

Here again the examples have been greatly extended and we see several inspired by the mathematical physics community. Vertex operators appear in §5 Ex. 8 while §6 Ex. 7 comes from Kirillov and Reshetikin’s treatment of the Bethe Ansatz and Young Tableaux. Although Chapter IV (“The Characters of  $GL_n$  over a Finite Field”) and Chapter V (“The Hecke Ring of  $GL_n$  over a Local Field”) are unchanged, their references and notes have been updated.

The final two chapters are new. Chapter VI, “Symmetric Functions with Two Parameters”, generalises the one parameter family of Hall–Littlewood functions to yield a family containing the Schur functions of Chapter I and Jack’s symmetric functions. This chapter, both in its text and examples, encapsulates much recent research and will be of significant interest not only to mathematicians but also to those workers (conformal field theorists, statistical mechanists and the like) who deal with multi-dimensional integrals and orthogonal polynomials. We find here various generalisations of Selberg’s integral

$$\begin{aligned} & \frac{1}{n!} \int_0^1 \dots \int_0^1 \prod_i x_i^{a-1} (1-x_i)^{b-1} \prod_{i<j} |x_i-x_j|^{2c} dx_1 \dots dx_n \\ &= \prod_{j=0}^{n-1} \frac{\Gamma(a+jc)\Gamma(b+jc)\Gamma((j+1)c)}{\Gamma(a+b+(n+j-1)c)\Gamma(c)}, \end{aligned}$$

many of which have been inspired by Macdonald. In this context Greg W. Anderson’s elegant proof [1] of the above formula was one of the few references I felt lacking.

The final chapter treats zonal polynomials via Gelfand pairs and zonal spherical functions. These polynomials have long been of interest to statisticians and more recently have found application by algebraic combinatorialists studying association schemes. The material here contains not only a useful compilation of results dispersed in the literature but many new results as well. Readers interested in such material should also note that Audrey Terras has a draft sequel (*Fourier analysis on finite groups and applications*) to her two volume *Harmonic analysis on symmetric spaces and applications* (Springer-Verlag, 1985, 1988) dealing with similar material, though with a different emphasis.

The second edition appears remarkably free of misprints, which is important for peripatetic workers wishing to use this as a reference volume. (The only misprint I am aware of, carried over from the first edition, is in (ii) p. 205 – a rather innocuous typographical error.) Macdonald should be congratulated for responding to his earlier reviewers and producing a more self-contained and better-referenced text; the extra examples will delight and aid many. This volume will undoubtedly become the standard reference on the subject until well into the next century and should be purchased by all research libraries. Knowing the impecunious nature of postgraduate life I am well aware that the £55 cost of the book represents much food and drink: this is one of those rare books I would recommend to students able to partake of its rich fare.

H. BRADEN

REFERENCE

1. G. W. ANDERSON, A Short Proof of Selberg’s Generalized Beta Formula, *Forum Math.* 3 (1991), 415–417.

KALUŻA, R. *Through a reporter’s eyes: the life of Stefan Banach* (Birkhäuser, Boston–Basel–Berlin, 1996), 167 pp., 3 7643 3772 9 (hardcover), £17.

Stefan Banach (1892–1945), one of the founders of modern functional analysis, was a leading figure in that group of extremely able Polish mathematicians who flourished in the period

between the First and Second World Wars and were to some extent inspired by a national identity which resulted from the creation of an independent Poland at the end of the First World War. In spite of his preeminent position, until the appearance of this book, there was a lack of generally available information about Banach; one may note with surprise that the *Dictionary of Scientific Biography* (Scribner's, New York, 1970) devotes only a page and a half to Banach. The idea of a biography of Banach apparently came from Teubner Verlag in the 70s and Roman Kałuża, a well-known Polish journalist, agreed to carry out the task. When he began Kałuża was unable to find a mathematician who was willing to collaborate with him; he tells us, "Thus I was forced to limit myself to the concept of a mathematician's biography from a reporter's perspective which only to a very modest degree analyzes his scientific work." Within this framework he has done a vast amount of research and presents us with a fascinating story, although some aspects remain vague or speculative because of failing memories or a lack of records or witnesses. The turbulent 80s and possibly also a reluctance to take Banach off his pedestal frustrated publication and it was not until 1992, appropriately the centenary of Banach's birth, that the Polish edition appeared. The present edition has been edited and translated by Ann Kostant (Birkhäuser) and Wojbor Woyczyński (Case Western Reserve University) and the text has been expanded with the author's approval; in particular an interesting set of photographs has been added.

Chapter I *Beginnings and Premonitions* takes us from Banach's birth through to his graduation from his gymnasium in 1910. Contrary to what is stated in the *Dictionary of Scientific Biography* it appears that the name Banach was his mother's surname (his father was Stefan Greczek). Hugo Steinhaus has been quoted as regarding Banach as his greatest discovery. Chapter II recounts how in 1916 Steinhaus came upon the then unknown Banach and a friend discussing Lebesgue measure in a park in Cracow. It also deals *inter alia* with Banach's studies at Lvov Polytechnic and in Cracow and his marriage to Łucja Braus in 1920. Chapter III *Lvov and Mathematics: 1919–1929* is concerned with Banach's thesis and early papers, the writing of textbooks (apparently to pay off debts), the Banach–Tarski paradox, Banach's sabbatical visit to Paris and his elevation at the age of 32 to *Professor Ordinarius*, the highest professorial rank, at the Jan Kazimierz University.

