PROBLEMS FOR SOLUTION

P. 153. Kuratowski has shown that at most 14 distinct sets can be constructed from a subset A of a topological space X by successive applications in any order of the closure operator f and the complementation operator g. Let us call sets thus constructed relatives of A. Defining the rim of A to be $r(A) = A \cap fg(A)$, prove the following:

(i) If the rim of A is nowhere dense then A has at most 10 relatives;

(ii) If the rim of A is dense in a regular closed subset of X, then A has at most 12 relatives.

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P. 154. Let n identical weighted coins, each falling heads with probability x, be tossed, and let $p_i(x)$ be the probability that exactly i of them fall heads. Evaluate

$$f_n = \min_{0 \leq x \leq 1} \; \max_{i = 0, 1, \ldots, n} p_i(x) .$$

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P. 155. If $a_1 < a_2 < \ldots < a_k \leq n$ is a sequence of positive integers such that $[a_i, a_j] > n$ for all $i \neq j$, show that $\sum_{i=1}^{k} \frac{1}{a_i} < 2$, where $[a_i, a_j]$ denotes the least common multiple of a_i and a_j.

Anonymous

SOLUTIONS

P. 145. If two disjoint subsets of a metric space have the property that every function lipschitz on each is lipschitz on their union, then every function continuous on each is continuous on their union. Prove this and give an example to show that this is false if the sets are not disjoint.

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