, and to Dr L. H. Aller who is currently studying the atmospheres of B-type stars. These data confirm the statement frequently made that turbulent velocities are usually greater in stars of high luminosity than in stars of low luminosity. Although our knowledge of curves of growth does not cover all classes of stars in the Hertzsprung-Russell diagram, this general relation between turbulent velocity and luminosity seems to hold and, in addition, it would appear that turbulence is greater in the atmospheres of early-type supergiant stars than in those of later type. The stars that have been studied have frequently been chosen either because they are non-normal stars, or because they appear bright, and therefore this diagram is strongly influenced by observational selection. The early work

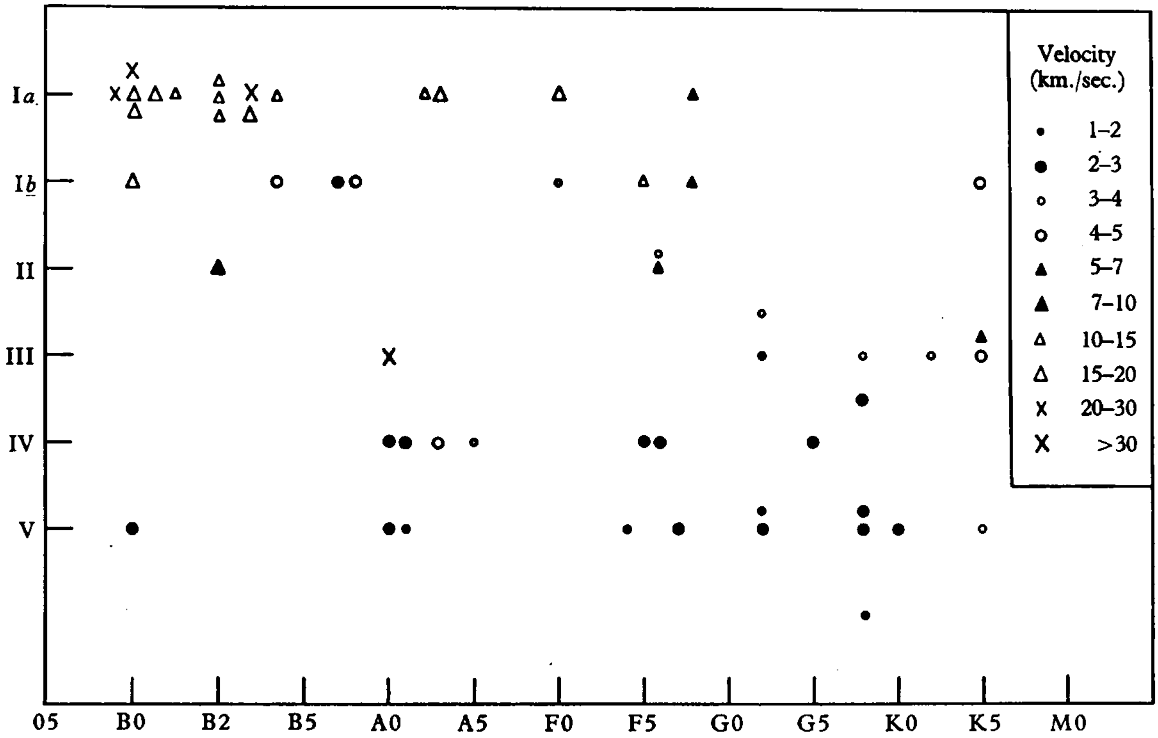

Fig. 5. The relation between turbulent velocity derived from curves of growth, spectral type and luminosity. 17 Leporis, A0, has been assigned luminosity class III and metallic-line A-type stars have been called class IV.

of Struve and Elvey (7) indicated the presence of large velocities in supergiant stars and therefore special attention has been given to stars of this class. 17 Leporis, the star having the largest published turbulent velocity (7), is a main-sequence star if the parallax ($\pi = 0{\cdot}023 \pm 0{\cdot}010$) (25) is correct, but its spectrum is variable and shows shell characteristics at times. The data for the B-type stars cannot be considered definitive because they are based almost entirely on measures of a few helium lines studied by Goldberg (26) (the O II lines were considered less trustworthy). The helium lines cannot be used for curve-of-growth studies for main-sequence stars because they are affected by Stark broadening.

