

THE FLEXURAL RESPONSE OF A TABULAR ICE ISLAND TO OCEAN SWELL †

by

D. J. Goodman*

(Physics and Chemistry of Solids Group, Cavendish Laboratory, Madingley Road, Cambridge, CB3 0HE, England)

and

P. Wadhams and V. A. Squire

(Sea Ice Group, Scott Polar Research Institute, Cambridge, CB2 1ER, England)

ABSTRACT

Measurements of surface strain and vertical heave responses to swell were made on a tabular ice island in Kong Oscars Fjord, east Greenland, in September 1978. At two sites surface strain was measured with a wire strainmeter of 2 m gauge length, and heave was measured with a vertical accelerometer. While the first site was occupied a simultaneous measurement of ambient wave energy was made with a wave buoy. The results show that the ice island flexes and heaves in response to the longest component only of the forcing wave field, at periods above 16 s, and with a mean strain amplitude of the order of 5×10^{-7} . The results are compared with theoretical calculations of the response of a thick floating beam. In the light of the theory, the flexural behaviour of tabular icebergs and sea-ice floes is considered and their critical size ranges in relation to sea state are estimated.

LIST OF SYMBOLS

A	wave amplitude under the ice block
A'	wave amplitude on the sea surface
c	typical crack length
D	$Eh^3/(1-\nu^2)$
E	Young's modulus
g	acceleration due to gravity
h	ice-block thickness
k	incident wave number
K_{IC}	stress intensity factor
λ	ice-block length in swell direction
p_n	normal mode period
T	wave period
ϵ	surface strain amplitude
ν	Poisson's ratio
ξ	vertical displacement of neutral axis
ρ_i	ice density
ρ_w	sea-water density
σ_0	average stress
ω	angular frequency

1. INTRODUCTION

Icebergs can disintegrate in a variety of ways; in this paper we explore how swell passing under an ice island causes it to bend and heave, and examine theoretically the wave height required to propagate a crack. This has important implications for the size distribution of icebergs and sea-ice floes.

2. OBSERVATIONS

2.1 Field work

During the course of a field programme to study the marginal sea ice off the east Greenland coast in September 1978, a landing was made on an ice island (Fig. 1) which had drifted into Kong Oscars Fjord. It was located at $72^\circ 12' N.$, $23^\circ 20' W.$, some 40 km from the fjord mouth. A belt of pack ice 65 km wide protected it from the open sea, but allowed the transmission of a detectable swell of long period. The extreme dimensions were 431 m (EF in Figure 1), and 179 m (CD), and from a measurement of freeboard, a thickness between 30 and 35 m was estimated. No crevasses were visible on the surface, but at some locations along the sides the top surface had curled down towards the sea, and a small piece, which had probably become detached by a Reeh-type calving process (Reeh 1968) could be seen to one side.

Two stations were occupied, A and B in Figure 1. At site A, the heave and the surface-strain changes in one direction were recorded, while simultaneous measurements were made of the local wave spectrum with a wave buoy deployed over the edge of the island. The equipment, excepting the wave buoy, was then moved to the centre, site B, and heave and surface-strain changes only recorded. Data were recorded for about 35 min at site A, and for 20 min at site B. The strainmeter was orientated with its axis

†This is a shortened version of the submitted paper. Copies of the full text may be obtained from the Sea Ice Group, Scott Polar Research Institute, Cambridge CB2 1ER, England.

*Present address: BP Trading Ltd., Moor Lane, London, EC2Y 9BU, England

pointing in the direction of the swell. In this direction the ice island had a length of 406 m. (Full details of the equipment used are given in the longer version of the paper and in Goodman (in press).)

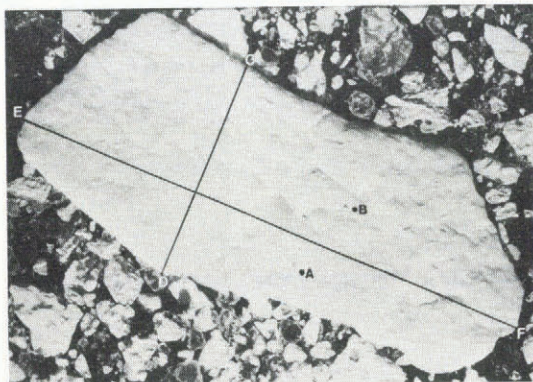


Fig.1. A vertical photograph of the ice island. A and B were the experimental sites. EF and CD are 431 m and 179 m respectively. The horizontal edge of the photograph is orientated along the fjord. G is a part of the ice island which has calved off by a Reeh-type mechanism.

