RELATIVITY EFFECTS

Radar echoes from Venus obtained at the Lincoln Laboratory, MIT, (1) and at the Jet Propulsion Laboratory, Cal. Tech., (2) have been interpreted by Clemence (3) to mean that the distance from the Earth to the Sun is about 75 000 km greater than is indicated by the dynamical method. He shows, by improving de Sitter's treatment on the relativity corrections of the orbital elements and by basing on Hansen's theory, that the relativistic effects previously neglected yields correction to the radius vector of a planet amounting at most to about a kilometre. While the correction to the longitude of Mercury is too small to be detected by any known optical technique, the correction to the radius vector in the principal periodic term may provide a new test of general relativity, by radar echo observations.

On the other hand Kustaanheimo (4, 5) deals with the possibility of demonstrating the relativistic curvature of space by the observations of satellites of large eccentricity. He proves that the period of revolution of a satellite is increased by an observable relativistic effect. If a Keplerian orbit and an orbit of the Schwarzschild metrics are defined by means of two orbital constants which have the same numerical values in both theories, then the sidereal periods of the two orbits are different. If the two orbital constants are detectable by observations, then the relativistic period is longer than the Newtonian period.

Briggs (6) discussed the steady-state distribution of meteoric particles under the operation of Poynting-Robertson effect.

Clemence (7) discussed controlled experiments in celestial mechanics for clearing the possible dependence of the Earth's gravitational field on its orbital velocity, the secular change of the gravitational constant, the orbital constants of the Earth, the mass of the Moon and the mechanical ellipticity of the Moon. He pointed out the work of Eckert on the motion of the Moon together with the motions of the near planets Mars and Venus, the measurement of the annual change of the radial velocity of hydrogen cloud, the measurement of the radial velocity of Venus on optical wave length, and finally the lunar probes, as the powerful means for the clarification. This is not a relativistic correction.

Schmidt-Kaler (8) discussed the free falls in Einstein's theory of gravitation.

BIBLIOGRAPHY

1. Pettengill, G. H., Briscoe, H. W., Evans, J. V., Gehrels, E., Hyde, G. M., Kraft, L. G. Price, R., Smith, W. B. *Astr. J.*, **67**, 181, 1962.
2. Muhleman, D. O., Holdridge, D. B., Block, N. *Astr. J.*, **67**, 191, 1962.
3. Clemence, G. M. *Astr. J.*, **67**, 379, 1962.
4. Kustaanheimo, P., Lehti, R. *Astr. J.*, **68**, 392, 1962. = *Pub. astr. Obs. Helsinki*, no. 97, 1963.
5. ,, ,, *Comm Phys. Math. Soc. Sci. Fennica*, **28**, 2. = *Pub. astr. Obs. Helsinki*, no. 98, 1963.
6. Briggs, R. E. *Astr. J.*, **67**, 710, 1962.
7. Clemence, G. M. *Astr. J.*, **65**, 272, 1960.
8. Schmidt-Kaler, Th. *Astr. Nachr.*, **286**, 121, 1961.

THREE-BODY PROBLEM

Klemperer (1) proved the existence of equilibrium configurations of n bodies for limited ranges of rhombic configurations and of arrays in the shape of hexagonal and octagonal regular rosettes, by extending the works of Dziobek and Bilimovič. Such arrangements comprise similar heavier bodies and an equal number of similar lighter bodies in regularly alternating fashion.

Miyahara (2) showed certain conditions, under which a contact transformation from a system of Hamiltonian equations with two degrees of freedom to that of the normal form is convergent in the neighbourhood of an equilibrium point.

Sibahara proved several theorems concerning the three-body problem after Chazy and Merman. He showed a possibility of a motion which is hyperbolic-elliptic for $t = -\infty$ and hyperbolic for $t = \infty$ (3), or is hyperbolic-elliptic for $t = \pm\infty$ (4), and derived a sufficient condition for a hyperbolic-elliptic motion of a collision-free system with negative energy (5). He found a lower limit of the shortest mutual distance among the three bodies for the same system to be stable in Lagrange's sense (6), and tried to extend Birkhoff-Merman's theorems to a system with positive energy. Also Sibahara and Yoshida (7) derived a condition for a triangular solution and a three-body collision.

Stumpff (8) attempted to bring the classical Lagrange's theory of the three-body problem in new trappings of modern mathematical representation. Lagrange's theory needs nine integrals for solving the problem by simple quadratures, which are invariant with respect to co-ordinate transformation. Stumpff used four vectors and five angles. He selected nine quantities expressed symmetrically by the velocities and the co-ordinates, and named them the fundamental invariants of the reduced three-body problem. The coefficients of Lagrange's quartic-equation are expressed symmetrically in terms of these fundamental invariants. He proposed three third-order differential equations as preferable for numerical integration.