Table 1. *Turbulent velocities of representative stars determined from curves of growth and from line-profiles*

Star	Spectral type and luminosity class	Velocity (km./sec.) From curves of growth	Velocity (km./sec.) From line-profiles	Reference
τ Scorpii	B0 V	<2·6		(27)
ρ Leonis	B1 I*b*	10	18	(47)
ε Canis Majoris	B2 II	8	15	(66)
			27	(29)
55 Cygni	B3 I*a*	30	36	(28)
			18	(66)
α² Canum Venaticorum	A0 *p*	2–4		(30)
17 Leporis	A0	67		(7)
γ Geminorum	A1 IV	1·7		(67)
		4	17	(24)
α Cygni	A2 I*a*	13	35	(24)
		≤14·8	34	(48)
8 Comae Berenices	A3	4·8		(68)
15 Vulpeculae	A5	3·8		(68)
ε Aurigae	F0 I*ap*	20		(7)
		2–26	33	(38)
		4–24	15–25	(49)
α Carinae	F0 I*b*	2·2–3·3		(69)
		3·5		(70)
δ Canis Majoris	F8 I*a*	4·9		(71)
			30	(36)
		4·3–8·2		(69)
		6·4–6·9	3–15	(44)
			22	(45)
W Virginis	F2 I*b*–G6 I*b*	4·6–12·3		(58)
η Aquilae	F6–G3	4	12·2	(37)
		4·0–5·8	7·4–10·4	(63)
		6–13		(72)
π Cephei	G2 III	1·1		(15)
Sun	G2 V	0·9–2·0	1·7	(73)
δ Leporis	G8 III	3·2		(68)
τ Ceti	G8 V	2·1		(68)
Groombridge 1830	G8 *sd*	1·3		(68)
70 Ophiuchi A	K0 V	2·5		(74)
α Bootis	K2 III*p*	3·4		(74)
		2·7–4·4		(75)
ξ Cygni	K5 I*b*	4·7		(74)
ζ Aurigae (chromosphere)	K5	6·5–20·6		(42)
(chromosphere)		7·3–13·0		(43)
31 Cygni	K5	5		(76)
(chromosphere)		10		(76)
(chromosphere)			10–20	(56)
(chromosphere)			20	(56)

However, studies have been made of τ Scorpii by Unsöld (27) and of 55 Cygni by Voigt (28), while Aller is now preparing data for several other B-type stars. Miss Underhill (29) has determined velocities from line-profiles of a few supergiant B-type stars and found values larger than those derived from the curves of growth. Several Cepheid variables have been included in Table 1; the turbulent velocities vary with phase and spectral type and studies of these stars may prove very valuable in future for more detailed discussions of turbulent phenomena. No data are yet available for white dwarfs.

Metallic-line stars and stars showing large magnetic fields are of special interest in studies of turbulence phenomena. Greenstein studied the star τ Ursae Majoris (16) and, with Miczaika and Deutsch, is studying 15 Vulpeculae and 8 Comae Berenices. All three stars show larger turbulent velocities than normal stars of the same luminosity. A similar effect was noted by Burbidge and Burbidge (30) for the magnetic star α^2 Canum Venaticorum; they remark that magnetic intensification produced by the Zeeman components of the lines, as suggested by Babcock (31), is not sufficient to account for the observed large velocity. They suggest that turbulence may be related to the magnetic and hydrodynamic phenomena to be expected for such a star.

Although most of the early evidence for turbulent motions in stellar atmospheres was obtained from curve-of-growth data, it was recognized by Unsöld (2) that important information could be obtained from the shapes of the line-profiles. The determination of an accurate profile for a stellar absorption line is a difficult procedure unless the line is quite wide, since the effects produced by the spectrograph must be removed. For most stellar spectra the observed line-profiles are produced mainly by the instrument, and that is one reason why the curve-of-growth method, which requires only total absorptions and not intensities at each point on the line, has been used so extensively. However, with the building of more powerful spectrographs and the availability of fast photographic emulsions, it is now possible to obtain sufficiently high-resolution spectra of many stars to study the profiles of some of the stronger lines. The study of true line-profiles, even of those in the solar spectrum, is just beginning. The interpretation of a corrected profile is often open to question since the radiation received from a stellar atmosphere traverses layers of varying temperature and pressure and also may be affected by Doppler motions and other characteristics of the stellar atmosphere such as rotation, limb darkening, etc.

