RESULTS FROM LUNAR LASER RANGING DATA ANALYSIS

J. O. Dickey
J. G. Williams
C. F. Yoder
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, CA 91109 USA

ABSTRACT. The lunar laser range data taken at McDonald Observatory between August 1969 and May 1980 has been analyzed. The simple rms residual for the 2954 ranges is 31 cm. Results of the analysis include GM_{earth} = 398600.45 ± 0.02 km^3/sec^2 and a secular acceleration of the lunar orbital mean longitude of $\dot{n} = -23.8 \pm 1.5''/century^2$ which yields a Q of 12.3 at semidiurnal frequencies. The lunar harmonic C_{30} is $(-8.7 \pm 1.1) \times 10^{-6}$ and the lunar rotational dissipation $k_{2m}T = (4.7 \pm 0.5) \times 10^{-3}$ day. Also resulting from the solution are geocentric coordinates of McDonald accurate to 30 cm, including the first value for the longitude with the new IAU constants and a dynamical equinox.

ANALYSIS

Laser ranges to the lunar quartz cube corner reflectors from the McDonald Observatory have greatly improved our knowledge of Earth dynamics. The past decade of decimeter accuracy ranges from McDonald Observatory permits accurate determinations of GM_{earth}, the secular acceleration of the

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moon due to Earth tides and the geocentric coordinates of McDonald Observatory. The results of this paper come from a 40 parameter solution. The residuals from such solutions provide the starting point for the more extensive studies of UT1 and polar motion given elsewhere in this volume.

The analysis that we present here includes lunar laser data obtained at McDonald Observatory between August 1969 and May 1980. Forty parameters were solved for simultaneously by a weighted least squares fit of 2954 lunar laser ranges. The solution is done in a two stage process. The first stage adopts the BIH 1979 system for UT1 and polar motion; its solution has a rms residual of 39 cm. Next, UT1 corrections (actualy UT0 corrections with polar motion held constant) are obtained from the residuals on 703 individual days. Then the significant Fourier components of UT1(LLR)-UT1(BIH) with periods of 70 days or longer are retained as corrections, the shorter periods being discarded as noise. These corrections indicate about 1.25 ms of noise in the BIH UT1 (see Fliegel *et al*, this volume). The final step involves taking the improved UT1 and the BIH polar motion and performing a new least squares solution for the parameters. This solution has a simple rms residual of 31 cm and a weighted rms residual $[(\Sigma_i(\text{observed-calculated})^2{}_i/\sigma_i{}^2/\Sigma 1/\sigma_i{}^2)^{1/2}$, i summed over the number of observations] of 27 cm. It is these final solution parameters that are presented here. We shall attempt to give realistic error estimates for the solution parameters not formal errors.

The analysis used the 1980 IAU nutation theory (Seidelmann *et al*, 1981) 1976 IAU precession expressions (Lieske *et al*, 1977; Lieske, 1979). As a test of accuracy, we examined the last four years of data (after August 1976) and selected days on which UTO corrections were avaiable. We then examined the weighted rms residuals (on the meridian) as a function of zenith angle and extrapolated to the overhead point to remove polar motion error. This gave a 13 cm estimate for modeling, fitting, and instrumental errors (Dickey et al, this volume).

RESULTS

The lunar laser ranging analysis for 40 parameters obtains $GM_{earth} + GM_{moon} = 403{,}503.25 \pm 0.02\ km^3/sec^2$, effectively by comparing Kepler's third law with the measured mean distance and period. Using the results of Ferrari et al (1980) for $GM_{moon} = 4{,}902.799 \pm 0.003\ km^3/sec^2$, we find that $GM_{earth} = 398{,}600.45 \pm 0.02\ km^3/sec^2$. Figure 1 shows a comparison of recent GM_{earth} determinations from three techniques. We note that the earth satellite and lunar laser results are consistent within their stated uncertainties.

