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THESIS ABSTRACTS

CHRISTIAN ROSENDAL, EDITOR

Beginning with this issue of the *Bulletin of Symbolic Logic*, The Association for Symbolic Logic will be publishing abstracts of PhD theses in logic. There are several aims of the newly created section. First of all, this will provide a permanent, organized and public record of the majority of theses in logic produced worldwide. This allows researchers to keep up to date with the research in the field and makes available work that might not always find its way into other publications. While there is no central repository for the actual theses themselves, links and internet addresses can be provided in the abstract thus ensuring wider circulation of the work. Secondly, the thesis abstracts offer recent PhD's with an additional outlet for communication of their ideas in a short and accessible format.

Initially, abstracts will be accepted for students having received their PhD within the past five years. Hence abstracts submitted in 2018 are limited to students having received their PhD since 2013. With time, this will become more restrictive to ensure timeliness of publication.

The Thesis Abstracts Section is edited by Christian Rosendal. Any abstract should formally be submitted by the thesis advisor though is expected to be prepared by the candidate. For detailed instructions for preparation and submission, including the required TeX template, please consult the URL below.

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Advisors should send thesis abstracts, including both a TeX file and a PDF file, to Christian Rosendal, rosendal.math@gmail.com, To be accepted, an abstract must be relevant and appropriate for *BSL* readers; that is, it must be in logic, and it must be neither slanderous nor libelous. Acceptance is solely the decision of the editor for thesis abstracts.

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Sebastien Vasey, ***Superstability and Categoricity in Abstract Elementary Classes***, Carnegie Mellon University, USA, 2017. Supervised by Rami Grossberg. MSC: Primary 03C48, Secondary 03C45, 03C52, 03C55, 03C75, 03E55, 18C35, 03B30, 03E30. Keywords: abstract elementary classes, nonelementary classes, infinitary logics, categoricity, superstability, stability, classification theory, forking, independence.

Abstract

We[1] study the model theory of abstract elementary classes (AECs). They are a family of concrete categories closed under directed colimits where all the morphisms are monomorphisms, containing in particular the category of models of an $\mathbb{L}_{\lambda,\omega}(Q)$-theory (with the natural notion of elementary embedding). This framework was identified by Shelah in 1977, when he proposed the far-reaching program of adapting his classification theory (originally developed for first-order logic) to AECs and hence to all "reasonable" infinitary logics. This thesis develops an analog of Shelah's first-order superstability theory to AECs. This involves studying forking-like notions of independence in this general framework, giving criteria for when they exist, and linking their properties to the stability spectrum and the behavior of chains of saturated models. We usually assume that the AEC has amalgamation, and also often that it is tame, a locality property of orbital (Galois) types introduced by Grossberg and VanDieren. It is conjectured that these properties should follow from categoricity.

We solve several open problems using the technology developed in this thesis. Shelah's eventual categoricity conjecture is the statement that an AEC categorical in *some* high-enough cardinal should be categorical in *all* high-enough cardinals. It is the main open question of the field. We apply the superstability theory to show that the conjecture holds in universal classes (a special kind of AECs: classes closed under isomorphisms, substructures, and union of $\subseteq$-increasing chains). Previous approximations were in a stronger set theory than ZFC and always assumed categoricity in a successor cardinal. Here, we eliminate the successor hypothesis and work in ZFC.

We also show that if an AEC $\mathbf{K}$ with amalgamation and no maximal models is categorical in $\lambda > \mathrm{LS}(\mathbf{K})$, then the model of cardinality λ is saturated (in the sense of orbital types). This answers a question asked by both Baldwin and Shelah.

In several cases, the arguments developed for the superstability theory are useful also in case the AEC is stable but not superstable. This thesis develops the theory in this case as well, proving for tame AECs the equivalence between stability and no order property, as well as an eventual characterization of the stability spectrum under the singular cardinal hypothesis.

Independence relations (like Shelah's notion of a good frame) are *the* central pillar of the (super)stability theory developed here. They are a deep generalization of linear independence of vector spaces and algebraic independence in fields, so we expect that they will have many other applications, both to the abstract theory and to concrete (algebraic or perhaps analytic) examples.

Another contribution of this thesis is the definition of a *quasiminimal AEC*: it is an AEC with countable Löwenheim–Skolem–Tarski number which has a prime model, is closed under intersections, and has a unique generic type over every countable model. We show that quasiminimal AECs are exactly the quasiminimal pregeometry classes that Zilber used to study pseudo-exponential fields, motivated by Schanuel's conjecture. In particular, an unbounded quasimimimal AEC is categorical in every uncountable cardinal. Along the way, we give new conditions under which a homogeneous closure operator has exchange and conclude that the exchange axiom is redundant in Zilber's definition of a quasiminimal pregeometry class.

