Mathematical Notes.

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EDITED BY P. PINKERTON, M.A., D.Sc.

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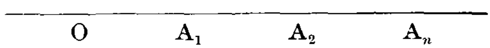
The Inequality Theorem $(x^m - 1)/m > (x^n - 1)/n$, $m > n$.—I have sometimes found the following geometrical method of attack useful in explaining this theorem to students of only average ability. The Lemmas introduced are almost self-evident, and admit of simple analytical demonstration.

(1) Let $a_1, a_2, \ldots, a_n$ be n positive quantities.

They may be represented graphically by n points A_1, A_2, ..., A_n



on an oriented straight line on which they arrange themselves in order of magnitude. For our present purpose we may speak indifferently of the n points, or the n quantities they represent.

Lemma I.—The average a of the n quantities furnishes a point A lying between A_1 and A_n.

Lemma II.—Let $A_1A_2 \ldots A_n \ldots A_m$ be m points in order on a straight line. The average of n consecutive points at the extreme left, or at the extreme right, lies on the same side, left or right, of the average of the m points.

(2) Consider the G. P.

$$1 + x + x^2 + \ldots + x^{n-1} + \ldots + x^{m-1}.$$

If $x > 1$ the corresponding points on the line are arranged from left to right, and the average of the first n points is to the left of the average of the m points.

Hence $$\frac{x^m - 1}{m(x-1)} > \frac{x^n - 1}{n(x-1)}.$$

Remove the factor $x - 1$ which is positive

$$\therefore \quad (x^m - 1)/m > (x^n - 1)/n.$$

But if $0 < x < 1$ the points are arranged from right to left, with the result

$$\frac{x^m - 1}{m(x-1)} < \frac{x^n - 1}{n(x-1)}.$$

But $x - 1$ is now negative, and its removal reverses the sign of inequality.

Hence in both cases

$$\frac{x^m - 1}{m} > \frac{x^n - 1}{n}$$

if x is positive, and $m > n$.

C. TWEEDIE

A Rule for Resolving Integral Algebraic Expressions into Factors.—Professor Chrystal remarks (Algebra, Chap. VII. §4) that "for tentative processes no general rule can be given." The tentative processes consist in arranging the terms in groups in such a way as either to manifest a factor common to these groups or aggregates of terms, or to bring the expression under one of the Standard Forms of which the factors are already known, such as $a^2 - b^2$, $a^3 - b^3$, $a^3 + b^3$.

The following Rule seems to me to possess sufficient generality and efficiency to be worth stating:

Let the expression be arranged according to powers of the letter which occurs in the simplest manner, *i.e.* let each group consist of the terms which contain one particular power of that letter.

The phrase "in the simplest manner" is to be interpreted usually as "in the smallest *number* of different powers," and when there is one letter occurring *only* in the first degree, that one ought to be chosen.