Our analysis yields an improved value for the tidally induced secular acceleration of the lunar orbital mean longitude of $\dot{n} = -23.8 \pm 1.5''/century^2$. This determination is a factor of two and a half times more accurate than our previously published value (Williams et al, 1978). Assuming a terrestrial potential Love number of $k_2 = 0.30$, this acceleration corresponds to the geometrical phase angle of 0.041 ± 0.003 radians and to a global Q value of 12.3 ± 0.8 for the semidiurnal tides of the earth. The lunar interior tidal dissipation, $k_{2m}T = (4.7 \pm 0.5) \times 10^{-3}$ day, is measured through its influence on the lunar physical libration and can affect the value of $\dot{n}$ by no more than 2%. Our value agrees with the Cappallo et al (1981) estimate of $(4.7 \pm 0.2) \times 10^{-3}$ from a similar analysis of lunar laser

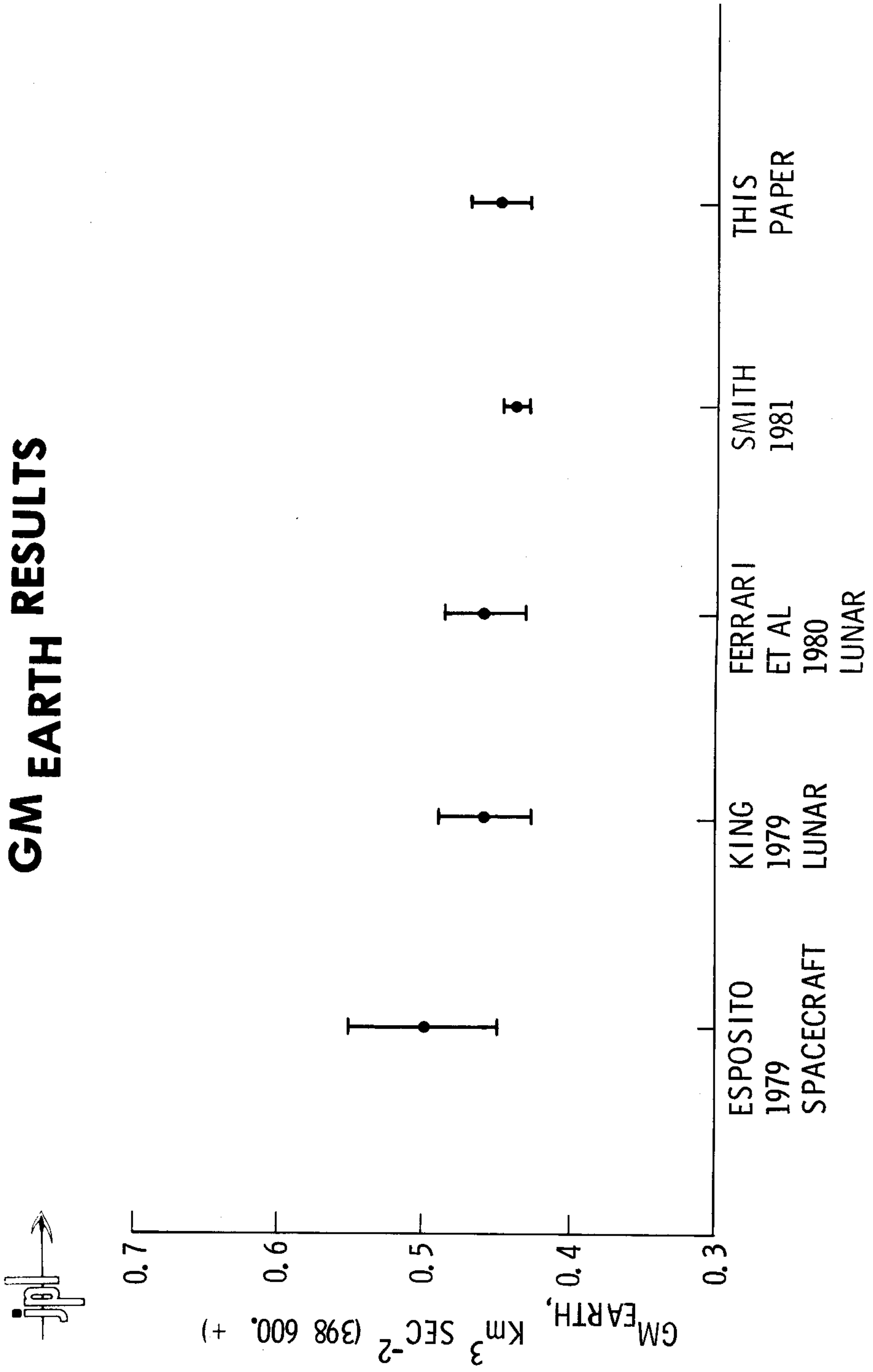
GM EARTH RESULTS
GM EARTH, Km^3 SEC^-2 (398 600. +)
0.3
0.4
0.5
0.6
0.7
ESPOSITO 1979 SPACECRAFT
KING 1979 LUNAR
FERRARI ET AL 1980 LUNAR
SMITH 1981
THIS PAPER

