RAFFAELLO CAVERNI

An Excerpt from *History of the Experimental Method in Italy**

We consider this merit,[1] however, to have almost no value in comparison to one which we wish to acquire from the offended worshippers of Galileo. We announce to them that after having identified and reordered the scattered writings which complete the sixth dialogue as far as percussion is concerned, we were also able to reintegrate the dialogue with regard to the use of a little chain to provide a rule for aiming artillery, without having to resort to laborious calculations.

Toward the end of the Fourth Day[2] Salviati says that little chains held loosely at their extremities hang in a curve very much resembling a parabola. He then alludes to a not unimportant use for such hanging chains which he promises the interlocutors to deal with later, diverting the conversation at first to the demonstration concerning streched rope. After this demonstration Simplicio reminds Salviati of his promise to explain "the utility that may be drawn from the little chain, and afterward give us those speculations made by our Academician about the force of percussion" (Galilei 1842–56, 13:266).[3] But the hour being so late there was not enough time to deal with the mentioned topics, Salviati suggests postponing the meeting to a more opportune time.

Apparently in the next meeting they intended to deal first with the little chain and then with percussion. The program was then changed, for whatever reason, by their discussing the second subject first. This, however, did not relieve Salviati of fulfilling his promise. That he really meant to keep it is evident from the fact that Salviati gauged the time of the conversation very well. When the discussion of the first subject ended, which also covered the theory of collision, it was only nine o'clock. It was then possible to spend what was left of the evening satisfying the curiosity of those who desired to know to what use the little chain might be put.

* This is a translation of pages 143–154 of volume 5 of Raffaello Caverni, *Storia del metodo sperimentale in Italia, 1891–1900*, 6 vols. Firenze: Civelli, 1891-1900. All footnotes are by the translator. The list of references can be found in a bibliographical section at the end of the appendix.

[1] In the preceding part of the chapter, Caverni argues that in a truly complete edition of Galileo's works a text written by Torricelli should have been published as a completion of Galileo's fragment concerning the force of percussion. Caverni claims to have the merit of having recognized the connection between the two texts.

[2] Caverni comments here on the fourth part of *Discorsi e dimostazioni matematiche intorno a due nuove scienze* (Galilei 1638).

[3] Caverni's bibliographical references have been standardized. For the accepted edition, see Galilei 1890–1909, 8:312. We adopt the translation by Stillman Drake (Galilei 1974, 259).

Notwithstanding, after more than a century and a half, their curiosity is still not satisfied, and it did not, and does not seem to matter at all to any of the most fervent Galileans. Therefore, we are the first and only ones among them to have searched industriously and finally found that second part of the Galilean dialogue, which, along with the first concerning percussion, gave the good Salviati and his friends subject matter to philosophize upon until evening. We shall abstain from narrating how we made this discovery among certain jumbled manuscripts given to us by a friend for examination because we believe that our readers would rather wish to learn without delay what we copied from it. This reads as follows:

SAGREDO. Your reasoning, Mr. Salviati, has completely convinced me that the forces of natural percussion and collision are infinite, so that you can now save yourself the trouble of further discussion. As far as I'm concerned, you may now keep your other promise, which was to tell us about the usefulness that our Academician hoped to obtain from little chains when applied to plot many parabolic lines on a flat surface. But I see Mr. Aproino looking quite surprised.

APROINO. You have understood me, Mr. Sagredo, because I find your proposal quite new.

SAGREDO. You are right. I had not realized that Your Lordship was not present on the evening of our last meeting when, before leaving, Mr. Salviati gave Mr. Simplicio and me to understand that, following the demonstration on the force of percussion, he would have added the explanation about the little chains. These, when held at their extremities, he said, naturally accommodate the curvatures of parabolic lines.

APROINO. To the first surprise you now add a further one, greatly arousing my curiosity, which is to see the purpose of something that has always been without any meaning to me or anyone else. I therefore join you in requesting Mr. Salviati to begin without further ado this new reasoning.