In 1937 Allen (32) studied the profiles of solar lines and, from the correlation between central intensity and equivalent width, came to the conclusion that the line-widths were greater than theory would permit if they were due only to thermal motions of the atoms. He concluded that turbulent velocities of the order of 1·6 km./sec. were present in the solar reversing layer. He also discussed, as did Unsöld (33), the effects of large-scale macro-turbulence and small-scale micro-turbulence on stellar spectra. The problem has been discussed in considerably more detail recently by Huang (34), and Wrubel (35). Although the present paper is concerned primarily with the observational material, a brief survey of the implications of macro- and micro-turbulence seems desirable.

It is considered that there is evidence for turbulent motions in stellar atmospheres if the spectrum indicates that Doppler motions are present that cannot be explained by thermal motions, rotation, pulsation, or other normal factors. Such Doppler motions can affect a spectral line in either of two ways: in the case of stellar rotation, each element of the atmosphere may be regarded as independent and behaving as in a static atmosphere; the absorbing medium moves as a whole relative to the source and the observer, and the radiation is absorbed by a layer of uniform velocity and is not further affected; hence the absorption coefficient is not changed and the equivalent width is unaffected. If large eddies are present in the atmosphere (large being defined as greater than the free path of a quantum of radiation, or the effective depth of the absorbing layer), they will have little effect on the radiation except to change the shape of the spectrum line produced by the whole star. Such a case is called macro-turbulence; although the shape of the line is changed, its equivalent width is not altered and the curve of growth is not affected. However, if small eddies, of diameter less than the free path of a quantum, are present in the atmosphere, then the radiation reaching the observer may have been absorbed and re-emitted several times by atoms moving with different velocities. These micro-turbulent motions will affect the line-absorption coefficient and hence both the equivalent width and the line-profile. Huang (34) has discussed the two cases indicated above, and has also considered the effect of non-Maxwellian distributions of velocities. The results indicate that the velocity derived from the curve of growth depends on the nature of the frequency distribution of the velocities in the atmosphere, a point which may become important when more accurate observational data become available. However, the general conclusions remain: the Doppler velocity determined from the curve of growth is a measure of the small-scale turbulence, but the velocity derived from the line-profiles is some mixture of large- and small-scale turbulence.

In view of the above remarks, it may seem surprising that the evidence for such motions from stellar spectra remained unnoticed for so long. Struve (36) pointed out this 'interesting phenomenon' in 1946 when he remarked that the velocity derived from the curve of growth for δ Canis Majoris was much less than that obtained from the line-profiles; he also pointed out that the strong lines were much broader than the weak lines, which increased the complexity of the problem. Further evidence for both macro- and micro-turbulence in stellar atmospheres was soon forthcoming. M. and B. Schwarzschild and Adams (37) found that for η Aquilae the velocity derived from the curve of growth was 4 km./sec. and that from the line-profiles was 12 km./sec.; Wright and van Dien (38) found similar results for ϵ Aurigae. Both papers pointed out that the line-profiles were not inconsistent in shape with those in the spectrum of a rotating star, but suggested that large-scale turbulence was a more likely interpretation in view of the extensive atmospheres of these giant stars. Recent observations of the spectrum of ϵ Aurigae by the author (39), combined with earlier data obtained before the appearance of any atmospheric-eclipse phenomena, show evidence of a real variation in the shape of the lines though the equivalent widths probably remain almost constant. As it seems most unlikely that the speed of rotation should vary, this and somewhat similar observations made by Struve (40) appear to indicate the presence of turbulent elements in the atmosphere of ϵ Aurigae. According to theory, the average velocity of the macro-turbulent elements should be larger than that in the micro-turbulence. This condition is satisfied in all cases that have been studied if it is agreed that the curve of growth gives the micro-turbulent velocity and that the line-profile is a combination of the macro- and micro-turbulence.