2.2 Data analysis and results

The analogue records recorded on a RACAL FM tape recorder were replayed using the same tape recorder into an A/D converter connected to a PDP-11 computer via an analogue filter with a cut-off at the Nyquist frequency. The time series was digitized at 0.512 s intervals, and the rezeroing jumps, mean, and trends removed. Then a segment which contained either 2 048 or 4 096 data points was used to generate a finite Fourier transform using a FFT package after the final tenths of the time series had been tapered to reduce side-lobe leakage using the cosine data window of Bingham and others (1967). To reduce the error in the spectral estimate, the energies were grouped in the frequency domain by a factor p , which reduces the error in each spectral estimate by a factor $1/(p)^2$ (Bendat and Piersol 1971).

Figure 2 shows the power spectral density functions for the wave buoy (a), the ice-island heave at site A (b), and the ice-island heave at site B (c). The units are $m^2 s$; the spectral estimates were divided by $16\pi^4 f^4$, where f is the frequency. The gain factor (Bendat and Piersol 1971) between the wave buoy and ice-island heave at site A and site B were also computed. The results are shown in Figure 3.

Figure 4 shows the power spectral density functions for the wave buoy (a), the strain record at site A (b), and the strain record at site B (c). In this case the units of the wave-buoy spectrum are as before ($m^2 s$), but the strain spectra have units of s (the spectral estimates have *not* been divided by $16\pi^4 f^4$). It must be noted that the strain shown in Figure 4 overestimates the true strain by approximately 10%. This is because the wire strainmeter, when placed on a heaving body, displays an acceleration response (Moore and Wadhams, in press). In Figure 5 the strain-spectral estimates have been divided by the wave-amplitude estimates, and are shown as a function of period.

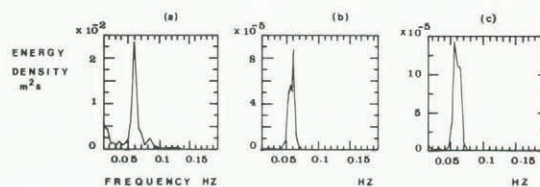


Fig.2a (left), b (middle), and c (right). Power spectral density functions for the wave buoy at site A (a), the ice-island heave at site A (b), and the ice-island heave at site B (c).

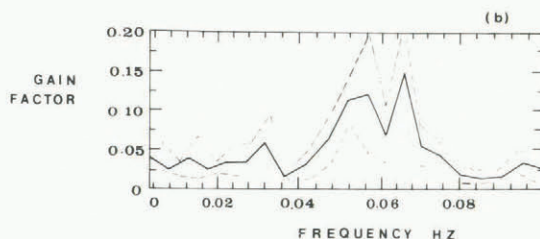
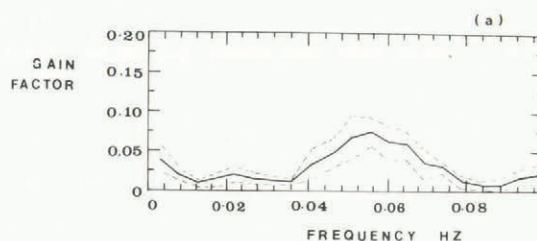


Fig.3a (upper) and b (lower).

The gain factor between the wave buoy and ice-island heave at site A (a), and at site B (b). The wave-buoy record from site A was used to compute (b). The dotted lines are the 95% confidence limits

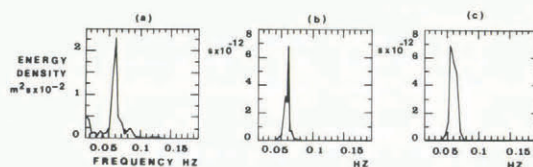


Fig.4a (left), b (middle), and c (right). Power spectral density functions for the wave buoy at site A (a), the strain record at site A (b), and the strain record at site B (c).