SALVIATI. Mr. Aproino, who entered too late into our conversation, perhaps does not know that in our previous meeting we read the demonstrations by the Academician concerning the new science of projectiles. This science was founded on the allegation that the projectiles, disregarding any air impediment and any other extrinsic causes, describe in the air a curved line indeed no different from the parabola. Henceforth some absolutely reliable rules concerning the marksmanship of bombardiers were unexpectedly suggested. Having at first established the impetus of an instrument, that is the force with which it shoots a projectile perpendicularly upwards with a given amount of gunpowder, it is possible to determine to what distance the instrument, held at different inclinations, would shoot the same ball merely by using mathematical calculations, presented by the Author in very exact tables for military use. But the use of these tables still required some knowledge of the doctrines, and in any case it was also necessary to consult a book

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and handle the instruments of learned men, requirements which were not always easily met in a military camp. Therefore the same Academician, having observed that the curvature of a little chain has the shape of a parabola, had the idea to reduce to a simple manual exercise what the Philosopher had written in his books. Suppose Mr. Aproino, you have two pins tacked at the extremities A and B of a horizontal line on a flat surface of wood or cardboard (*figura 46* [fig. 1]). A very thin chain loosely suspended from the pins will hang along lines ACB in the figure of a parabola, the height of which will be CD and the width AB. In order to obtain higher or lower parabolas passing through a given point, for instance E, whilst retaining the same width, you have only to pull one end of the chain. Imagine now that these curves represent the paths traced in the air by a projectile passing through B. You are now easily able to understand how it is possible to measure the angles DBF and DBG by tracing the tangents BF and BG, and in this way find out the elevation of the device required for a certain width and height of a shot. Consequently a quadrant, correctly divided and applied to the board with the center at B, would be enough to resolve this and other similar problems.

APROINO. I understand well how such a device would be very convenient to soldiers. To them it would be of no less service than the proportional compass that the same inventor described and divulged in order to facilitate the geometrical and arithmetic operations for people lacking the endurance needed to follow the rules taught in books, because they are occupied with and distracted by many other tasks. But I have some difficulties in performing the operations conceived above. The first one is how to get the little chain to leave a mark on the surface when it touches it.

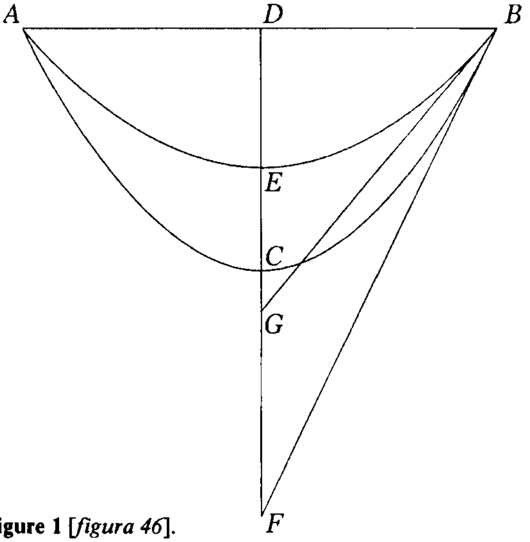

Figure 1 [*figura 46*].

SALVIATI. The easiest way, that does not even depart too much from the required precision, is to dot with a stylus or pen. But our Academician, since he wanted to have a drawing and keep it for use as a print, used to puncture the cardboard with a pin along the contour of the chain. By pouncing he could then reproduce the same drawing elsewhere, as many times as he liked. Do you see this cardboard, so punctured and blackened along the three lines, over which the feather duster full of charcoal powder had been swept? It was prepared in order to determine the degree of elevation for parabolas of various heights but with the same width of 465. On finding the Author in his studio one day, intent on these activities, I asked him for this cardboard. It was no longer of any use to him since he had made another similar and more precise one. Although it is a cheap thing for common people, philosophy and friendship induce me to hold it in high esteem.

APROINO. Mr. Salviati, by reason of friendship I certainly would not hold it in any less esteem. But as far as philosophy is concerned, for my part, I would not be content in prizing the invention's value until it has been demonstrated to me that the line described by the curvature of a chain is really a parabola. And since you assert it with such conviction, I cannot believe that you do not have some demonstrative evidence. I beg you to disclose such evidence to me, so that I am then able to place the same value as you do and I would like to, upon the invention of our common friend.

SALVIATI. The demonstration you are asking for consists of factual evidence. Delineate, with the devices suggested by geometricians and according to the rules they teach, the parabolas ACB and AEB as in the preceding figure or in any other you like, and then place a little chain over them. You will find that the chain coincides to a hair's-breadth with each of the geometric parabolas that you have drawn.