The conclusions of the author (38, 41) that turbulent velocities derived from curves of growth of giant stars are different for ionized and neutral atoms and, for Fe I, for lines of different excitation potential, have been confirmed for ζ Aurigae by Wilson and Abt (42, 43) and for δ Canis Majoris by Huang and Struve (44), using their correlation between measured half-width and equivalent width. The curves of growth are somewhat uncertain because the range in intensity for each excitation potential is not as large as might be desired and, in addition, sufficient account of the variation in the continuous absorption coefficient has not always been taken. However, there is reason to believe that the range in turbulent velocities is large enough to be of significance. The implications of this result have been discussed by Unsöld and Struve (45). They suggested that the almost wingless lines observed in supergiant spectra might be an observational effect and that the true continuum might be some 30% higher than that usually drawn in the λ4050 region of the spectrum of δ Canis Majoris. This suggestion needs checking by measurement of lines in a region where blending is less severe. Struve and Unsöld pointed out that an alternative explanation in terms of the turbulent motions of a field of prominences, whose nature was not clearly specified, was equally probable. The variation of the turbulent velocity with ionization and excitation can be explained qualitatively on model-atmosphere theory: in upper layers of the atmosphere where the temperature and pressure are low, ionized atoms and low excitation states are favoured, whereas neutral atoms and higher excitation states are found in deeper layers. Further, if the atmosphere is turbulent, the higher layers at low pressures will have larger turbulent velocities than the lower dense layers, according to the aerodynamic equation of continuity.

An interesting application of the theory of macro-turbulence has recently been made by Gathier (46) in his study of the equivalent width of Hδ in F- and G-type stars of different luminosities. He found that the observed equivalent widths in many cases require higher excitation temperatures than given by his calculations for the various models. This result was interpreted as being produced by macro-turbulent elements ascending from great depths in the atmospheres, which were able to maintain pressures and temperatures greater than the average value for the surrounding atmosphere, the temperatures being greater by as much as 1100° for supergiants, 800° for giants, and 300° for the Sun.

Recent work by Huang and Struve (44, 47, 48) has shown that further information concerning Doppler motions in stellar atmospheres can be obtained from high-dispersion stellar spectrograms. For such spectra, where the shape of the lines is not determined primarily by the instrument, they showed that estimates of the turbulent velocities of both large and small eddies could be obtained by fitting the relation between half-width and equivalent width to theoretical curves. Their results for δ Canis Majoris confirmed the conclusion of Wright and van Dien (38) that the Doppler velocity decreased in supergiant atmospheres for Fe I lines of increasing excitation potential. Their study of line-profiles in the spectrum of ρ Leonis (47) included profiles calculated for rotation and also for turbulent motions. Their observations could not distinguish between two possible models, one, a rotating star where the rotation was not shared by the outermost layers in which turbulence exists, and the other, where there is a field of prominences in large-scale turbulent motion high in the atmosphere. Somewhat similar results were obtained by the same authors for α Cygni (48) when they studied the relation between central depth and equivalent width. The correlation between line-width and equivalent width was also used by Mrs Kilby (49) to study the lines in ϵ Aurigae; her results confirmed the relation between turbulent velocity and excitation potential found by Wright and van Dien (38). In all cases of giant stars studied in this way, the observational material and the theoretical methods are not yet sufficiently precise to separate three possible models: the mechanism of line-formation may be different for lines of different ionization and excitation; there may be a large spread in the frequency distribution of the turbulent velocities, both in microscopic and macroscopic elements; or there may be several distinct layers, e.g. a lower one showing rotation or convection and an upper one made up of a large-scale field of prominences.

Since we are interested in the structure of stellar atmospheres in relation to the probable motions of different layers, the study of stars such as ζ Aurigae is important. In this group, which includes VV Cephei, 31 and 32 Cygni and a few other fainter stars, with ϵ Aurigae a possible border-line case, a hot, early-type star revolves about a cool, giant, late-type star in a period of several years. There is a total eclipse and 'chromospheric' absorption lines are observed in the spectra for some time before and after totality. Wilson and his co-workers (50, 42, 43) at Mount Wilson Observatory have observed ζ Aurigae at three eclipses. Goedicke (51) studied spectra of VV Cephei, but the variability of this star makes the analysis difficult. Changes in the K-line in spectra of 32 Cygni have been examined by Colacevich and Fracastoro (52) and by Wright (53). For 31 Cygni, studies have been made by Larsson-Leander (54) and by McKellar *et al.* (55), and further analyses of the Victoria material are being made by McKellar and by Wright.

