NEUTRON STAR MODELS[†]

V. Canuto
NASA Goddard Institute for Space Studies, New York, N.Y., USA
and Department of Physics, CCNY, New York, N.Y., USA

Richard L. Bowers
Lawrence Livermore National Laboratory, Livermore, CA, USA

ABSTRACT

The current state of neutron star structure calculations is reviewed. Uncertainties in the equation of state for matter at and above nuclear density remain. The role of the delta resonance, pion condensates, and quark matter is reviewed. Despite uncertainties, we find that recent models yield stable neutron star masses which are consistent with observational estimates.

1. INTRODUCTION

It must be clear to everybody that the peak of activity in modeling neutron star (NS) structure is behind us. We are at present gliding on the tail of a Gaussian distribution whose maximum occurred around 1975-1976. The flurry of activity began in 1970, when theorists from nuclear, low temperature and high energy physics and relativity converged on the problem.

Although the pioneering work on neutron stars is usually traced to the papers of Oppenheimer and Volkoff (1939) and work by Cameron, Wheeler and others (Harrison et al., 1965 and references therein), the nature of cold equations of state for densities above about 10^8 g cm^{-3} had not been thoroughly or systematically investigated. Many of the theorists who participated in this effort have moved on to new problems in physics or astrophysics, but they have left behind a great deal of work that has elucidated, certainly beyond the initial expectation, one of the most complex "nuclei" nature has ever devised.

The density gradient from the surface to the core of a neutron star varies by eight to nine orders of magnitude. The material in

---

[†]This article was read in absence of the authors by R.N. Manchester

W. Sieber and R. Wielebinski (eds.), Pulsars, 321–330.

various regions of the star consists of electrons, nuclei (many of which would be unstable in the laboratory), neutrons, protons, more massive baryons (hyperons) and possibly pions and possibly even quarks. Furthermore the liquid, solid, superconducting, superfluid and perhaps even boson condensate states are believed to occur within the expected density range. This explains why the effort of theorists from many branches of physics was needed. There exist several extensive reviews of the problems associated with equations of state (Canuto, 1974, 1975; Baym and Pethick, 1975) and of the resulting neutron star structure (Arnett and Bowers, 1977; Hartle, 1978; Ruderman, 1972). No attempt will be made here to duplicate those efforts, though we will summarize some recent results.

The physics relevant to static neutron star structure is complex and of course we have not reached a perfect understanding of every detail. However, we have pinned down the range of variation of the quantities of interest well enough that a tolerable comparison between theory and observational data can now be made (Arnett and Bowers, 1977).

Let us review the constraints which can be placed on neutron star structure. We note that because internal temperatures in present day neutron stars are expected to be below 1 MeV or so, and the chemical potentials of the fermions in the regions comprising most of the mass exceed this value by perhaps a factor of 100 or more, neutron star matter may be considered to be in its ground state. The mass, radius and moment of inertia are therefore determined by the central density ρ_c and the equation of state $P(\rho)$ relating pressure to density. The maximum neutron star mass M_{max} which is gravitationally stable is particularly important, since general relativity predicts that a configuration with $M > M_{max}$ will gravitationally collapse to a black hole.

The following general conclusions can be obtained from fundamental physical constraints (Rhodes, 1971; Nauenberg and Chapline, 1973; and Sabbadini and Hartle, 1973), or by investigating the sensitivity of structure calculations to a variety of theoretically based equations of state (EOS) (Arnett and Bowers, 1977; Borner and Cohen, 1973; Canuto, 1975).

Assuming that the energy density is positive and that the mass configuration is stable Sabbadini and Hartle obtain $M_{max} \cong 5\ M_{\odot}$. By further imposing causality (speed of sound less than the speed of light), Rhodes and independently Nauenberg and Chapline showed that $M_{max} \lesssim 3\ M_{\odot}$. These estimates further assume that our knowledge of EOS around nuclear density is not in error by an order of magnitude. It therefore appears impossible to obtain $M_{max} \lesssim 3$ to $5\ M_{\odot}$ for non-magnetic static configurations. The maximum observed rotation rates and predicted magnetic field strengths of pulsars are not large enough to increase this limit by much.