[1]The results of this thesis were written up in 22 separate articles submitted for publication in refereed journals, 21 of which have already been accepted, and 18 of the accepted ones have already appeared in print. While a majority of the articles are single author, some were written with collaborators (Boney, Grossberg, Kolesnikov, Lieberman, Rosický, Shelah, and VanDieren). Details and credits appear in Section 1.6 of the thesis and at the start of every chapter.

We also study a more general notion than AECs: μ-AECs, which are only required to be closed under μ-directed (rather than $\aleph_0$-directed) colimits. We generalize some basic arguments from the theory of AECs and show that μ-AECs are exactly the accessible categories whose morphisms are monomorphisms (this is joint work with Will Boney, Rami Grossberg, Michael Lieberman, and Jiří Rosický).

Finally, the thesis contains a chapter on simple first-order theories. We present a new proof of the existence of Morley sequences in such theories which avoids using the Erdős-Rado theorem and instead uses only Ramsey's theorem and compactness. The proof shows that the basic theory of forking in simple theories can be developed using only principles from "ordinary mathematics", answering a question of Grossberg, Iovino, and Lessmann, as well as a question of Baldwin.

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Alessandro Vignati, ***Logic and*** C****-algebras: Set Theoretical Dichotomies in the Theory of Continuous Quotients***, York University, Toronto, Canada, 2017. Supervised by Ilijas Farah. MSC: Primary 03E50, 46L05. Keywords: C*-algebras, forcing axioms, automorphisms of quotients.

Abstract

This thesis focuses on interactions between logic and C*-algebras (Banach self-adjoint subalgebras of $\mathcal{B}(H)$, the algebra of bounded operators on a complex Hilbert space). As abelian C*-algebras are dual to locally compact spaces, the study of C*-algebras is considered as noncommutative topology. Our interest is primarily on corona C*-algebras, nonabelian analogs of Čech-Stone remainders (i.e., $\beta X \setminus X$), and on how set theory has an influence on their automorphisms. The motivation comes from two sources: (1) the search for outer automorphisms of the Calkin algebra; (2) the study of homeomorphisms of $\beta X \setminus X$.

(1) The seminal work in [1] developed extension theory and defined analytic K-homology for C*-algebras, to make a major breakthrough in extending Weylvon Neumann theory and understanding index theory for normal[1] elements of the Calkin algebra $\mathcal{C}(H)$ (if H is separable, $\mathcal{C}(H) = \mathcal{B}(H)/\mathcal{K}(H)$, $\mathcal{K}(H)$ being the ideal of compact operators). Few questions were left open. Since, for $S, T \in \mathcal{B}(H)$, $\dot{S}$ and $\dot{T}$ (their images in $\mathcal{C}(H)$) are unitarily equivalent if and only if there is an automorphism of $\mathcal{C}(H)$ mapping $\dot{S}$ to $\dot{T}$, can the same be said if S and T are essentially normal (S is essentially normal if $\dot{S}$ is normal)? In particular, is it possible to map the image of the unilateral shift to its adjoint via an automorphism of $\mathcal{C}(H)$? Such an automorphism cannot be inner (induced by a unitary), but it was not known whether an outer automorphism of $\mathcal{C}(H)$ would exist at all. Set theoretic axioms entered play: under CH outer automorphisms exist ([7]) while under OCA don't ([5]). It is still open if there can be an automorphism mapping the shift to its adjoint.

(2) In studying the homogeneity properties of spaces of the form $\beta X \setminus X$, the following question arose: is every homeomorphism of $\beta\mathbf{N}\setminus\mathbf{N}$ induced by an almost permutation? CH gives a negative answer ([8]), while a positive answer is consistent ([9]), and in fact follows from Forcing Axioms ([10]). This question was generalized in two, linked, ways. First, replacing the notion of almost permutation with "permutation up to compact subsets of X", one can ask about homeomorphisms of $\beta X \setminus X$ for a metrizable locally compact X. Second, as homeomorphisms of $\beta\mathbf{N}\setminus\mathbf{N}$ correspond to automorphisms of $\mathcal{P}(\mathbf{N})/\mathrm{Fin}$, the theory of automorphisms and relative embeddings of quotients of the form $\mathcal{P}(\mathbf{N})/\mathcal{I}$, where $\mathcal{I} \subseteq \mathcal{P}(\mathbf{N})$ is an analytic ideal, was extensively studied in [4].

The goal is to extend the results in (1) and (2) to corona C*-algebras. In the same way $\mathcal{K}(H)$ is related to $\mathcal{B}(H)$ and $\mathcal{C}(H)$, to a nonunital C*-algebra A one can associate its

[1] S is normal if $SS^* = S^*S$.

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