3. DISCUSSION

3.1 Heave response

The data shown in Figures 2 and 3 indicate that the ice island behaves as a low pass filter. The local wave spectrum is predominantly a monochromatic swell, with a peak of 17 s, but there is also a secondary swell, with a peak at 12 s, and a slight but detectable sea

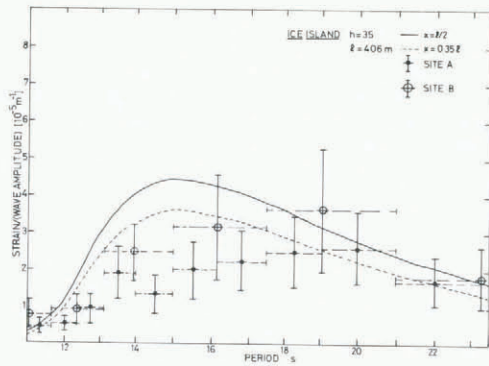


Fig. 5. The strain amplitude divided by the wave amplitude at the two sites plotted as a function of period. The solid and dotted lines are the theoretical predictions.

with frequencies up to 0.14 Hz (7 s). In the full version of the paper, an outline of a possible method for deriving the heave response is given.

3.2 Bending response

In a similar way to the heave-response spectra, the strain spectra illustrated in Figure 4 show a low pass characteristic (although it should be noted that the units of the vertical axis are different).

In the full version of the paper, two solutions are given for the bending response of a section in plane strain from a rectangular ice block subject to a pressure field $P(x, t)$ applied along its bottom surface. Both solutions use Froude-Krylov theory whereby the diffraction of waves is ignored. One of us (V.A.S.) is now working on a numerical solution where this restriction is lifted. The first solution finds an analytic expression for the strain amplitude on the surface of the block at its centre when it is subject to a pressure field

$P(x, t) = \xi \rho_w g + A \rho_w g \text{Re}[\exp(i(kx - \omega t))]$. ξ is the profile of flexure of the block, and A the amplitude of the water particle motion at the bottom surface. A is related to A' , the wave amplitude on the surface, by the expression $A = A' \exp(-kh \rho_i / \rho_w)$. Then, subject to a set of assumptions, the solution for the strain is:

$$\epsilon(l/2, t) = \frac{A \rho_w g k^2 h \cos \omega t}{2 D \Delta (k^4 + 4 k'^4)} \left[\Delta - \Psi \cos \frac{k l}{2} - \frac{k}{k'} \sin \frac{k l}{2} \sin \frac{k' l}{2} \sinh \frac{k' l}{2} \right] \quad (1)$$

where

$$k' = \left[\frac{\rho_w g - \rho_i h \omega^2}{4 D} \right]^{1/4}$$

$$\Delta = \frac{1}{2} (\sin k' l + \sinh k' l), \text{ and}$$

$$\Psi = \sin \frac{k' l}{2} \cosh \frac{k' l}{2} + \cos \frac{k' l}{2} \sinh \frac{k' l}{2}.$$

The second solution finds an expression for the surface strain anywhere on the block, subject to a general forcing function $P(x, t)$ as a sum of overlap integrals of $P(x, t)$ and the normal modes of vibration of the block. The solution is not given here, but in Figure 6a

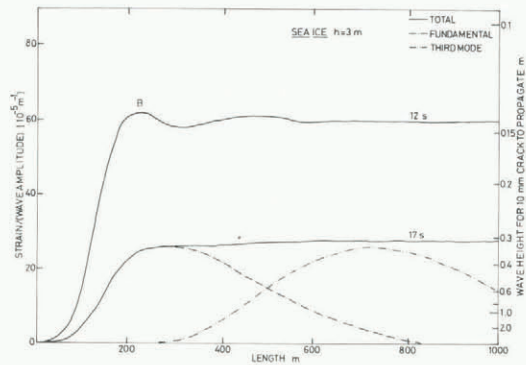
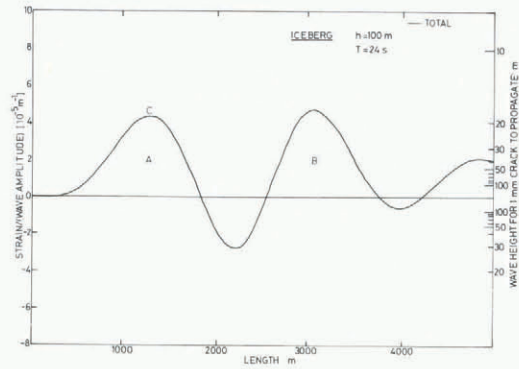
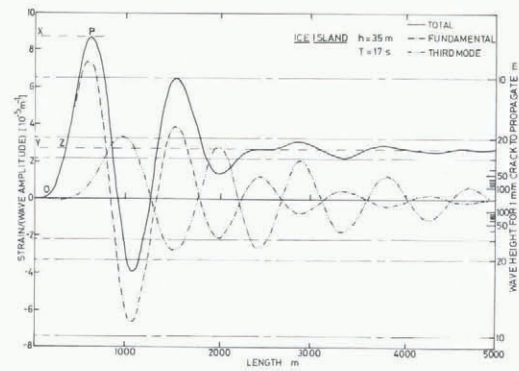