For a wide range of EOS based on detailed micro physics input, including some examples which are likely to be extremes of stiffness or softness, M_{max} was found to be in the range 1.6 $M_\odot$ to 3 $M_\odot$, and moments of inertia lie between 10^{44} and several times 10^{45} g cm^2 (Figures 1 and 2). Radii are generally of the order of 6 - 20 km, though low density envelopes can extend these values without significantly modifying the mass. Generally the greater M_{max} (as a function of EOS), the lower the central density of the mass peak. Typically ρ_c at M_{max} is in the range of one to several times 10^{15} g cm^{-3}. Few "realistic" EOS yield $\rho_c >$ 6 x 10^{15} g cm^{-3}, or $\rho_c < 10^{15}$ g cm^{-3}.

Finally for configurations with $\rho_c > 10^{15}$ g cm^{-3}, much of the mass is at densities comparable to ρ_c. This implies that a knowledge of the upper portion of the $M(\rho_c)$ curve requires a knowledge of the EOS at densities nearly four times nuclear density ($\rho_n \cong 2.5 \times 10^{14}$ g cm^{-3}).

Perhaps the most significant parameters obtained from model neutron stars are their maximum mass, moments of inertia, whether or not they

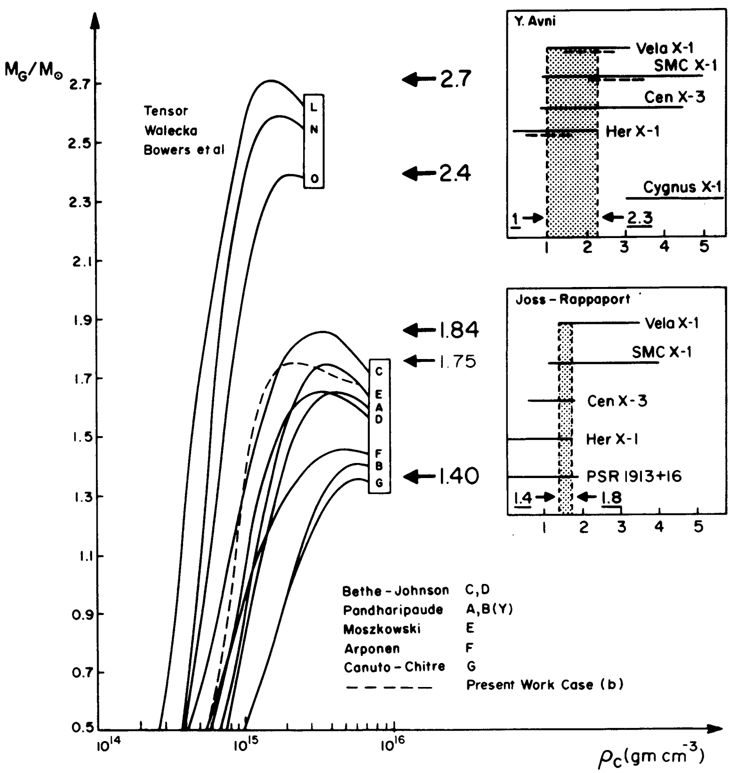

Figure 1a: Neutron star mass vs. central density. Letters A through O refer to equations of state and models discussed by Arnett and Bowers (1977). Inserts show observational bounds on neutron stars (pulsars or X-ray sources) given by Joss and Rappaport (1976) and Avni (1977). The dashed curve is a recent result by Canuto et al. (1978). Figure adapted from Canuto (1977).

https://doi.org/10.1017/S0074180900093074 Published online by Cambridge University Press

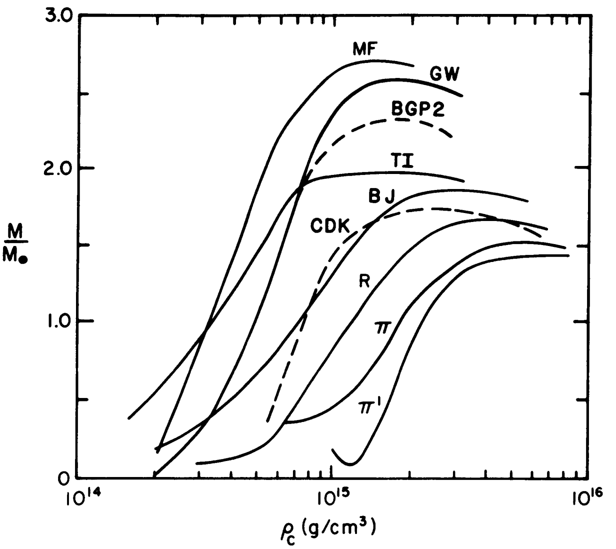

Figure 1b: Neutron star mass vs. central density for recent equations of state. The lower dashed curve is the spin two exchange model of Canuto et al. (1978); BGP2 is a spin two exchange model of Bowers et al. (1978) modified for neutron star matter (the results quoted by BGP applied to pure neutron matter). The curve GW is a relativistic meson exchange model which includes a pion condensate (Wheeler and Gleeson, 1980). The remaining curves have been adapted from Baym and Pethick (1979).

contain quarks or pion condensates (primarily for cooling estimates), and what regions are expected to be crystalline.