Szebehely (9) studied the relations between the zero-velocity curves and the orbits in the restricted three-body problem. A condition which the force function must satisfy is derived giving a criterion for identifying certain periodic orbits with the zero-velocity curves. This kind of study is important for proving the existence of the surface of section in Birkhoff's sense of Poincaré's invariant-point theorem on the existence of periodic solutions in the three-body problem. Szebehely (10) then extended the application of zero velocity or Hill's curves in the dynamical systems of two degrees of freedom, the one to more general dynamical systems and the other to much wider applications than to establish various regions of possible motions in Hadamard's theory. A general set of dynamical problems, which possess zero-velocity curves was presented and the general problem of using the Hill curves for orbit generation was solved. Szebehely (11) further studied the general relation between the zero-velocity curves and the orbits for certain classes of dynamical systems including the restricted three-body problem and established the conditions for giving precise analytical meaning to the similarity, which the periodic orbits numerically obtained show to the Hill curves. Szebehely's extension of the Hill curves is called isotach since those curves are described with constant speed. It is shown that the totality of separable potential function, both of the product and of the additive types, possessing isotach orbits is generated by the solution of a second-order ordinary differential equation, and that a necessary condition for such extended Hill curves to be orbits for a dynamical system performing small vibrations is the equality of the characteristic values of the system. The result is applied to the restricted three-body problem in a particular case.

Eckstein (12) has examined the bounded Hill's curve in the $3 + 1$ body problem, after Lindow and Schaub.

Arenstorf (13) has derived a new regularizing transformation of the co-ordinates and the time in the planar restricted problem of three bodies. This transformation, a generalization of Birkhoff's transformation of 1915 and of Thiele's transformation of 1895, transfers the Hamiltonian function to a rational function of the new canonical variables and the original co-ordinates are rational functions of the new co-ordinates of order four.

Contopoulos (14) found the third integral, besides the energy and the angular momentum integrals, in the motion of stars under the gravitational potential of a galaxy as a whole. He calculated explicitly this third integral by means of von Zeipel's method. The question is the convergence of the integral in general cases, for example, in the three-body problem when expanded in series form as is usually done. Search for such a kind of integrals might throw some light on the solution of the classical problem in celestial mechanics.

Arnold (**15**) has obtained a basic result for presenting the solution of the problem of disturbed movement of planets in a trigonometric form for all initial data, with the exception of a certain manifold whose relative measure is small together with the disturbing mass. For the planar restricted problem of three bodies it gives a stability of periodic solutions in the sense of Liapounov and that of any solution in the sense of Lagrange.

Sitnikov (**16**) has proved the existence of oscillatory movements in the problem of three bodies. Alekseev (**17**) has shown a possibility of exchange in the problem of three bodies, that is, the change of hyperbolic-elliptic motion in Chazy's sense. Merman (**18**) has established the non-stability of periodic solutions in the case of a main resonance, according to Liapounov and Levi-Civita.

Ovenden and Roy (**19**) have formulated the Jacobi integral and the angular momentum integrals in the elliptical restricted problem of three bodies in terms of certain auxiliary functions depending on time. It is shown that long-range prediction on the Jacobi integral in the circular case would give wrong result for the elliptical case.

It is known that, when there is a cyclic co-ordinate, a Hamiltonian system posesses an integral common to the whole class of potential functions. Schmeidler (**20**) obtained a general criterion for the existence of such an integral on the basis of Lie's idea of transformation groups and function groups. He applied this criterion to problems of stellar dynamics and showed the impossibility of the existence of such an integral.

BIBLIOGRAPHY

1. Klemperer, W. B. *Astr. J.*, **67**, 162, 1962.
2. Miyahara, S. *Pub. astr. Soc. Japan*, **14**, 311, 1962.
3. Sibahara, R. *Pub. astr. Soc. Japan*, **13**, 108, 1961.
4. ,, ,, , **13**, 113, 1961.
5. ,, ,, , **14**, 90, 1962.
6. ,, ,, , **13**, 229, 1961.
7. Sibahara, R., Yoshida, Y. *Pub. astr. Soc. Japan*, **15**, 86, 1963.
8. Stumpff, K. *NASA Technical Note*, D–1417, 1963
9. Szebehely, V. *Astr. J.*, **68**, 147, 1963.
10. ,, *J. Franklin Inst.*, **275**, 371, 1963.
11. ,, *Arch. for rat. Mech. and Analysis*, **13**, 192, 1963.
,, *Space Science Rev.*, 1963, in press.
,, *Astr. J.*, **68**, 1963, in press.
12. Eckstein, M. C. *Astr. J.*, **68**, 535, 1963.
13. Arenstorf, R. F. *Astr. J.*, **68**, 548, 1963.
14. Contopoulos, G. *Astr. J.*, **68**, 1, 1963.; **68**, 1963, in press.
15. Arnold, V. I. Small denominators and the problem of stability in classical and celestial mechanics. *Problems of Movement of Artificial Celestial Bodies*, 7, Acad. Sci. U.S.S.R. Publishing House, Moscow, 1963.
16. Sitnikov, K. A. *Dokl. Ak. N. SSSR.*, **133**, 303, 1960.
17. Alekseev, V. M. *Astr. Zu.*, **39**, 1102, 1962.
,, Final movement in the problem of three bodies. *Problems of Movement of Artificial Celestial Bodies*, 50, Acad. Sci. U.S.S.R. Publishing House, Moscow, 1963.
18. Merman, G. A. On the non-stability of a periodic solution for the canonical system with one degree of freedom in the case of a main resonance. *Problems of Movement of Artificial Celestial Bodies*, 18, Acad. Sci. U.S.S.R. Publishing House, Moscow, 1963.
19. Ovenden, M. W., Roy, A. E. *M. N. RAS*, **123**, 1, 1961.
20. Schmeidler, F. *Astr. Nachr.*, **287**, 1, 1963.

https://doi.org/10.1017/S0251107X00014437 Published online by Cambridge University Press