# ORBITS OF TROJAN ASTEROIDS

G. A. CHEBOTAREV, N. A. BELYAEV, and R. P. EREMENKO
*Institute of Theoretical Astronomy, Leningrad, U.S.S.R.*

**Abstract.** In this paper the orbital evolution of Trojan asteroids are studied by integrating numerically the equations of motion over the interval 1660–2060, perturbations from Venus to Pluto being taken into account. The comparison of the actual motion of Trojans in the solar system with the theory based on the restricted three-body problem are given.

## 1. Introduction

Since the orbits of Trojan asteroids are typical examples of the motion in the vicinity of the libration triangular points $L_4$ and $L_5$ in the restricted three-body problem, many investigations have been made for these asteroids, especially for libration orbits about the triangular points. In Table I the minimum angular distance of the longitudes of Jupiter and the asteroid, $\alpha_1$ the maximum angular distance, $\alpha_2$, and the libration period in years, $T$, are given for seven libration orbits based on the restricted problem of three bodies. Orbit No. 1 corresponds to the exact triangular solution, whereas No. 7 corresponds to the asymptotic solution, for which the longitude difference becomes as large as 180°.

TABLE I
Libration orbits near triangular points $L_4$ or $L_5$

| No. | $\alpha_1$ | $\alpha_2$ | $T$ |
|---|---|---|---|
| 1 | 60°.00 | 60° | 147.42 |
| 2 | 51°.32 | 70° | 148.9 |
| 3 | 39°.54 | 90° | 153.9 |
| 4 | 29°.79 | 120° | 172.6 |
| 5 | 25°.25 | 150° | 215.7 |
| 6 | 24°.05 | 170° | 294.0 |
| 7 | 23°.91 | 180° | $\infty$ |

Applying the results to actual Trojans maximum and minimum deviations and the maximum possible deviations of the mean motion from that of Jupiter $(n-n_1)$ are obtained and are given in Table II.

The purpose of this paper is to compare the actual motion of Trojans in the Solar System with the theory based on the restricted three-body problem.

## 2. Evolution of Orbits of 15 Catalogued Trojans

In Table III the orbital elements of 15 Trojans given in *The Ephemerides of Minor Planets for 1972* are also tabulated. The asteroids are arranged in increasing order of

*Y. Kozai (ed.), The Stability of the Solar System and of Small Stellar Systems*, 63–69. 

mean longitude. Ten asteroids move ahead of Jupiter, describing libration orbits around the point $L_4$. Five asteroids follow Jupiter (libration point $L_5$). Table III includes along the orbital elements the difference of the mean longitudes of these

TABLE II

Libration orbits of Trojans

| Asteroid | $\alpha_1$ | $\alpha_2$ | $(n-n_1)$ |
|---|---|---|---|
| 588 Achilles | 54°63 | 65°86 | 2''36 |
| 617 Patroclus | 53°96 | 66°65 | 2''66 |
| 624 Hector | 45°22 | 79°13 | 7''03 |
| 659 Nestor | 49°07 | 73°11 | 5''01 |
| 884 Priam | 49°71 | 72°20 | 4''69 |
| 911 Agamemnon | 46°72 | 76°68 | 6''23 |
| 1143 Odysseus | 51°37 | 69°95 | 3''88 |
| 1172 Aeneas | 51°08 | 70°31 | 4''02 |
| 1173 Anchises | 40°61 | 87°72 | 9''65 |
| 1208 Troilus | 52°05 | 69°05 | 3''55 |
| 1404 Ajax | 44°01 | 81°21 | 7''69 |
| 1437 Diomedes | 35°64 | 99°61 | 12''83 |

asteroids and Jupiter $\Delta\lambda = \lambda - \lambda_1$. For this epoch $\lambda_1 = 342°8$ and the positions of libration points are $\lambda_4 = 42°8$ and $\lambda_5 = 282°8$.

For studying the orbits of Trojans a numerical integration of equations of motion has been made taking into account the perturbations by eight major planets (Venus–Pluto) covering the time interval of 400 years (1660–2060). The initial osculating elements are taken from *The Ephemerides of Minor Planets for 1972.*

Table IV summarizes the changes of orbital elements of Trojans for 400 years, and the maximum deviations $(n-n_1)$, as well as the maximum and minimum angular distances from Jupiter $\alpha_1$ and $\alpha_2$. Comparison of Tables II and IV demonstrates a good agreement between the theory and the real motion of Trojans. Changes of inclinations and eccentricities of the orbits are insignificant. The nodal lines are nearly stationary. The lines of apsides turn in direct direction by angles from 1°2 (1437) to 49°2 (659). During the time interval of 1660–2059 the approach of Anchises (1173) to Jupiter is the nearest; its minimum distance is $\Delta = 2.6$ AU. Small eccentricities are characteristic for the orbits of the Trojans.

