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# A CHARACTERIZATION OF SEMIPRIME IDEALS IN NEAR-RINGS

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**Abstract**

It is well known that in any near-ring, any intersection of prime ideals is a semiprime ideal. The aim of this paper is to prove that any semiprime ideal $I$ in a near-ring $N$ is the intersection of all minimal prime ideals of $I$ in $N$. As a consequence of this we have any seimprime ideal $I$ is the intersection of all prime ideals containing $I$.

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## 1. Preliminaries

A near-ring is an algebraic system $(N, +, \cdot)$ satisfying (i) $(N, +)$ is a group, (ii) $(N, \cdot)$ is a semigroup and (iii) $(x + y)z = xz + yz$ for all $x, y, z$ in $N$. We abbreviate $(N, +, \cdot)$ by $N$.

If $S$ and $T$ are subsets of $N$, we denote the set $\{st | s \in S, t \in T\}$ by $ST$. A normal subgroup $I$ of $(N, +)$ is called an ideal of $N(I \trianglelefteq N)$ if $IN \subseteq I$ and for all $n, n' \in N$ and for all $i \in I$, $n(n' + i) - mn' \in I$. An ideal $P$ of $N$ is called a prime ideal if for any ideals $I$ and $J$ of $N$, $IJ \subseteq P$ implies either $I \subseteq P$ or $J \subseteq P$. An ideal $I$ of $N$ is called a semiprime ideal if for any ideal $J$ of $N$, $J^2 \subseteq I$ implies that $J \subseteq I$. An ideal minimal in the set of all prime ideals containing some given ideal $I$ is called a minimal prime ideal of $I$ in $N$.

If $x$ is an element of $N$, then the principal ideal generated by $x$ is denoted by $(x)$. If $S$ is a subset of $N$, we write $N \setminus S = \{n \in N | n \notin S\}$. A subset $M$ of a near-ring $N$ is called an $m$-system if for any $a, b \in M$ there exists $a_1 \in (a)$ and





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$b_1 \in (b)$ such that $a_1 b_1 \in M$. A subset $S$ of a near-ring $N$ is called an *Sp*-system if for any $s \in S$, there exists $s_1 \in (s)$ and $s_2 \in (s)$ such that $s_1 s_2 \in S$.

## 2.

In this section we prove the main theorem. Before proving this theorem, we state a lemma.

LEMMA 2.1. *Let $I$ be an ideal in a near-ring $N$ and $M$ be an m-system in $N$ such that $I \cap M$ is empty. Then*

(i) *There is an m-system $M^*$ maximal relative to the properties: $M \subseteq M^*$, $I \cap M^*$ is empty.*

(ii) *If $M^*$ is an m-system maximal relative to the properties: $M \subseteq M^*$, $I \cap M^*$ is empty, then $N \setminus M^*$ is a minimal prime ideal of $I$ in $N$.*

PROOF. Immediate from 2.75, 2.80 and 2.81 of G. Pilz (1977).

Now we prove the main theorem.

THEOREM 2.2. *If $I$ is any semiprime ideal in a near-ring $N$, then $I$ is the intersection of all minimal prime ideals of $I$ in $N$.*

PROOF. Let $I$ be any semiprime ideal of $N$. Then by 2.89(b) of G. Pilz (1977), $N \setminus I$ is an *Sp*-system. So by 2.92 of G. Pilz (1977), we have that for each $s \in N \setminus I$ there exists an $m$-system $M$ in $N$ such that $s \in M \subseteq N \setminus I$. So $M \cap I$ is empty. Now by the above lemma, there exists a minimal prime ideal $P$ of $I$ such that $P \cap M$ is empty. But $s \in M$. Hence $s \notin P$. Therefore $I = \bigcap_{P \supseteq I} P$, where $P$ ranges over all minimal prime ideals of $I$. Thus every semiprime ideal $I$ is the intersection of all minimal prime ideals of $I$.

COROLLARY 2.3. *If $I$ is a semiprime ideal in a near-ring $N$, then $I$ is the intersection of all prime ideals containing $I$.*

PROOF. By the above theorem, $I = \cap P$, where $P$ ranges over all minimal prime ideals of $I$, $\supseteq \bigcap_{P \supseteq I} P$, where $P$ ranges over all prime ideals containing $I$. Therefore $I$ is the intesection of all prime ideals containing $I$.

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## References

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