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The P-W problem

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This thesis investigates the properties of two systems of pure implication, with axioms and rules corresponding to the principles of *transitivity* of implication in the one system, and these axioms and rules plus the law of *identity* in the other.

The main part of the text is devoted to a semantic analysis of these two systems. A class of models, called *S-models*, is defined and studied. The original construction of these *S*-models is due to Meyer [2]. They derive their strength and interest from the fact that they employ a three-valued interpretation of the metalogical operations. By suitably determining the notion of validity using these three values, both of the systems mentioned above can be accommodated at once within the same class of models.

The results obtained include soundness and completeness, decidability, and, most importantly, the solution to a problem posed by Anderson and Belnap in their treatise [1]. Briefly, it is shown that, in the system of pure transitivity axioms (and corresponding rules), no instance of the axiom of identity is provable. When the axiom of identity is added to the system, one obtains as a corresponding result that no two distinct formulas co-entail each other.

References

[1] Alan Ross Anderson and Nuel D. Belnap, Jr., *Entailment. Volume I: The logic of relevance and necessity* (Princeton University Press, Princeton, New Jersey; London; 1975).

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[2] R.K. Meyer and E.P. Martin, "S (for Syllogism)", in preparation.

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