Rabe (1967) suggested that in the case of the circular restricted three-body problem the limiting eccentricity for Trojans has the value $e = 0.19$; when allowance is made for Jupiter's orbital eccentricity this limit increases up to $e = 0.24$. From the catalogued Trojans the orbit of Achilles (588), with $e = 0{,}15$, has the maximum eccentricity.

In accordance with the theory for Trojans the difference of the longitudes of perihelia is also 60°, $(\pi - \pi_1) = 60°$. As $\pi_1 = 14°$, for the Trojans near the libration point $L_5$, the mean perihelion longitude should be equal to 74° and for those about the libration point $L_4 \pi = 314°$. Comparison with real Trojans proves that the perihelia of all known Trojans get into the interval of $\pi = 314° \pm 45°$, while the perihelia of the

TABLE III

Orbital elements of the Trojans. Epoch $T = 1950$ Nov. 15.0 ET

| Asteroid | $\lambda$ | $\Delta\lambda$ | $n$ | $a$ | $e$ | $\omega$ | $\Omega$ | $\pi$ | $i$ |
|---|---|---|---|---|---|---|---|---|---|
| 1437 Diomedes | 22°9 | +40.1 | 305″4 | 5.130 | 0.048 | 128°9 | 315°3 | 84°2 | 20°6 |
| 1583 Antilochus | 23°5 | +40.7 | 292″7 | 5.276 | 0.054 | 186°3 | 221°1 | 47°4 | 28°3 |
| 588 Achilles | 37°9 | +55.1 | 299″1 | 5.202 | 0.150 | 128°4 | 316°4 | 84°5 | 10°3 |
| 624 Hector | 40°4 | +57.6 | 306″1 | 5.122 | 0.025 | 180°0 | 342°1 | 162°1 | 18°3 |
| 911 Agamemnon | 47°7 | +64.9 | 305″1 | 5.133 | 0.066 | 79°5 | 337°3 | 56°8 | 21°9 |
| 1749 Telamon | 49°7 | 767°0 | 293″5 | 5.267 | 0.111 | 106°1 | 340°8 | 86°9 | 6°1 |
| 659 Nestor | 51°2 | +68°4 | 295″6 | 5.243 | 0.110 | 333.7 | 350°5 | 324°3 | 4°5 |
| 1647 Menelaus | 51°7 | +68°9 | 298″1 | 5.214 | 0.026 | 286°6 | 239°8 | 166°4 | 5°6 |
| 1143 Odysseus | 53°1 | +70°3 | 300″3 | 5.187 | 0.092 | 233°3 | 220°6 | 93.9 | 3°1 |
| 1404 Ajax | 63°5 | +80°7 | 301″9 | 5.170 | 0.113 | 57°6 | 332°3 | 29°9 | 18.1 |
| 884 Priam | 269°8 | −72°9 | 298″3 | 5.211 | 0.120 | 331°6 | 301°0 | 272°7 | 8°9 |
| 1172 Aeneas | 273°2 | −69°6 | 300″6 | 5.184 | 0.103 | 46°4 | 246°8 | 293°2 | 16°7 |
| 1208 Troilus | 282°3 | −59°5 | 302″6 | 5.161 | 0.093 | 292°9 | 48°0 | 340°9 | 33°7 |
| 617 Patroclus | 287°9 | −54°9 | 298″8 | 5.206 | 0.142 | 303°8 | 43°9 | 347°7 | 22°1 |
| 1173 Anchises | 290°8 | −51°9 | 308″4 | 5.097 | 0.137 | 30°9 | 284°2 | 315°1 | 7°0 |

https://doi.org/10.1017/S007418090007039X Published online by Cambridge University Press