Wilson's analysis (42) of spectra of ζ Aurigae obtained at the 1939–40 eclipse suggested that the micro-turbulence, derived from curves of growth, increased with the height in the atmosphere of the K-type star through which the light of the B star passed, from 6·5 km./sec. at 8×10^5 km. to 13·0 km./sec. at $20{\cdot}6 \times 10^5$ km. However Wilson and Abt's work (43) on the 1948 eclipse indicated that this result was somewhat uncertain, although they confirmed a difference of turbulent velocity between lines of neutral and ionized atoms. Wright's unpublished curves of growth for 31 Cygni give values for the turbulent velocity of 10 km./sec. for the chromospheric lines a few days before totality, and of 5 km./sec. for the normal K-type spectrum observed during eclipse. These values indicate that the motions in the chromosphere are larger than the average throughout the atmosphere. Studies of the profile of the Ca II K-line during the atmospheric eclipse of 31 Cygni by McKellar *et al.* (56) give a turbulent velocity of 20 km./sec. a few days before totality and this agrees with Underhill's results (57) for the Fe I lines. The values for the K-line decrease to 10 km./sec. at distances a diameter away from the edge of the star. These results, combined with the observations of McKellar *et al.* (55) of satellite lines that appeared and disappeared in an apparently erratic manner, particularly during egress, are consistent with Wilson and Abt's model (43) for this type of star of many clouds or condensations of relatively small size and high density, moving with different velocities above the so-called surface of the primary, cool star.

A few brief references may be made to other observational evidence for motions in stellar atmospheres. Some of these as well as some of those already discussed may not be related directly to turbulence as understood by physicists, but until we have more information, both observational and theoretical, it seems that all data of possible value should be included in the discussion.

Abt's work (58) on W Virginis included data on curves of growth as well as on radial velocities of this population II Cepheid; the Doppler velocities derived from them range from 4·8 km./sec. near the photosphere to 12·3 km./sec. during expansion and contraction; similar results were obtained in his recent study (59) of U Monocerotis. As a model, he suggests a photosphere that expands periodically, sending off a shell that rises through a relatively rarefied region before it falls and meets the next shell. In order to pass through each other, the shells are considered to consist of many cloud-like condensations of such a number and size that collisions are improbable. A somewhat similar hypothesis has been suggested by Odgers (60) to explain the double lines observed by Struve (61) in the β Cephei-type star, H.D. 199140 (BW Vulpeculae).

Struve's observations of close eclipsing binary stars suggest that streams of gas may flow from one star to another. As the stars are faint and many of the observable phenomena are quite transient, only the gross changes have been examined. However, many of the lines vary in intensity throughout the cycle and Struve has suggested (62), particularly for RW Persei and AQ Pegasi, that when the hydrogen lines are strong the profiles are characteristic of turbulent motions.

The large Doppler displacements and broad lines observed in the spectra of novae are undoubtedly produced by large-scale motions in the atmospheres of these stars. Pulsating stars such as the Cepheids also show the effects of turbulence, as indicated by the Schwarzschilds and Adams (37) and η Aquilae, and by the asymmetries of faint lines found by van Hoof and Deurinck (63). Other variable stars, such as R Coronae Borealis and *o* Ceti, also show large changes that, it would seem, can only be explained by turbulent motions as Rosseland (1) suggested.

The general picture concerning stellar atmospheres is still far from complete. The difficulty of the problem is illustrated by the fact that we cannot describe mathematically all the phenomena observed on the Sun. Menzel (64) concluded that turbulent motions played a large part in the support of the chromosphere, and Unsöld (65) considered that the concept of a 'field of prominences' or a 'burning prairie' was a good first approximation for the Sun. However, there is no proof that observations of solar phenomena can be related to stellar atmospheres, particularly to those of giant stars where conditions of excitation, pressure and gravitational attraction may be much different. Indeed, Wilson's (42, 43) and McKellar's (55) studies of the ζ Aurigae-type stars indicate that in these stars the prominences may often be large discrete clouds of fairly high density and quite widely separated from their nearest neighbours.