SAGREDO. I made this experiment several times and found that it works, especially in the case of parabolas with an elevation of less than 45 degrees. However, I confess to you Mr. Salviati that I was never convinced of this method of mechanically drawing curves as I would be by a proper mathematical demonstration. Such a demonstration is required, I think, in order to make of the little chain a military instrument perfectly suited to ballistic operations as the compass is suited to arithmetic and geometrical operations. Therefore, I also share Mr. Aproino's perplexities.

SALVIATI. Lucky for me that I am able to amply satisfy both of you, having received from our Academician the mathematical demonstration you are wishing for. Indeed, I will tell you for your relief, that the Academician himself confessed to me several times that he was not content with entrusting such an important conclusion to mere eyesight, which could be suspected of some fallacy. Moreover, matter does not always match the purposes of experimental art. For that reason, our Academician proposed the use of his new military instrument only when he was able to demonstrate that the line along which the chain links arrange themselves is the same as the one traced by projectiles in the air. Likewise I would not have promised to expose you to this matter, was I not scientifically certain about it myself.

SAGREDO. I suppose that this certainty cannot depend on anything but the doctrines concerning the new science of motion which have already been demonstrated.

SALVIATI. It could not be other than as you say. Such doctrines are derived in particular from one of those propositions that, — you will remember —, you heard me reading whilst treating the resistance of solids to break. Imagine that all the links of a chain are threaded through a bar suspended horizontally at both ends. The bar suddenly yields at the points where the weights rest while only its extremities remain immobile. All the other links in the middle are now loosened and will fall. They will not be able to arrange themselves in a new state of equilibrium unless each link has fallen as much as its own momentum requires. It is the disposition of those fallen links, beginning from the second to the middle one, that determines the line of curvature of half of the chain, which is, of course, identical to the other half. You understand that everything depends on knowing with what momentum the links gravitate according to the various distances from each of the supports, presuming the links to be identical along the whole length of the bar.

APROINO. Allow me, Sir, to help my weak intelligence with a bit of drawing. Let CD be the bar resting on its extremities (*figura 47* [fig. 2]); assuming that the weights of two links, one at B and the other at A, are represented by the equally heavy bodies H and F hanging from the bar at those same points B and A, you propose to resolve the question of what is the ratio of the momentum of weight H at B to that of the same weight, or of its equal F, at A. I do not find clear principles to resolve this question in the mathematical science that I have learned up to now through teachers and books. Neverthe-less, it seems to me that those principles are not different from the mechanical speculations concerning the balance. Therefore, I would not see what pro-positions concerning the resistance of solids to breaking have to do with this question even if I had had, like Mr. Sagredo, the luck of attending your past meetings.

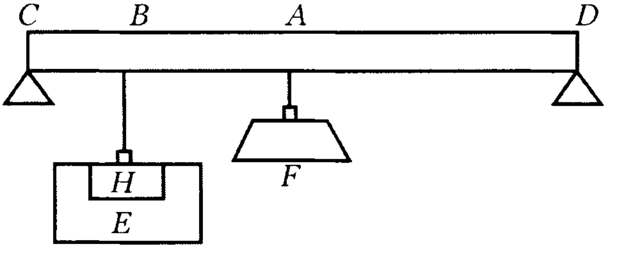

Figure 2 [*figura 47*].

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SALVIATI. You need to know, though, that the new science of resistances depends upon nothing but on Archimedes' ancient science of balance, if you consider the geometric line at the extremities of which the weights are added to be a stiff rod that can break. Let the balance AB be supported at C (*figura 48* [fig. 3]). You say that according to the doctrine of weights in equilibrium the balance will be in equilibrium when the weight B resists being lifted adequately to the power of the weight A to heave. But the same ratio of power and resistance can be applied to the instrument, if we consider the line AB to be a stiff rod, which will remain in equilibrium every time the power of A to break equals the resistance of B against breaking. If those two opposite powers of acting and resisting are the strongest in producing their effects, any minimal addition to the one or detraction from the other would be enough to unsettle the equilibrium, that is, to bend the rod by pulling it down and turning it around the center C, as it happens with the simple balance.

