108.09 A visual proof that $b^e < e^b$ when $b > e$

In a recent visual proof ([1]), the author provided a visual proof of the inequality $\pi^e < e^\pi$. However, their visual proof can be used to show the more general inequality $b^e < e^b$, where $e < b$.

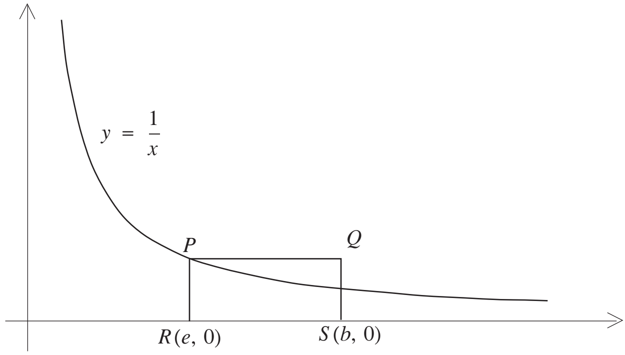

FIGURE 1

$$\ln b - 1 = \int_e^b \frac{dx}{x} < \frac{1}{e}(b - e) = \frac{b}{e} - 1$$

and so $$b^e < e^b.$$

Reference

1. Bikash Chakraborty, A visual proof that $\pi^e < e^\pi$, *Mathematical Intelligencer* **41** (2019) p. 60.

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108.10 Proof without words: $\tan\frac{\pi}{12} = 2 - \sqrt{3}$, $\tan\frac{5\pi}{12} = 2 + \sqrt{3}$

The standard proof of $\tan\frac{\pi}{12} = 2 - \sqrt{3}$ is to use the less well-known formula

$$\tan\alpha = \frac{-1 + \sqrt{1 + \tan^2 2\alpha}}{\tan 2\alpha}$$

for $\alpha = \frac{\pi}{12}$ and the well-known value $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$. Using only the last fact,

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