Huang (34) has studied the possible interpretations of the observational evidence from curves of growth and from line-profiles, and has indicated how uncertain are the precise values that can be ascribed to the velocities derived from these observations since they depend on the method of line formation, the assumed velocity distribution, and many other variables. However, the concept of turbulence as consisting of large eddies breaking down step by step into smaller and smaller eddies seems to be consistent both with the physical theory and with the stellar observations. We cannot define the size of these eddies precisely, and the approximate nature of the spectroscopic results so far has prevented a more quantitative treatment of the subject. Nonetheless, it would seem that from the curve-of-growth and line-profile data obtained from stellar spectra, combined with the measurements of prominences, granules, etc., observed on the Sun, a few points in the spectrum of turbulence may be determined within reasonable limits and it may be possible in the future to apply the physical theories of turbulence to these observations.

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Discussion

Dr Minnaert:

It was agreed upon with Dr Wright that I would be allowed to say a few words on some general effects of turbulence on stellar spectra. Turbulence may become manifest by the following effects.

(1) Doppler effects of micro- and macro-turbulence, to be found by the broadening of spectral lines; these are the phenomena, treated in a very complete way by Dr Wright.

(2) Turbulent expansion. If a stellar atmosphere is stirred, there will develop in each layer a kinetic temperature which adds itself to the thermal temperature, the layers will expand, the density will decrease everywhere in the proportion: $T/(T+T_{\text{kin}})$. The practical effect will be as if the gravitation had decreased in that same proportion. The star mimics a star with a smaller gravitation than it really has. This effect may be important in the determination of spectroscopic parallaxes. It may amount even to a factor of 1/10 in g.

(3) Temperature inhomogeneity. Unsöld, de Jager, and Gathier have already pointed out which interesting effects can be obtained when the stellar atmosphere is composed out of elements which, in the same horizontal layer, have different temperatures. These will appear every time when a spectral feature is determined as a function of temperature by a function of the shape indicated on Fig. 6*a*. Instead of assuming an atmospheric

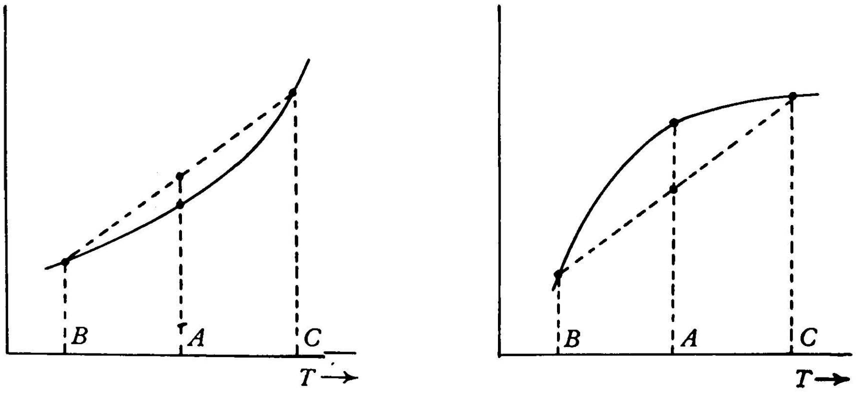
temperature A, we introduce two regions of temperatures $B=A-\Delta T$ and $C=A+\Delta T$; clearly the spectral feature studied will be stronger in the mean of the spectra $B+C$ than in the original spectrum A. Reversely a spectral feature described by a curve like the one in Fig. 6*b* will become less pronounced when temperature inhomogeneity is introduced.

It might very well be that even the old Adams-Russell effect can be explained in this way.

In any case, the introduction of turbulent expansion and of temperature inhomogeneity gives more flexibility to our stellar models, which thus may be adapted to the explanation of spectral phenomena, otherwise not understandable.

(Note added after the meeting.) A fourth effect needs to be mentioned:

(4) Density inhomogeneity. This will in many cases be correlated with the temperature inhomogeneity; it may enhance or modify the effect of the temperature fluctuations.



a Fig. 6 *b*

Miss Underhill:

(1) I wish to draw attention to the fact that the He I lines are unsuitable in early-type main-sequence stars for giving evidence about turbulent effects. It seems clear from observational and theoretical grounds that large non-adiabatic effects due to the *varying* electric fields occur. At present no theory is available to describe these effects, thus the interpretation of the line-profiles and strengths is gravely restricted.