SAGREDO. Now, Mr. Salviati, you make me conjecture that the proposition of the treatise on resistances you have just mentioned could be the twelfth, which, if I remember right, you formulated in this way: "If two places are taken on the length of a cylinder at which the cylinder is to be broken, then the resistances at those two places have to each other the inverse ratio [of areas] of rectangles whose sides are the distances of those two places [from the two ends]."[4] However, I must confess that regarding this proposition I am assailed from two sides: the first attack comes from considering the proposition in itself, and the second from applying it to the momentums of the same weight placed at various distances from the middle of the rod. In fact, I never doubted the truth of the mentioned proposition, but the way you demonstrated it. You based your demonstration on the assumption, dubious in my opinion perhaps because I do not understand it well, that the momentums of heavy bodies hanging from a balance are to each other in the ratio compounded from the distances from the support and the weights.[5] So much for

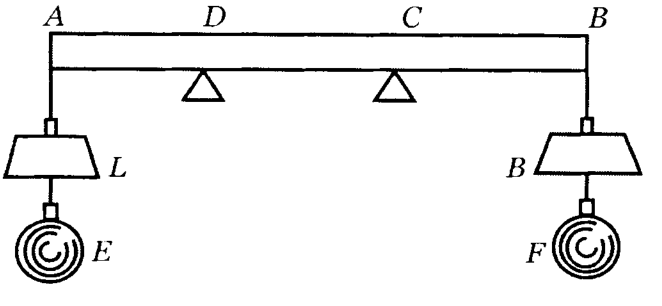

Figure 3 [*figura 48*].

[4] The quoted sentence is taken from *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* (Galilei 1890–1909, 8:176). We adopt the translation by Stillman Drake, who numbered the proposition as the eleventh (Galilei 1974, 133).

[5] In accordance to the context the word "weights" renders Caverni's "moli," although in Galileo's language "mole" means generally "volume."

the proposition itself. As to its intended application to the momentums of weights hanging from a balance supported at its ends, my doubt is due to the consideration that in the twelfth proposition you set the cylinder in which the breakage has to take place as having the supports at middle points.

SALVIATI. Do not doubt, Mr. Sagredo, that I will find the way to satisfy your mind as to both of your doubts. Starting with the first, I will not deny that the ratio of momentums as it shines through the twelfth proposition of the treatise on resistances leaves something to be desired. We could, however, easily compensate for this fault by referring to the definition of momentum given by the authors of mechanical science and to the known laws of weights in equilibrium on a balance. From these laws derives, in fact, that the machine remains in equilibrium when, as in the preceding figure, the weight of A multiplied by the distance AC from the support is the same as the weight of B multiplied by the distance BC. If you give the name *momentum* to the tendency or impetus of going downwards, compounded from gravity and position, you will have already concluded that the momentums in the balance have the ratio compounded from the distances and the weights.

Because of these considerations, the Author of the treatise on resistances did not deem it necessary to demonstrate something that can be so easily concluded from Archimedes' ancient theorems. But then the Academician wanted to expound the propositions he had ultimately demonstrated for serving as fundamental to the new little treatise on the use of little chains. Since he wanted to start by introducing the momentums, according to the ratio of which the links fall down more or less, he thought it better to formulate the proposition that I will read to you from this sheet, in the original form in which it was written. For us, too, this proposition will be the first of all subsequently appearing in our reasoning.

Proposition I.[6] The momentums of weights hanging in the balance have the ratio compounded from the ratio of the weights itself and from the ratio of the distances.

Let the weights DE and F hang at the distances AB and BC (*figura 49* [fig. 4]). I say that the momentum of weight DE have to the momentum of weight F

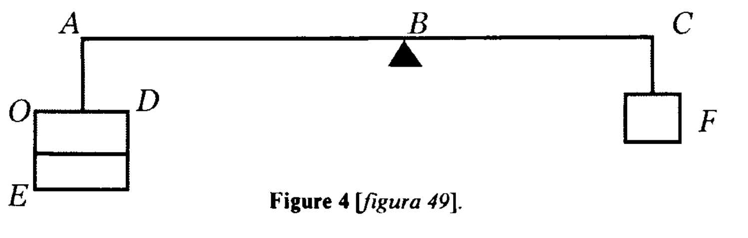

Figure 4 [*figura 49*].

[6] Proposition I and its demonstration are in Latin. The original Galilean fragment and figure has been edited by Favaro in 1898 in Galilei 1890–1909, 8:367–68. The figure there differs from that reproduced here by Caverni.

the ratio compounded from the ratio of weight DE to weight F and the ratio of distance AB to distance BC. As AB is to BC, so let weight F be to weight DO. Therefore, since weight F and weight DO have the inversed ratio of the distances AB and BC, the momentum of weight F will be equal to the momentum of weight DE. Thus, whatever the three weights ED, F and DO may be, the ratio of weight ED to weight DO will be compounded from the ratios of ED to F and of F to DO. Moreover, as weight ED is to weight DO, so is the momentum of ED to the momentum of DO, since they hang from the same point. Therefore, since the momentum of DO is equal to the momentum of F, the ratio of the momentum of ED to the momentum of F will be compounded from the ratio of weight ED to weight F and from that of weight F to weight DO. Moreover, weight F has been set to weight DO like distance AB to distance BC. Therefore it follows that the momentum of weight ED has to the momentum of weight F the ratio compounded from the ratios of weights ED and F, and from those of distances AB and BC.