Fig. 6a (upper), b (middle), and c (lower). Theoretical predictions of the flexural response of the ice island (a), an iceberg (b), and a sea-ice floe (c) as a function of length in the swell direction for a given thickness and wave period. The wave height (peak to trough distance) to make a crack of a given size propagate is shown on the right-hand axis.

contribution from each normal mode is shown. For ice islands less than 500 m long, or sea-ice floes smaller than 300 m, only the first mode contributes significantly. The normal mode solution also identifies the periods at which there will be a bending resonance between the sea and the ice block. The normal mode periods, P_n , are given by the expression

$$P_n^2 = (k_n^4 D + \rho_w g) / \rho_i h$$

where $k_n \approx (n + \frac{1}{2}) \pi$ (Timoshenko and others 1974) ($n = 1, 3, 5, \text{etc.}$).

3.3 Comparison of bending response theory and observations

If representative values of E (8.72 GN m^{-2}), ν (0.3), l (406 m, the dimension of the ice island in the direction of the swell), h (35 m), and T (17 s, the dominant swell period) are inserted into Equation 1, the bending response (strain amplitude/wave amplitude) as a function of period can be computed. The result is shown in Figure 5 for the centre of the ice island (solid line), and $0.35l$ (dotted line). Also shown on the figure are the experimental observations; the agreement is good.

4. THE LIKELIHOOD OF SWELL-INDUCED FRACTURE

4.1 Critical strain at failure

An ice block has no unique strength; the stress at which failure occurs is a function of the distribution of the cracks that the block contains. This distribution is invariably controlled by the microstructure, which is itself determined by the rate of growth and the strain history. A single crack of length c will propagate when the stress-intensity factor (K_{IC}) reaches a critical value. The stress-intensity factor is a measure of the stress concentration near a crack, and is related to the local average stress σ_0 and the crack length by the relation

$$K_{IC} = \Omega \sigma_0 (\pi c)^{\frac{1}{2}}$$

where Ω is a constant which depends on the geometry of the free surfaces and is of order one. A critical discussion of how and when elastic fracture mechanics can be applied to ice is presented by Goodman (1980), who gives a representative value for K_{IC} for pure ice as $115 \text{ kN m}^{-3/2}$. Then, if c is taken to be equal to the grain size, and, considering the worst case where the period of the forcing function is so short that there is no time for creep to relax the stresses (a purely elastic response), the critical strain ϵ_{crit} which will cause the crack to propagate is:

$$\epsilon_{crit} = K_{IC} \frac{1}{\Omega \sqrt{\pi c}} \frac{1-\nu^2}{E}$$

which, for $c = 1 \text{ mm}$ (a typical grain size), $\Omega = 1$, $E = 8.72 \text{ GN m}^{-2}$, and $\nu = 0.31$ is

$$\epsilon_{crit} = 2.14 \times 10^{-4}$$

For sea ice, any crack will be filled with salt water and K_{IC} will be reduced. As no satisfactory measurements of K_{IC} exist for sea ice, suppose $K_{IC} = 50 \text{ kN m}^{-3/2}$, $\nu = 0.31$ and $E = 6 \text{ GN m}^{-2}$ (this will depend on brine volume), and suppose $c = 10 \text{ mm}$, then

$$\epsilon_{crit}^{sea\ ice} = 4.3 \times 10^{-5}$$

(During the field experiment a direct observation of the strain at failure of a sea-ice floe was made; the value observed was close to 3×10^{-5} .)