As for the NS masses, the comparison is usually made with data derived from observations of compact X-ray sources (Joss and Rappaport, 1976) and the binary pulsar. X-ray observations give $M/M_{\odot} \approx 1.33 \pm .2$, $1.5 \pm .2$ for Her X-1 and Vela X-1 respectively. Mass estimates for the binary pulsar give for each component, $M/M_{\odot} \approx 1.4 \pm 0.2$ (Taylor and McCulloch, 1980). The possibility of an independent mass measurement based on gravitationally red shifted gamma ray lines (Leventhal, 1977; Leventhal et al., 1977) has also been suggested (Brecher, 1977; Bowers, 1977). These are particularly important since they may be used to constrain the mass of isolated neutron stars. For example, a 0.4 MeV line observed from the Crab Nebula (Leventhal et al., 1977) gives $1.3 \leqq M/M_{\odot} \leqq 1.9$ (Bowers, 1977). Such data fall within the range of theoretical values and are consistent with the remnant mass predicted by supernova models. A second quantity of interest is the NS moment of inertia I, an estimate of which can be obtained from the observed secular rates of change of the spin periods. We recall that pulsars are believed to be rotating magnetized neutron stars formed by the core collapse and supernova explosion of massive stars. The result $I \gtrsim 1.5 \times 10^{44}$ g cm^2, is compatible with the theoretical predictions.

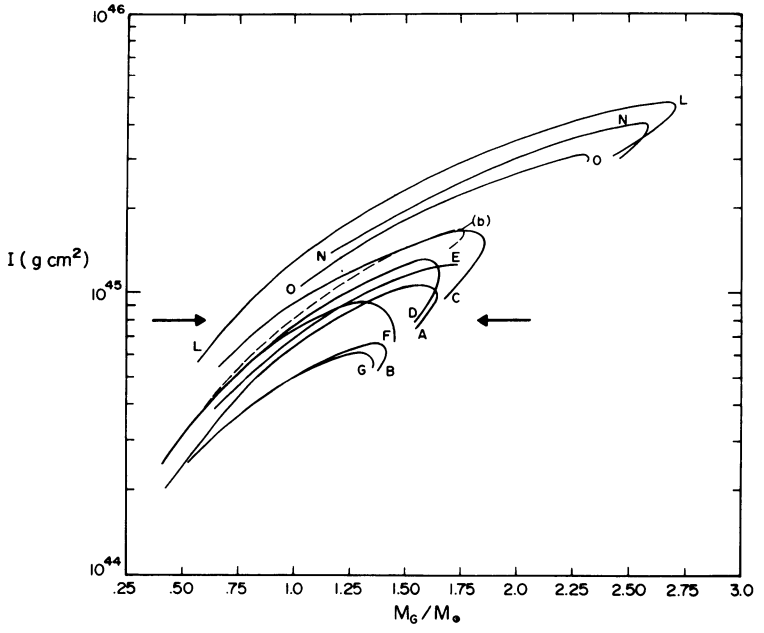

Figure 2: Neutron star moment of inertia for models shown in Figure 1a. The horizontal arrows represent an observational lower bound on the moment of inertia for the Crab pulsar. Figure adapted from Arnett and Bowers (1977).

These results are arrived at through a laborious buildup, layer by layer, of this gigantic nucleus, 10 km in radius, with a mass $M \sim M_\odot$ and a corresponding average density around or above nuclear density, $\rho_n = 2.5 \times 10^{14}$ g cm^{-3}. Although the density drops to values of order 10^4 g cm^{-3} at the surface, most of the mass in a neutron star is expected to be more dense than about 10^{14} g cm^{-3}. A trip from the surface down to the interior will start from the "crust", most likely solid, 1 km thick, at the beginning of which we encounter rather familiar nuclei both in kind as well as in size. The situation changes rapidly with depth as the neutronization process ($p + e^- \rightarrow n + \nu$) enriches the neutron content of every nucleus, thus moving it away from the valley of beta-stability characterized by $N \approx Z$. The nuclei become fat.