APROINO. I thank you, Mr. Salviati, and at the same time bless Mr. Sagredo's doubts which gave the opportunity to expose a theorem, which I do not remember having ever met when reading what has been written on similar matter by other authors. Moreover, the principles from which the conclusion follows are so clear that they enable me to glimpse many other useful consequences for the doctrine of motion.

SALVIATI. Sir, you will soon see the applications we will make of these principles proving the usefulness that you have shrewdly perceived, but now it is better to proceed in resolving the other doubt of Mr. Sagredo. And it seems to me that on his serene countenance I can read the satisfaction he has already felt regarding the first doubt.

SAGREDO. You should say not just satisfaction but delight because this demonstration of the ratio of momentums is for me like for Mr. Aproino something completely new. And even if I could perhaps be able to understand by myself the reasons for the step from the cylinder sustained in the middle to the cylinder supported at its ends, to the point of yielding because they are both overburdened by the same weights, I am waiting that you alleviate my labor and convince me, better than I can myself, of having seen the truth.

SALVIATI. Very willingly I would leave to you the whole pleasure of finding out how it is true that we have the same conditions of equilibrium in the geometric balance and in the rigid rod near to breaking, whether the supports are in the middle or at the ends, being that indeed very easy to demonstrate. But since you want me to lighten your burden, I will again call your attention to the balance AB just drawn in *figura 48* [fig. 3]. As you well know, it remains in equilibrium around point C when weight A is to weight B as distance BC is to AC. By composition we will find that weights A and B together are to single weight A or to single weight B as BC and AC together, that is AB, are to BC or to AC. Whence it is evident that the balance remains in equilibrium when the support is at C and the weights at A and B as well as when the supports are set at A and B and the sum of those two same weights at C. Proceeding then from the geometric balance to the solid cylinder, you will understand that if A and B are the maximum forces to which the cylinder sustained at C resists without breaking, the sum of the two weights at C will give the measure of the maximum force to which the solid can resist being broken at that same point when it is instead sustained at A and B.

Let us now remember the twelfth proposition about resistances: With it we demonstrated that if forces A and B are the minimum ones for breaking at C, and forces E and F are equally the minimum ones for breaking at D, the forces A and B have to E and F reciprocally the same ratio as the rectangle ADB has to the rectangle ACB. But according to what was already said and agreed upon, it is the same to keep the supports at C or D, and the weights at A and B or at E and F, as to move the supports to A and B, and the weights A and B together to C or the other weights E and F together to D. We will say therefore, and let this be the second proposition, that for a cylinder supported at its ends A and B the weight that can break at C is to the weight that can break at D, that is, the resistance at C is to the resistance at D, as the rectangle ADB is to the rectangle ACB. Thus, the demonstration would now be the same one that was already given and should be repeated only in favor of Mr. Aproino, who was not present then.

APROINO. Sir, with your learned reasoning you have prepared the way for me so well that I do not doubt of being able to trace that demonstration by myself. Anyway, in order not to delay for too long in deducing the rest, — which is the purpose of our conversation, — I will presume as true the proposition that you have put as the second one in the row.

SALVIATI. If so, there is nothing left but to make one step in order to achieve our main purpose which was to know with what various momentums the links gravitate on the bar through which we imagined they were threaded, hence to deduce the ratios of descents to lastly conclude what is the line in which the chain curves. At first I enounce, with reference to the figure drawn for that first purpose,[7] a third proposition that says: The momentum of weight F at A is to the momentum of the same weight or of an equal weight H at B as the rectangle CAD is to the rectangle CBD.

SAGREDO. Therefore, the momentums are to each other in the inverse ratio as the resistances, and the chain's link at B will have less impetus in falling than the link at A because the former encounters in the bar, that resists it more, a greater impediment. Similarly, I understand why the chain from the first link through to middle one diverges more and more from the horizontal arrangement it had when it was threaded through the bar, having been abandoned to its own weight. It seems to me, also, that I can distinctly see the

[7] The figure referred to is *figura 47* [fig. 2].