4.2 The break-up of ice islands by the action of swell

The bending-response theory, and the critical strain given in the last section can be combined to find a critical wave height at which cracks (in this case 1 mm in length) will propagate. For $h = 35 \text{ m}$, and $T = 17 \text{ s}$ the bending-

response theory has been used to plot in Figure 6a strain/(wave amplitude) as a function of length (in the swell direction) using $E = 8.72 \text{ GN m}^{-2}$, $\nu = 0.31$, $\rho_i = 910 \text{ kg m}^{-3}$, and $\rho_w = 1025 \text{ kg m}^{-3}$. Then, on the right hand axis the wave height required to propagate a one-millimetre crack is shown.

Cracks of this size are certain to occur on the surface of an ice island. For a given wave height the curve predicts the range of dimensions at which break-up is likely to occur. For instance if ice islands with a wide distribution of sizes pass through a region where the maximum wave height is 10 m, ice islands with any dimension between 490 and 760 m will not survive the passage. If the wave height is 20 m, the critical range where failure might occur is extended to 380-840, 1 020-1 140, and 1 350-1 760 m. Finally, if there is a likelihood of 30 m waves, only ice islands with dimensions within the range 0 to 340 m and 1 840 to 2 160 m will survive. Then, if a calving glacier generates a wide distribution of ice-island sizes, after a short exposure to sea swell the distribution would exhibit some of the features in Figure 6a. Dimensions close to 915 m and 1 246 m are particularly favourable. But the majority of ice islands of this thickness would continue to calve until none of their dimensions was greater than about 400 m.

The ultimate failure will be a type of fatigue process. During part of each wave cycle, cracks will grow, reducing ϵ_{crit} . For each successive cycle the cracks will grow for a larger proportion of the cycle until eventually one crack will propagate unstably. The increasing hydrostatic pressure with depth will tend to arrest downward-growing cracks, but water-filled cracks at the bottom do not suffer this disadvantage and have a lower K_{IC} . Consequently the failure will probably originate at the bottom.

The solution for the bending response may also be used to examine the effect of change of thickness on the survival of the ice island. In Figure 7 the strain/(wave amplitude) is plotted as a function of thickness for various lengths in the swell direction using the same values of E , ν , ρ_w , ρ_i , and T as Figure 5a. The curves show that, as the ice island thins

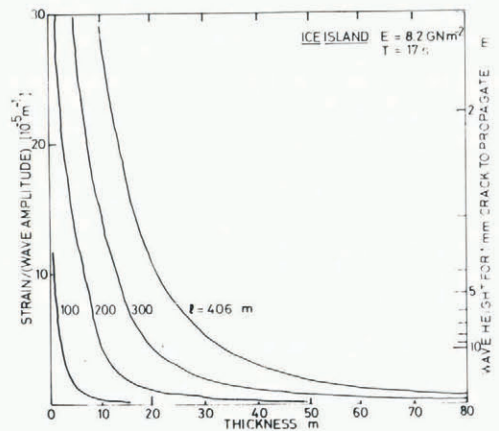


Fig.7. The strain amplitude/(wave amplitude) plotted as a function of thickness for a given length and period.

by melting, there is a rapid rise in the surface strain for a fixed wave amplitude (the beam cross-section is smaller for the same load, and so the strain is larger). Again on the right-hand axis the wave height required to propagate a one-millimetre crack is shown.

4.3 Extension of the theory to icebergs and ice floes

The theory developed in the longer paper can be very appropriately applied to sea-ice floes. Figure 6c shows the strain/(wave amplitude) as a function of length in the swell direction for a three-metre thickness, and wave period of 12 and 17 s (calculated using $E/(1-\nu^2) = 6 \text{ MN m}^{-2}$, $\rho_w = 910 \text{ kg m}^{-3}$, $\rho_i = 1.025 \text{ kg m}^{-3}$). On the right-hand axis the wave height required to propagate a ten-millimetre crack is shown. Only small wave heights are required to fracture a floe; large floes will rapidly break up into a size range given by OB (0 to 220 m) for wave heights greater than 0.15 m of 12 s period. This is in good accord with observations of floe size distributions.

