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On the timing histories of the Crab, PSR1509-58 and PSR0540-69

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Abstract. A simple model of a combined magnetic dipole, gravitational and/or magnetic quadrupole and particle accelerator is applied to simulate the time behaviour of the timing parameters of PSR 0531+21, PSR 1509-58 and PSR0540-69, as well as their energy budget. The model restricts the allowed values for the second braking index and is therefore testable through measurements of $\dddot{\nu}$.

1. Introduction

Pulsars are often described in terms of a rotating magnetic dipole (Pacini 1968) which predicts a braking index $n = 3$. Although this is not observed, it is not too far from the index measured in the youngest pulsars, suggesting the simple model is not far from adequate. The term for gravitational radiation to model the $P\dot{P}$ diagram was included by Ostriker & Gunn (1969). Adding this term can only give indexes between 3 and 5, and therefore an extra term, with weaker dependence on the pulsar frequency ν is needed. Theoretical models of γ-ray emission suggest particle acceleration contributes towards a $n = 2$ index, but often put only a small fraction of the rotational losses into accelerated particles (Dermer & Sturner 1994), guided by the low fraction of $\dot{E}_{\rm rot}$ that is observed as gamma-ray radiation. Evidence exists that a large fraction of the energy goes into particles, for example the synchrotron emission of the Crab and PSR0540-69 nebulae, and we assume here that an arbitrary fraction of $E_{\rm rot}$ can be lost through this process.

2. Model

We model the dynamics of the pulsar through the differential equation:

$$\dot{\nu} = -g\nu^5 - r\nu^3 - s\nu^2 \,, \tag{1}$$

the coefficients g, r, s been assumed constant in time. They can be directly related to measured frequency, first derivative and braking indexes $n \equiv \ddot{\nu}\nu/\dot{\nu}^2$ and $m \equiv \dddot{\nu}\,\nu^2/\dot{\nu}^3$. Physically, g, r, s must be between 0 and 1, and therefore this model bounds the second braking index to satisfy, $\mathrm{Max}(n^2{+}4n{-}6; n^2{+}7n{-}15) \leq m \leq n^2 + 6n - 10$ which can be tested through measurements or bounds on the third frequency derivative. The absence of quadrupolar radiation ($g = 0$) forces the second braking index to have its lowest value, $m = n^2 + 4n - 6$.

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3. Direct application of the model

3.1. PSR1509-58

From the timing parameters given by Kaspi et al. (1994), the model requires $13.40 \leq m \leq 15.07$, consistent with the measurement $m = 14.5 \pm 3.6$. Assuming $m = 14.5$, the energy output is 18% gravitational, 29% magnetic dipole and 53% particle acceleration.

Integrating eq.1 backwards, and assuming an age of 1200 years, we obtain ν at birth of 15.4 Hz, or about 65 milliseconds. Maximizing the contribution from the quadrupole component gives 53 milliseconds, still slower than the Crab today. The dynamical age $\nu/(-\dot{\nu})$ correlates well with model estimated age, suggesting PSR1509-58 is some 300 years older than the Crab pulsar.

3.2. Crab pulsar

The model requires $10.33 \leq m \leq 11.35$, predicting $\dddot{\nu} = (-6.5 \pm 0.3) \times 10^{-31}\,\mathrm{s}^{-4}$. Given that we know the actual age of the pulsar we can integrate 1 to its birth. The frequency at birth is rather insensitive to the input value of m and, averaged over the allowed values, gives $\nu_0 = 54.1 \pm 1.2\,\mathrm{Hz}$. We note the observed frequency $\dot{N}$ and strength $\Delta\nu$ of the glitches of the Crab should not affect this value, as $\dot{N}\tau\Delta\nu << \nu$, where τ is the age of the pulsar.

The Crab gives also an opportunity to estimate how good is $\nu/(-\dot{\nu})$ as an age indicator: for $m \leq 11$ an linear fit gives $\nu/(-\dot{\nu}) \simeq 0.9t + 500$.

3.3. PSR0540-69

Although the timing information for PSR 0540-69 is somewhat more restricted (no date of birth or $\ddot{\nu}$), the braking index $n = 2.04 \pm 0.02$ (Gouiffes et al. 1992) highly restricts the parameter space of the model: m has to lie in a narrow interval $[6.32; 6.40]$, with a predicted triple frequency derivative of $\dddot{\nu} = -(1.090 \pm 0.007) \times 10^{-31}\,\mathrm{s}^{-4}$. If PSR 0540-69 continues to be a stable rotator a third derivative of a sizeable fraction of $10^{-30}\,\mathrm{s}^{-4}$ should be measurable in the forthcoming years.

The slope between dynamical age and pulsar age is typically ~ 0.6 (i.e. $\nu/(-\dot{\nu})$ goes slower than time) and $\nu/(-\dot{\nu})$ might actually *underestimate* the actual age of the pulsar. In particular, an assumed age of 2000 years gives almost the same initial frequency as for the Crab, $\nu_0 = 52.4 \pm 1.2\,\mathrm{Hz}$.

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