TABLE IV

Orbital evolution of Trojans for 400 years

| Asteroid | $\alpha_1$ | $\alpha_2$ | $(n-n_1)$ | $\Delta e$ | $\Delta\pi$ | $\Delta\Omega$ | $\Delta i$ |
|---|---|---|---|---|---|---|---|
| 1437 Diomedes | 40°1 | 93°0 | 11″6 | −0.004 | + 1°2 | −1°9 | −0°3 |
| 1583 Antilochus | 37°6 | 84°8 | 8″9 | +0.013 | + 8°7 | −1°1 | −0°1 |
| 588 Achilles | 54°7 | 66°1 | 3″2 | 0.000 | +25°9 | −1°1 | 0°0 |
| 624 Hector | 47°4 | 81°4 | 7″4 | −0.026 | +19°0 | −1°3 | −0°2 |
| 911 Agamemnon | 43°5 | 76°0 | 6″0 | +0.007 | +10°0 | −0°9 | −0°1 |
| 1749 Telamon | 48°2 | 71°9 | 6″2 | −0.005 | +24°4 | −1°9 | 0°0 |
| 659 Nestor | 52°0 | 70°0 | 4″1 | +0.030 | +49°2 | −1°8 | 0°0 |
| 1647 Menelaus | 56°2 | 69°0 | 4″0 | −0.033 | +38°0 | −1°4 | +0°1 |
| 1143 Odysseus | 52°1 | 70°4 | 4″6 | −0.009 | +20°9 | −1°5 | +0°1 |
| 1404 Ajax | 46°5 | 81°9 | 8″7 | +0.020 | +24°4 | −2°2 | +0°1 |
| 884 Priam | 54°2 | 73°1 | 5″2 | +0.026 | +31°2 | −2°4 | 0°0 |
| 1172 Aeneas | 51°5 | 69°6 | 4″6 | +0.016 | +24°7 | −1°0 | −0°1 |
| 1208 Troilus | 48°4 | 69°2 | 3″9 | −0.003 | +12°4 | −0°1 | −0°1 |
| 617 Patroclus | 54°8 | 63°6 | 2″2 | −0.011 | +21°3 | −0°8 | 0°0 |
| 1173 Anchises | 43°6 | 89°4 | 10″4 | +0.010 | +29°2 | −3°5 | +0°1 |

three Trojans about the libration point $L_4$ (624, 659 and 1647) lie beyond the limit of $\pi = 74° \pm 45°$.

According to the orbital inclination, the Trojans may be divided into two groups (see Table V). In this table, besides $i$, the value $Z_0 = a \sin i$ is given, which indicates the height (in AU) over the plane of the ecliptic to which the Trojans can rise.

Table V gives also the size of Trojans. These dimensions have been determined

TABLE V

Distribution of Trojans by the angle of inclination

| Asteroid | $Z_0$ (AU) | $i$ | $d$ (km) |
|---|---|---|---|
| 1208 Troilus | 2.86 | 33°7 | 43 |
| 1583 Antilochus | 2.50 | 28°3 | 43 |
| 617 Patroclus | 1.96 | 22°1 | 57 |
| 911 Agamemnon | 1.92 | 21°9 | 65 |
| 1437 Diomedes | 1.80 | 20°6 | 57 |
| 624 Hector | 1.61 | 18°3 | 71 |
| 1404 Ajax | 1.61 | 18°1 | 34 |
| 1172 Aeneas | 1.49 | 16°7 | 52 |
| 588 Achilles | 0.94 | 10°3 | 52 |
| 884 Priam | 0.80 | 8°9 | 41 |
| 1173 Anchises | 0.62 | 7°0 | 37 |
| 1749 Telamon | 0.55 | 6°1 | 23 |
| 1647 Menelaus | 0.52 | 5°6 | 20 |
| 659 Nestor | 0.41 | 4°5 | 45 |
| 1143 Odysseus | 0.28 | 3°1 | 52 |

from their brightness by the formula (Combes, 1971):

$$\log d(\mathrm{km}) = 3.592 - 0.2g.$$

It is interesting to note that among the Trojans with high orbital inclinations the asteroids of larger dimensions prevail.

## 3. Orbital Evolution of 15 Noncatalogued Trojans

In 1970 the Palomar and Leiden observatories completed an extensive collective work (van Houten *et al.*, 1970) on taking the photographs and determining the orbits of faint minor planets. Among the discovered asteroids 15 typical Trojans were detected whose orbits are nearly circular. The orbital elements of these Trojans are given in Table VI. Photographs of Trojans were taken in the vicinity of libration point