The theory can also be applied to tabular icebergs, although a Froude-Krylov theory introduces large errors in this case. Further, only the longest swells penetrate deeply enough to cause an oscillating pressure field on the iceberg bottom. Figure 6b is the result of solving the theory for an iceberg 100 m thick, deformed by a 24 s swell, which is typical of the longest swells found in the Southern Ocean. To propagate a one-millimetre crack requires a wave height of about 20 m, which is a rare event even in the Southern Ocean for a 24 s swell (a 17 s swell requires a fetch of 2 000 km, and a wind speed of 30 m s^{-1} to reach 20 m). Therefore, the question can be reversed, and instead the crack size that a three-metre wave will propagate calculated. This is given by the expression:

$$e_{\text{crit}} = \frac{1}{\pi} \left(\frac{K I_c}{A'} \frac{1}{\Omega} \frac{(1-\nu^2)}{E} \left(\frac{A}{\ell} \right) \right)^2 .$$

If $2A'$ is 3 m, and ℓ for the iceberg is 1 km, e_{crit} has the value of 66 mm. Cracks of this size would not be uncommon in a large iceberg. Thus icebergs would continue to calve until their maximum dimension lies somewhere in the range of the early part of the curve, OC (0 to 1 km). This is in agreement with the observations by Landsat imagery in the open water of the Southern Ocean that most icebergs have dimensions no greater than 1.3 km (Hult and Ostrander 1974, Weeks and Mellor 1978).

5. CONCLUSIONS

We conclude that wave-induced flexural failure could well be the cause of the scarcity of icebergs with diameters greater than about 2 km in the open Southern Ocean. Very large icebergs are likely to suffer multiple fractures of this nature in a stormy sea or heavy swell, until all the fragments are less than a critical diameter. A long narrow iceberg several km long and 1 to 4 km wide is likely to fracture in a lesser sea if it is allowed to come beam on to the waves so that the length presented to the sea falls into one of the unstable regions A and B of Figure 6b.

The towing of an iceberg or ice island in open water is likely to be hazardous because of the likelihood of wave-induced fracture, and it is recommended that continuous monitoring of surface-strain changes should be made.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the assistance of S.C. Moore and S. Overgaard in the field work, and of the helicopter pilot, B. Andersson of Greenlandair. We also wish to acknowledge support from the Office of Naval Research under Contract N00014-78-G-0003, and the Natural Environment Research Council. We are grateful to R.J. Horne for developing the computer package used for the data analysis, and for the assistance of R. Weintraub and A.M. Cowan. We thank C.S. Neal for his assistance during the construction of the amplifier system. D.J. Goodman is grateful for research fellowships, from the Science Research Council and Girton College, for research grants from the Science Research Council and the Royal Society, and to Professor A. Higashi and the Japan Society for the Promotion of Science for allowing him to visit Hokkaido University where part of this work was completed.

REFERENCES

- Bendat J S, Piersol A G 1971 *Random data: analysis and measurement procedures*. New York, John Wiley
- Bingham C, Godfrey M D, Tukey J W 1967 Modern techniques of power spectrum estimation. *IEEE Transactions on Audio and Electro-acoustics* AU-15(2): 56-66
- Goodman D J 1980 Critical stress intensity factor (K_{Ic}) measurements at high loading rates for polycrystalline ice. In Tryde P (ed.) *IUTAM Physics and Mechanics of Ice, symposium Copenhagen, 1979*. Berlin, Springer-Verlag: 129-146
- Goodman D J In press. Surface strain measuring instruments for use on sea ice. In Allan A (ed.) *Workshop on sea ice field measurements, C-CORE, St John's, Newfoundland, 29th April to 1st May 1980*
- Hult J L, Ostrander N C 1974 Applicability of ERTS to Antarctic iceberg resources. In Freden S G, Mercanti E P, Becker M A (eds.) *Third Earth Resources Satellite-1 Symposium, Washington, DC, 1973, Vol.1B*. Washington, DC, National Aeronautics and Space Administration: 1467-1490 (NASA SP-351)
- Moore S C, Wadhams P In press. Recent developments in strainmeter design. In Allan A (ed.) *Workshop on sea ice field measurements, C-CORE, St John's Newfoundland, 29th April to 1st May 1980*
- Reeh N 1968 On the calving of ice from floating glaciers and ice shelves. *Journal of Glaciology* 7(50): 215-232
- Timoshenko S, Young D H, Weaver W Jr 1974 *Vibration problems in engineering, 4th ed.* New York, John Wiley and Sons
- Weeks W F, Mellor M 1978 Some elements of iceberg technology. *CRREL Report 78-2*