As the density increases (decreasing radius) the nuclear density grows until just below ρ_n where the nuclei touch each other. A further increase in density results in a phase transition where all the nuclei merge into a sea or liquid of neutrons, and protons.

The theoretical understanding of the physics of the crust is rather satisfactory and no great surprise may be expected in the future even though the thin outer layers where the strong magnetic field has the upper hand can distort the atoms in such a way as to pose brand new problems and require new theoretical methods. However, since the magnetic regime does not alter the gross properties (mass, radius, etc.) of the star, we shall not dwell on them any longer (see Ruderman, 1972; and Baym and Pethick, 1975).

2. THE LIQUID REGIME

The liquid regime begins at $\rho \sim \rho_n$ where the dominant species are electrons and free baryons. Here nuclear physics is dominant. To fully describe the properties of this liquid, we need at least two ingredients: 1) a model of the nuclear forces (a nuclear potential for densities up to a few times ρ_n); and 2) a many body technique capable of handling a situation in which the inter-particle separations may be comparable to the size of the nucleons.

As for the two-body potential, the first thing everybody did in the early 70's was to employ phenomenological potentials constructed to fit the scattering data of free nucleons at energies up to 200 MeV. The difficulties with such a potential were immediately apparent. When applied to the only other many body system we know, namely symmetric nuclear matter (N = Z), one does not get either the correct binding energy, saturation density or compressibility. It is usually concluded that the Reid potential, for example, is too soft.

Green and Haapakoski (1974) pointed out that in a medium, the Pauli principle excludes intermediate states (already occupied) thus eliminating some attraction. When dispersion effects are properly accounted for, some extra attraction is again eliminated. The list is not exhausted however. Perhaps the most relevant process is the inclusion of the Δ-resonance in the intermediate state. This effect, because of isotopic spin constraints is more important in neutron matter than in symmetric nuclear matter. Detailed calculations show that for $\rho \gtrsim \rho_o$, the inclusion of the Δ-resonance switches the NN, V(T = 1) potential from attractive to repulsive.

All these effects stiffen the equation of state, with the net result that at a given density the radius of the star becomes larger. The increased stiffness in the equation of state yields a higher NS mass. The two effects can be seen in Figure 1. Here again, the usual way to double check these results is to apply the same potential to symmetric nuclear matter. Unfortunately, such computations do not yield the correct saturation density and binding energy. We can only conclude that until the nuclear matter case is settled in a satisfactory way, one cannot fully trust the NS calculations, and the Δ-resonance effect cannot be considered understood. More work is clearly needed in the regime of nuclear density.

Two other phenomena that have been discussed over the years are the possibility of superfluid neutrons, superconducting protons and a pion condensate. Superfluidity in the laboratory is a relatively low density phenomenon involving the electromagnetic coupling between electrons. These interactions are dominated by the $^1S_0$ state. In neutron star matter superfluidity results from the strong interactions. At densities near nuclear density the $^1S_0$ states dominate, but at higher densities the $^3P_2$ state may also be involved. This introduces a possible anisotropic energy gap whose implications are by no means fully understood.

The pion condensate is a much newer phenomenon of perhaps greater theoretical interest. The pionic field is usually treated as a fluctuating field about nucleons which mediates some aspects of the nuclear force. A pion condensate is one in which such fluctuations develop a non-zero expectation value, so that the number density of physical pions in the matter is non-zero.

Such phenomena are currently believed to soften the equation of state, thus counterbalancing the stiffening due to the presence of the Δ-resonance.

The computations done so far for these exotic effects have illustrative rather than quantitative value, the uncertainties being still too many and difficult to quantify. Furthermore, it has been proposed that a pion condensate may help the formation of a solid core, a physical situation more nearly like a liquid crystal than a crystalline solid.

A recent relativistic model, which reproduces some bulk properties of nuclear matter, and which includes a relativistic mean field model of the pions, indicates that a condensate does form, but at a density $\rho \cong 6 \times 10^{14}$ g cm^{-3}, too high to have much effect on the structure of gravitationally stable neutron stars (Wheeler and Gleeson, 1980). The two phenomena most likely to have a noticeable effect on neutron star structure (the Δ-resonance and the pion condensate) may very well cancel each other to such an extent that the exclusion of both may not result in a substantial change in theoretical masses, radii or moments of inertia.

3. THE HIGH DENSITY REGIME

When we pass the regime of nuclear density and penetrate the high density core $\rho \gtrsim 10^{15}$ g cm^{-3}, we encounter a fundamental difficulty that is represented by the largely unrelated pieces of work that have been published so far.