TABLE VI

Orbital elements of 15 noncatalogued Trojans. Epoch 1950 Nov. 15.0 ET

| Astroid | $\lambda$ | $\Delta\lambda$ | $n$ | $a$ | $e$ | $\omega$ | $\Omega$ | $\pi$ | $i$ |
|---|---|---|---|---|---|---|---|---|---|
| 9607 | 34°.7 | 51°.9 | 307″.6 | 5.105 | 0.111 | 256°.2 | 34°.1 | 290°.4 | 5°.0 |
| 6629 | 49°.3 | 66°.5 | 312″.4 | 5.053 | 0.008 | 299°.6 | 162°.1 | 101°.7 | 4°.2 |
| 6020 | 57°.1 | 74°.3 | 297″.0 | 5.226 | 0.093 | 90°.3 | 189°.0 | 279°.3 | 1°.4 |
| 4523 | 59°.2 | 76°.4 | 305″.8 | 5.125 | 0.050 | 129°.5 | 208°.5 | 338°.0 | 0°.9 |
| 4139 | 64°.0 | 81°.2 | 306″.7 | 5.115 | 0.003 | 309°.5 | 2°.7 | 312°.2 | 17°.6 |
| 2008 | 68°.6 | 85°.9 | 302″.8 | 5.159 | 0.112 | 167°.6 | 197°.2 | 4°.8 | 16°.8 |
| 6844 | 69°.0 | 86°.2 | 301″.1 | 5.178 | 0.104 | 24°.0 | 17°.6 | 41°.6 | 8°.2 |
| 4572 | 69°.1 | 86°.4 | 304″.7 | 5.138 | 0.058 | 42°.0 | 16°.0 | 58°.0 | 9°.3 |
| 4655 | 70°.6 | 87°.9 | 297″.7 | 5.217 | 0.032 | 93°.5 | 5°.8 | 99°.3 | 17°.1 |
| 6591 | 71°.1 | 88°.3 | 292″.8 | 5.276 | 0.043 | 57°.0 | 18°.6 | 75°.6 | 7°.4 |
| 4596 | 72°.1 | 89°.3 | 294″.9 | 5.251 | 0.070 | 331°.0 | 43°.9 | 14°.8 | 4°.0 |
| 6581 | 72°.3 | 89°.5 | 291″.6 | 5.291 | 0.031 | 54°.3 | 350°.6 | 45°.0 | 4°.9 |
| 6540 | 72°.6 | 89°.6 | 296″.6 | 5.231 | 0.059 | 67°.3 | 24°.0 | 91°.3 | 9°.1 |
| 6541 | 73°.3 | 90°.5 | 297″.3 | 5.223 | 0.088 | 263°.4 | 152°.9 | 56°.3 | 8°.1 |
| 2706 | 73°.3 | 90°.6 | 313″.4 | 5.042 | 0.120 | 355°.9 | 98°.8 | 94°.7 | 1°.2 |

$L_4$ and, therefore, all 15 asteroids are in the orbit ahead of Jupiter. The lack of Trojans with great inclinations is explained by the fact that the photographs were taken in the region of the ecliptic. Nevertheless, in the case of faint Trojans it is also possible to distinguish two groups of asteroids with considerable and moderate inclinations (see Table VII). In this case the largest asteroids will again get into the first group.

Only for 9 asteroids out of 15 the perihelion longitude comes into the interval of $\pi = 74° \pm 45°$.

The lines of nodes are so located that they form two groups enclosed within the intervals of 350°.6–43°.9 (9 asteroids) and 152°.9–208°.5 (5 asteroids). The asteroid 2706 has a longitude of node $\Omega = 98°.8$ which is very close to the longitude of Jupiter's node $\Omega_1 = 100°.1$. However, the lacuna 208°.5–350°.6 is completely filled with 13 catalogued Trojans whose longitudes of node are enclosed within the limits 220°.6–350°.5. Thus,

TABLE VII

Distribution of Trojans according to the angle of inclination

| Asteroid | $Z_0$ (AU) | $i$ | $d$ (km) |
|---|---|---|---|
| 4139 | 1.57 | 17°6 | 13.5 |
| 4655 | 1.53 | 17°1 | 17.0 |
| 2008 | 1.50 | 16°8 | 30.0 |
| 4572 | 0.84 | 9°3 | 11.0 |
| 6540 | 0.82 | 9°1 | 11.5 |
| 6844 | 0.74 | 8°2 | 6.8 |
| 6541 | 0.73 | 8°1 | 11.5 |
| 6591 | 0.67 | 7°4 | 14.0 |
| 9507 | 0.45 | 5°0 | 16.0 |
| 6581 | 0.44 | 4°9 | 15.5 |
| 6629 | 0.38 | 4°2 | 13.5 |
| 4596 | 0.36 | 4°0 | 14.0 |
| 6020 | 0.13 | 1°4 | 10.0 |
| 2706 | 0.11 | 1°2 | 6.2 |
| 4523 | 0.08 | 0°9 | 13.5 |

in the distribution of the line of nodes we observe (Table VIII) a lacuna 48°0–152°8 wide, into which gets only one asteroid (2706). The middle of the lacuna, $\Omega = 100°4$, exactly coincides with the longitude of Jupiter's node. To the descending node longitude of Jupiter ($\Omega_1 = 280°1$) corresponds the second lacuna of 246°8–301°0, in which one asteroid, Anchises (1173), is located. This planet has the greatest angular velocity of the line of nodes ($\Delta\Omega = 3°5$).