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dawning of that light of truth that you will soon reveal to our eager eyes, and since it is unpleasant to wait, proceed, Mr. Salviati, to demonstrate that the momentums of weights F and H have to each other the same ratio as the rectangles whose sides are respectively the distances of those points [from the two ends].

SALVIATI. After all that has been said and agreed upon by you and Mr. Aproino, the demonstration is easy and speedy. For, keeping in sight the same figure, let us suppose that weight F is the measure of the resistance at A, and that the measure of the resistance at B is weight H increased to E. For the second proposition the resistance at A will be to the resistance at B, that is, the weight F will be to weight E as the rectangle CBD is to the rectangle CAD. But since the weights H and E are attached at the same point of the balance the ratio of their momentums is the same, that is, the momentum of H is to the momentum of E (which is equal to the momentum of F for having the same power to break the bar) as the weight F is to weight E. Therefore the momentum of H is to the momentum of F as the rectangle CBD is to the rectangle CAD, which is what I wanted to demonstrate.

Through this ordered series of propositions we came finally to the point where we can find what we were seeking from the beginning of our reasoning and was said to sum up everything. That is, we can learn with what momentum the different links of a chain tend to fall when they are dropped from the bar which held them. Let the bar be represented by the horizontal line HD (*figura 50* [fig. 5]). Let us assume that the link at F because of its impetus or momentum can fall to E for the entire perpendicular line FE and that likewise the link at N can fall for the entire line MN. Since the descents must be proportional to their momentums, and accordingly to what was already demonstrated, FE will be to NM as the rectangle HFD is to the rectangle HND. Now, in order to conclude that the points E, M and all the others corresponding to the links of a chain are really in a parabola, what else is left but to invoke a theorem, that you will not find written by any ancient or modern author but has been demonstrated by our Academician by reason of his treatise on resistances? Now I want to propound to you that theorem which says: The

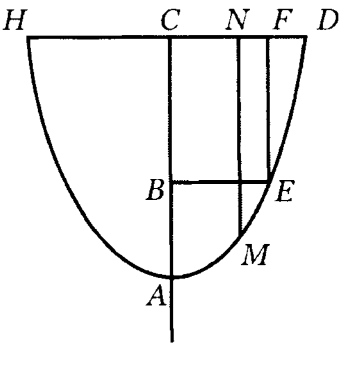

Figure 5 [*figura 50*].

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parallels to the diameter of a parabola, whose base they cut perpendicularly, have to each other the same ratio as the rectangles having as sides the segments. Hence, e.g., in the drawn figure the parallel NM and FE to the diameter AC are to each other as the rectangles HND and HFD.

APROINO. When I recently visited father Bonaventura Cavalieri in Bologna and, in relation with my instrument to strengthen hearing, we entered into a conversation on the conics, he told me this very same theorem but I did not quite understand whether he presented it as his own invention or Mr. Galileo's.

SALVIATI. It could very well be that father Bonaventura as well, whom our friend uses to call the Archimedes of our times, had encountered this very same property of the parabola, so useful for many demonstrations of Mechanics and Geometry. But I can assure you that I got news of it during my conversations with the Academician, many years before Cavalieri's genius was ripe to produce such fruits.

SAGREDO. You have now reminded me of having heard the same theorem in Padua when our mathematician was teaching at our University. Now, since truth does not deny itself to anyone who seeks it with desire and along the same right ways, solace us, Mr. Salviati, by showing it anew unveiled to our eyes.

SALVIATI. I am pleased to be able to completely satisfy you this time, too, since you do not, actually, need any other knowledge but that which you had already when from the simple generation of the parabola I immediately concluded that the diameters are to each other like the squares of the ordinates.[8]

APROINO. I remember well the demonstration given by Apollonius in his *Conics* and therefore I also do accept as known that the line AC is to AB as the square of CD is to the square of BE.

SALVIATI. Being it indeed so, let us divide and we will obtain that AC minus AB, that is BC as well as its equal EF is to AC like the square of CD minus the square of BE is to the square of CD. But, as can be easily deduced from the fourth proposition of Euclid's second book, the difference of two squares is equal to the rectangle obtained from the sum and the difference of the roots. Therefore, the square of CD minus the square of BE will be equal to the line CD plus BE, i.e. HF, multiplied by the line CD minus BE, i.e. FD. Said in another way, the difference of the two mentioned squares will be equal to the rectangle HFD. Hence, EF will be to AC as the rectangle HFD is to the square of CD. In the same way we will demonstrate that NM is to AC as the rectangle HND is to square of CD. Hence, since the two ratios have equal consequents and therefore the antecedents must be proportional, we conclude that FE and MN are to each other like the rectangles HFD and HND, as I promised you in order to satisfy your wish.