Banach's best known and most influential publication is undoubtedly his book *Théorie des Opérations Linéaires*, which was published in 1932, although a Polish edition had already appeared in 1931. This work and the founding in 1929 by Banach and Steinhaus of the important periodical *Studia Mathematica* are discussed in Chapter IV. Lvov's *Scottish Café*, where Banach and his cronies met, and the *Scottish Book*, in which their problems were recorded, have become part of mathematical folklore. A vivid account of these activities is given in Chapter V.

Chapter VI *Banach Privately and in Daily Life* consists of various reminiscences from former colleagues and students and from Banach's half-sister. Chapter VII *The Last Years* encompasses the period of the Soviet occupation of Lvov in September 1939, the German conquest in 1941 and the Soviet return in July 1944. Banach's position was secure and possibly even enhanced under the Soviets, who greatly admired his work, but in the middle period he had to endure the humiliation of working as a feeder of lice in a bacteriological institute. The chapter ends abruptly with Banach's death from lung cancer at the age of 53. Extracts from various tributes to Banach which were delivered at a conference organised by the Institute of Mathematics of the Polish Academy of Sciences in 1960 form the material of the final Chapter VIII *In the Eyes of Friends and Followers*; these come from Steinhaus, Sobolev, Mazur, Szökefalvi-Nagy, M.H. Stone and Ulam.

There are three appendices: *Mathematics in Stefan Banach's Time*; *Selected Publications of Stefan Banach*; *Selected Bibliography*. The first of these has been contributed by Wojbor Woyczyński and sets the scene for an understanding of the significance of Banach's achievements.

Kostant and Woyczyński state in their preface that ". . . the Banach that emerges from the book's pages is a sympathetic, warmly presented, thoroughly human, and obviously fascinating genius, anything but a stilted, traditional, European-style university professor and ivory-tower

academician.” With this I agree, but I wonder if this is the whole story. There seem to be some gaps; for example: What about Banach as a family man? Mrs Banach apparently provided the original book in which the deliberations in the *Scottish Café* were recorded – before then they were written in pencil on the marble tabletops and were usually lost when the janitor cleaned up – but we learn very little more about her; Banach’s son Stefan, who became a neurosurgeon in Warsaw, is only mentioned in connection with later ownership of the *Scottish Book*. Nevertheless, I found the book totally absorbing and can recommend it to anyone with an interest in modern mathematics. The author and editor-translators have produced a very readable text, which has been well presented by the publisher.

I. TWEDDLE

SACHKOV, V. N. *Combinatorial methods in discrete mathematics* (Encyclopedia of Mathematics and its Applications, Vol. 55, Cambridge University Press, Cambridge, 1996), xiii + 306 pp., 0 521 45513 8 (hardback), £45 (US\$69.95).

This is a translation of an extended version of a book which first appeared in Russian in 1977. It is one of two books by Sachkov which have recently been added to the series, the other volume being entitled *Probabilistic methods in discrete mathematics*. Its aim is to present enumerative methods in a unified way, concentrating on generating functions and emphasising asymptotic results.

In enumeration theory generating functions play an important role. They can be considered either as formal power series or as analytic functions. From the latter point of view contour integration and the saddle point method can be used to obtain asymptotic estimates for the coefficients. A number of examples are given; for example, asymptotic formulae for Stirling numbers of the second kind and the Hardy–Ramanujan asymptotic formula for the number of partitions of an integer are presented.

A chapter on graphs and mappings counts various types of graph and then proceeds to a discussion of the cycle structure of permutations.

The key chapter, which precedes an account of Polya’s theory, is entitled “The general combinatorial scheme”. Enumeration problems are viewed as concerning mappings from a set  $X$  to a set  $Y$  with permutation groups  $G$  and  $H$  on  $X$  and  $Y$  which give equivalence classes of elements, equivalent elements being regarded as indistinguishable. The problem is then to construct the generating function for enumerating distinguishable elements with respect to some “weight”. In Polya’s theory the required generating function  $\Phi$  is represented in terms of the basic generating function  $F$  as a cumbersome polynomial of several variables which depend on  $G$  and  $H$ . “Therefore it is natural to separate several simple cases in terms of  $G$  and  $H$ , and to present the method of finding generating functions for the enumeration of objects possessing certain characteristics (the so-called primary and secondary specifications of the corresponding mappings) that are common in combinatorial problems . . . . Let each of  $G$  and  $H$  be either the identity group or the symmetric group. Using the primary and secondary specifications as the characteristics, we can give a method for constructing the generating function for the enumeration of non-equivalent mappings in each of the four possible cases under various restrictions on these specifications. The described formalization of certain classes of problems, together with the method for finding the corresponding generating functions, is called the general combinatorial scheme.”

It is not an easy book to read. The subject matter is quite technical and is in the style of Riordan’s classic *An introduction to combinatorial analysis* (Wiley, 1958; Princeton U. P., 1980). Further, some of the terminology is old-fashioned; there are invariant subgroups and substitutions. But for the serious student of generating functions and asymptotic techniques it provides an account of work of Kolchin (who did the translation), the author and others which is not otherwise readily available in English.

I. ANDERSON