Table IX provides the general characteristic of orbital evolution of faint Trojans

TABLE VIII

Distribution of Trojans by the longitudes of nodes

| Asteroid | $\Omega$ | Asteroid | $\Omega$ |
|---|---|---|---|
| 4139 | 2°7 | 4523 | 208°5 |
| 4655 | 5°8 | 1143 Odysseus | 220°6 |
| 4572 | 16°0 | 1583 Antilochus | 221°1 |
| 6844 | 17°6 | 1647 Menelaus | 239°8 |
| 6591 | 18°6 | 1172 Aeneas | 246°8 |
| 6540 | 24°0 | 1173 Anchises | 284°2 |
| 9507 | 34°1 | 884 Priam | 301°0 |
| 4596 | 43°9 | 1437 Diomedes | 315°3 |
| 617 Patroclus | 43°9 | 588 Achilles | 316°1 |
| 1208 Troilus | 48°0 | 1404 Ajax | 332°3 |
| 2706 | 98°8 | 911 Agamemnon | 337°3 |
| 6541 | 152°9 | 1749 Telamon | 340°8 |
| 6629 | 162°1 | 624 Hector | 342°1 |
| 6020 | 189°0 | 659 Nestor | 350°5 |
| 2008 | 197°2 | 6581 | 350°6 |

for 400 years. The eccentricity value of asteroid 4139 has passed through zero; the longitude of perihelion, therewith, has changed from $\pi = 187\overset{\circ}{.}7$ to $\pi = 312\overset{\circ}{.}2$. The perturbations of orbital inclinations and eccentricities are insignificant.

TABLE IX

Orbital evolution of 15 noncatalogued Trojans

| Asteroid | $\alpha_1$ | $\alpha_2$ | $(n-n_1)$ | $\Delta e$ | $\Delta\pi$ | $\Delta\Omega$ | $\Delta i$ |
|---|---|---|---|---|---|---|---|
| 9507 | 51°9 | 96°1 | 11″5 | +0.004 | +50°7 | −3°5 | 0°0 |
| 6629 | 42°8 | 103°7 | 13″8 | −0.022 | −26°1 | −2°9 | +0°1 |
| 6020 | 52°7 | 74°9 | 4″5 | +0.002 | +61°6 | −2°5 | +0°1 |
| 4523 | 40°9 | 84°0 | 6″5 | +0.029 | +64°9 | −3°7 | +0°1 |
| 4139 | 32°5 | 93°0 | 10″1 | −0.018 | — | −2°3 | −0°1 |
| 2008 | 41°5 | 87°8 | 9″4 | +0.036 | +38°4 | −2°2 | +0°2 |
| 6844 | 50°3 | 86°4 | 9″5 | +0.020 | +26°5 | −2°7 | −0°1 |
| 4572 | 37°6 | 92°0 | 10″0 | +0.005 | +10°7 | −2°6 | 0°0 |
| 4655 | 45°0 | 87°9 | 9″6 | −0.018 | − 2°1 | −2°2 | +0°1 |
| 6591 | 38°7 | 88°3 | 10″9 | −0.008 | − 1°5 | −3°1 | 0°0 |
| 4596 | 42°1 | 89°3 | 8″8 | +0.035 | +28°9 | −2°5 | 0°0 |
| 6581 | 36°2 | 93°2 | 12″5 | +0.009 | −26°4 | −4°1 | 0°0 |
| 6540 | 46°1 | 89°8 | 9″7 | −0.016 | +15°1 | −2°5 | 0°0 |
| 6541 | 47°7 | 90°5 | 9″9 | +0.009 | +22°1 | −2°3 | +0°2 |
| 2706 | 37°6 | 121°0 | 16″9 | −0.014 | +29°7 | +1°3 | 0°0 |

## References

Combes, M. A.: 1971, *L'Astronomie*, Décembre, p. 413.

Rabe, E.: 1967, *Astron. J.* **72**, 10.

van Houten, C. J., van Houten-Groeneveld, I., Herget, P., and Gehrels, T.: 1970, *Astron. Astrophys. Suppl.* **2**, 339–448.