FIGURE 1

data. The difference in the uncertainty is partially due to the fact that in an attempt to account for systematic error, we have multiplied the formal error for this parameter by a factor of 5 while Cappallo _et al_ multiplied by 3. Solid body friction is not the only mechanism which can cause the observed libration signature (a phase _lead_ of ~ $0''.23$ in the 18.6 yr precession of the lunar spin axis). Eddy viscosity at a core-mantle interface now appears to be a more plausible explanation for this effect, requiring only a liquid, lunar core some 330 km in radius (Yoder, 1981). Our analysis yields an improved value for the lunar harmonic C_{30} of $(-8.7 \pm 1.1) \times 10^{-6}$. Both the $k_{2m}T$ and C_{30} estimates have shifted outside the errors given in Ferrari _et al_ (1980), which were based on a joint solution using lunar laser and lunar orbiter 4 data. In that paper it was stated that the lunar laser data alone gave significantly different estimates for the two parameters, and this analysis with an additional three years of data is concordant with those solo results. It appears that there may have been some difficulty with the odd zonal harmonics in the lunar orbiter data analysis.

Resulting from the solution are geocentric cylindrical coordinates of the intersection of axes of the McDonald telescope.

$$r_s = 5{,}492{,}414.4 \pm 0.2 \text{ m}$$

$$\lambda_{EAST} = 255.9779936 \pm 0.0000006^\circ \ (0.06\text{m})$$

$$z \approx 3{,}235{,}697.5 \pm 0.3 \text{ m}$$

The above longitude is based on the DE114 ephemeris with the 1976 IAU precession expressions and Fricke's (1981) equinox drift added to Sidereal Time. Using the formulation of Williams and Melbourne (this volume), the known 0".553 offset of DE114 from the dynamical equinox (Standish, 1981, private communication), and the catalogue equinox offset of 0".525 (Fricke, 1981), one can reference the longitude of McDonald to a dynamical equinox to get

$$\lambda = 255.9780014^{\circ}.$$

The 6 cm longitude uncertainty is an internal error that does not include the error in linking the lunar laser UT1 to BIH or the error in establishing the equinox offset of the ephemeris. Altogether 0.2 to 0.3 m would be a better absolute uncertainty, with the definition dependent UT1 drift dominant as discussed by Williams and Melbourne.

SUMMARY

The analysis has calculated improved values for:

1. tidally induced secular acceleration of the lunar orbital longitude, $\dot{n} = -23.8 \pm 1.5''/\text{century}^2$,
2. Lunar tidal dissipation,
3. Lunar harmonic C_{30},
4. $GM_{earth} = 398\ 600.45 \pm 0.02\ \text{km}^3/\text{sec}^2$,
5. coordinates of McDonald.

The residuals from such lunar laser solutions have provided the starting point for UT1 analysis (Fliegel *et al*, this volume) in combination with polar motion from other techniques.

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