[8] The proposition referred to can be found in Galilei 1890–1909, 8:270–71.

The dialogue breaks off at this point but the treatise on the use of the small chains is, in any case, complete; what we feel might be missing is only the more or less ceremonious farewell of the interlocutors. Anyway, even if our readers agree that the entire argument is included in the transcript, they could ask for the reasons which induced us to ascribe this text to Galileo. In this respect one has to distinguish between form and content. To prove with certainty that the latter is a genuine Galilean one would be enough to adduce the fact that the theorem concerning the momentums compounded of distances and weights read by Salviati is in Galileo's handwriting, in the codex and on the sheet we referred to in the IVth paragraph of chapter VIII of the previous volume;[9] that also in Galileo's handwriting is the proposition, likewise quoted by us at the same place,[10] concerning equal weights that operate on a bar supported at its extremities with momentums homologically proportional to the rectangles having as sides the distances from the supports; and finally that in Galileo's hand is the drawing reproduced by us in the mentioned volume and chapter.[11] As he hinted, Galileo in this drawing wanted to apply the last mentioned proposition to the chain's links, with the manifest intention of concluding that the chain's curvature is parabolic.

And we do not want to carry on our exposition without pointing out that the discovery of the dialogue on small chains, which luckily befell us these last few days, dispelled some of our doubts and made clear some facts still obscure to us when we were relating our story in the mentioned chapter VIII. There we wondered how the handwritings reported herein before could have been left among other useless papers, as if their author, although he could have enriched with these results his treatise on resistances, wanted to leave it with this defect so that later Mssrs. Cavalieri, Torricelli, and Viviani, in order to satisfy the needs of science, could compete in emending it. Now we did understand that the propositions left handwritten were meant for a treatise quite different from that concerning resistances. Far from having been demonstrated only to be then rejected, as it seemed to us when we found them so neglected, these propositons had rather to serve as rich warp upon which the rest of the conversation would have been woven, so leading into evening the day begun with the treatment of percussion.

Let us now resume to tell the reasons proving that the discourse on the use of small chains transcribed by us is shaped upon Galileo's concepts. We can add that a cardboard punctured with a pin along parabolic lines in order to reproduce by pouncing the same figure, this very cardboard with black smudges left by the feather duster and in the conditions described by Salviati, bearing at its opposite corners the repeated handwritten words *amplitudo tota 465*, is still kept sewn in

[9] Caverni 1891–1900, 4:484. For the standard edition of the mentioned fragments and figures, see Galilei 1890–1909, 8:367–70. One of the fragments is reproduced there in *facsimile* (ibid., Appendice, car. 43r).

[10] Caverni 1891–1900, 4:485.

[11] Ibid., 495.

place of folio 41 in Volume II of Part V of Galileo's Manuscripts.[12] But the most authoritative confirmation of what we intend to prove is given by Viviani's testimony, to whom we believe we should attribute the writing of the dialogue or rather of the dialogue's fragment, found by us in a copy that must be of that time.

In the margin of page 284 of the Leiden edition, at the place where Sagredo suggests that it is possible to dot with a little chain many parabolic lines and Salviati replies: "*That can be done, and with no little utility, as I am about to tell you*," Viviani appended a similar note: "By means of this small chain Galileo perhaps found the elevations to hit a given target" (Mss. Gal., P. V, T. IX).[13] Furthermore, in one of those notes written by Viviani on folio 23[14] of Volume IV of the same Part V of the collection, he expressed a similar doubt in this other way: "See at page 384,[15] the last sentence, which utility Galileo meant, whether in measuring the parabolic line or in finding the propositions concerning projectile motion."