(2) With regard to the analysis by curve of growth of the chromospheric spectrum of such eclipsing stars as 31 Cygni, it is important to ascertain whether a *continuous* absorption arising in the chromosphere is present as well as the line-absorption. There are some photometric results for ζ Aur suggesting that such continuous absorption occurs, and also certain results by Underhill for 31 Cygni suggest continuous absorption diminishing the B star before actual eclipse occurs. The presence of even weak continuous absorption can be readily shown to affect the derived equivalent widths of the chromosphere lines seriously.

Dr Wright, in reply to Miss Underhill:

(1) The O II, N II lines can be used for B stars, as stated by Unsöld (τ Scor) and Voigt (55 Cygni), but the curve of growth for these stars is still not well defined because the range of intensity is small.

(2) In 31 Cygni and other stars, the chromospheric spectrum may show continuous absorption which has not usually been accounted for (the absorption model has usually been adopted for the curves of growth). It seems that this continuous absorption may be important just prior to eclipse, but probably need not to be allowed for in the early stages.

Dr Goldberg:

I wish to make two points:

(1) Dr Aller has recently found that the turbulent velocities derived from curves of growth for early-type stars are strongly dependent upon the model from which the theoretical curve is derived.

(2) It is necessary to take into account the fact that there are probably temperature fluctuations in the photospheres of other stars as well as the Sun. Such fluctuations may play a major role in the interpretation of Dr Wright's results on the apparent correlation between turbulent velocity and excitation potential. Imagine, for example, a situation in which a group of lines are strong in a 'cold' region and weak, or absent, in a 'hot' region. The effect on the total equivalent width is roughly similar to that produced by scattered light in the spectrograph. All values of the equivalent width are reduced by a constant multiplying factor and the result is an apparent decrease in turbulent velocity. But the half-widths are not changed and hence the turbulent velocity derived from the curve of growth is less than that from the profile. Also, of course, the modification of the curve of growth will be different for lines of different ionization and excitation potential.

Dr Greenstein:

I believe that a more sophisticated view of the velocity field in the giant star's reversing layer is needed. It is known that there is about 2 or 3 km./sec. differential velocity between excited and resonance lines in ordinary red giants. In supergiants, line-profiles change, and even become double. The effectiveness of a velocity gradient in elevating a curve of growth is like that of turbulence.

I would suggest the usefulness of a two-stream model, before consideration of whether we should adopt fully developed aerodynamical turbulence.

I would also like to know whether the macro- or micro-turbulence velocity is the relevant one in computation of the effective support of the reversing layer by turbulence.

Dr Minnaert, in reply to Dr Greenstein's last question:

There is no doubt that micro-turbulence produces turbulent expansion. I would not dare to say definitively that macro-turbulence plays the same role. In a private conversation Dr McCrea, who suggested this effect many years ago, expressed the opinion that both should be treated in the same way.

(Note added after the meeting.) An argument favouring that last-mentioned opinion is that micro- and macro-turbulence differ only when we compare the size of their elements to the thickness of the transparent atmospheric layers; but that is a question of pure optics, and it is clear that it has nothing to do with the turbulent expansion, which is a purely mechanical effect.

Dr Unsöld, in reply to Dr Greenstein:

(1) The largest turbulence elements should have diameters of the order of the atmosphere's scale height. Therefore we should expect macro-turbulence in addition to micro-turbulence, if the optical depth corresponding to one scale height H is $\kappa H > 1$. Calculations made by K. H. Böhm (*Physik der Sternatmosphären*, p. 462) indicate that for normal atmospheres of any temperature and surface gravitation κH is, by chance, practically always of the order of unity. Macro-turbulence should therefore play a major role only in atmospheres, e.g., made up of huge prominences, etc.

(2) In general, quite good accuracy can be obtained using the 'schematic' curve of growth in connexion with the theory of weight functions for calculating the effective number of atoms (or optical depth) for weak absorption. Aiming at higher accuracy, one must in many cases also take into account the variation of turbulent velocities with height, the inhomogeneity of the atmosphere, and other effects which complicate the subject enormously. It does not help to increase the accuracy only in one direction!

3. INTERPRETATION OF THE MOTIONS IN STELLAR ATMOSPHERES

By L. Biermann, *Max Planck Institut für Physik, Göttingen*

In a theoretical interpretation of the motions observed in stellar atmospheres, which are often characterized as turbulent, it seems fitting to start by stating more precisely the essential physical features of turbulence.

For this it is convenient to regard the velocity field decomposed into its Fourier