All Viviani's doubts in this respect were solved when he came upon the handwritten slips bearing the propositions concerning momentums exerted by equal weights on a balance supported at its extremities, propositions from which one can deduce the impetus' strength and the descent's length of each link of a chain. While arranging these dispersed propositions Viviani remembered what he had heard his master saying in the refuge of Arcetri, and then he himself rewrote this little treatise on the use of small chains, about which nothing was known but the allusions made by Salviati in the evening of the Fourth Day. In this way both parts of the last conversation would come to an end, in accordance with the given promises. It is, therefore, natural that Viviani worked out, on the basis of newly found documents, what remained to be said on the use of little chains for military purpose, in order to add this part to the dialogue and so completing it, while he intended to publish the part concerning percussion among the posthumous works that were to be dedicated to the king of France, in attachment to his biography of Galileo. But since he failed in his hopes of gathering in a book the works that his master ultimately considered writing, Viviani was content to satisfy the public on this matter with this information, in the "Summary" he appended to the *Universal Science of Proportions*:

> Now it remains to be said what I know about the use of small chains, concerning which Galileo made a promise at the end of the Fourth Day. I will relate it as Galileo intimated it when, he being present, I was studying his

[12] Caverni refers to a document kept in the Galilean Collection at the Biblioteca Nazionale Centrale, Florence, call-nr.: Ms. Gal. 72, folio 42r. See Drake 1979, 238–39; Damerow et al. 1996, 5; Renn et al. 1998, 9–10.

[13] Caverni is referring to a copy of Galilei 1638, with handwritten notes by Viviani, kept in the Galilean Collection, call-nr.: Ms. Gal. 79. For the standard edition, where Viviani's notes are not entirely reported, see Galilei 1890–1909, 8:310. For Salviati's sentence we adopt the translation by Drake (Galilei 1974, 257).

[14] Viviani's note is on folio 33 (Ms. Gal. 74, folio 33r).

[15] Viviani writes "page 284" and not 384, referring to Galileo 1638.

science of projectiles. It seemed to me then that he intended to make use of such very thin chains hanging from their extremities over a flat surface in order to deduce from their different tensions the rule and practice of shooting with artillery at a given target. Our Torricelli, however, wrote adequately and ingeniously about that at the end of his treatise on projectiles, so that the loss is compensated.

If I remember well, Galileo deduced that the natural bend of such small chains always fits with the curvature of parabolic lines from a reasoning similar [to this]: Heavy bodies must naturally fall according to the proportion of the momentums which they have at the places from which they hang, and the momentums of equal heavy bodies, hanging from points of a balance supported at its extremities, are to each other in the same ratio as the rectangles having as sides the parts of that balance, as Galileo himself demonstrated in the treatise on resistances. And, according to the theory of conics, this ratio is the same as the one existing between the straight lines which from the points of that balance, taken as base of a parabola, can be drawn in parallel to the diameter of that parabola. And, finally, since all the links of a small chain, — which are like as many equal weights hanging from points on the straight line that connects the extremities where the chain is attached and serves as base of the parabola, — must fall as much as allowed by their momentums and there must stop, these links must, therefore, stop at those points where their descents are proportional to their momentums at the places from where they hang in the last instant of motion. These then are the points adapting to a parabolic curve [which is] long as much as the chain and whose diameter, which rises from the middle of the said base, is perpendicular to the horizon. (Viviani 1674, 105–06)

It is easy to see summarized in these words the dialogue we have transcribed, the loss of which Viviani thought was compensated for by Torricelli. But Torricelli, in fact, at the end of his treatise on projectiles[16] ingeniously describes a new type of square that could be effectively used by bombardiers; he does not, however, say a word about the instrument conceived by Galileo, nor about the order of the propositions which should give a greater certainty of mechanical science to Galileo than to the instruments to measure the force of percussion [which he] imagined and described.[17] The dialogue as published by Bonaventuri[18] lacks, therefore, its

[16] Reference to the last part of "De motu proiectorum" in Torricelli 1644, 204–43. For a modern edition, see Torricelli 1919–1944, 2:197–232.

[17] The sentence is difficult to understand. We interpret it in the sense that Torricelli did not give any information about the logical order of the propositions concerning the alleged connection between catenary and parabola summarized by Viviani. After that Caverni seems, in our opinion, to suggest that Galileo worked on these propositions in order to acquire scientific certainty as far as the mentioned connection, and not in order to give scientific foundation to the use of devices described in "Della forza della percossa" (Galilei 1890–1909, 8:319–46; Galilei 1974, 281–306).

[18] Reference to "Della forza della percossa" in Galilei 1718, 2:693–710.

›est part which the devoted public would never have expected could be restored by us, sacrilegious offenders of the Numen. But that is, it seems, how things go in the religion of science as well as in all mundane affairs, whose care we leave to others while we come back to the thread of our previous reasoning.

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