At higher than nuclear densities, the meson cloud surrounding each baryon overlaps the one associated with neighboring baryons and the simple representation of the interaction via a non-local, velocity independent potential is no longer valid. In effect the inter-baryon separation becomes small enough that we can no longer distinguish between the interactions between particles and the interactions which give them their structure. In fact it may be necessary to replace a description of the matter as baryons by a description based on quarks.

Possible descriptions of matter at super nuclear density have been attempted by several authors. One model consists of considering a gas of relativistic "bare" nucleons interacting through a meson field. These models are intrinsically relativistic, include finite density effects not only on the baryons but on intermediate meson states as well, and can be extended to finite temperature. If spin zero and one mesons are exchanged, the resulting equation of state has the limit $p \to \rho$, i.e. $c_s^2 = 1$, in the $\rho \to \infty$ limit (Walecka, 1974; Bowers et al., 1975; Wheeler and Gleeson, 1980). The underlying fields and their couplings reproduce nucleon-nucleon scattering (Bowers et al., 1977b). If however, one includes spin two exchange (such as the $f°$ meson) an unusual asymptotic density dependence occurs. In the high density limit, the pressure softens until a density ρ_a is reached (Canuto et al., 1978; Bowers et al., 1978) when the attractive nature of the spin-2 particles takes over: the pressure becomes a tension and a gravitational instability sets in. This regime ($\rho_c > \rho_a$) would clearly lead to collapse. However, the transition density is too high to be relevant to stable neutron stars.

The NS masses obtained from these models are shown in Figure 1. As one can see, the N, σ, ω case yields the highest NS mass so far; the inclusion of the $f°$-meson produces results closer to the ones obtained with the static potentials. The high mass is reduced by the inclusion of the attraction due to the spin-2 meson exchange.

Another possibility discussed several years ago but which has not found experimental support, is "abnormal matter", abnormal in the sense that a scalar field is assumed with an expectation value which provides enough negative energy to make the effective nucleon mass $m_* = m + g\ \phi(x)$ (where $\phi(x)$ is the scalar field) vanish as the density $\rho \to \infty$ (Lee and Wick, 1974). Since the energy difference between the normal and abnormal states is very small, one must be very confident in the reliability of the computation in order to be sure that a transition to the lower energy "abnormal state" has indeed been achieved. A preliminary result indicating that an abnormal state could occur in NS was subsequently contradicted when it was found that upon fitting the parameters of the model to NM, the transition no longer occurred within the density region relevant to NS. This seems to be the generally accepted viewpoint.

4. QUARK MASSES

Today the most successful fundamental theory for strong interaction is based on quarks. It is thought that strongly interacting particles (like the nucleons, mesons and hyperons) will dissolve into quarks at densities a few times ρ_n. Quarks have spin 1/2, and baryon number 1/3, and come in six flavors u,d,s,c,b,t. The first four of these have electric charges 2/3, -1/3, -1/2, 2/3 and strangeness 0, 0, -1, 0 respectively. Corresponding to each quark state is an analogous anti-quark state. A proton is a bound state of three quarks in the states uud. A neutron is an udd combination, whereas a meson is a $q\bar{q}$ combination (q stands for quark and $\bar{q}$ for an anti-quark). The quarks possess internal degrees of freedom called "color": two quarks of the same color repel, those of a different color attract. Finally, the masses of the u, d, s quarks are estimated to be in the vicinity of 10 MeV, 10 MeV, and 300 MeV respectively. The species which occur in quark matter models are the u, d and s quarks.

As for the application to NS, the simple but rather complete MIT bag model has been used with the result that the estimated quark matter transition density falls in the range (14-40) x 10^{15} g cm^{-3}, too high a density to be of interest to NS interiors. More refined computations than the ones based on the bag model have also been carried out with the result that depending on the assumed coupling constant strength, the transition could well occur within the density range of interest to NS.

At present, the value of such coupling constants is not known with sufficient accuracy to allow a definite conclusion to be made as to whether quarks do indeed exist in the NS cores.

The possibility of a quark matter phase transition in neutron stars raises several interesting questions. If the transition density ρ_a is low enough so that the cores of NS contain quarks, will there be observational consequences? Brecher (1977) has suggested that the surface red shift could be used to distinguish between quark and hyperon matter. Neutrino emission from hot neutron stars is an efficient cooling mechanism. Quark models predict that the direct beta decay reactions $u + e^- \rightarrow d + \nu_e$ and $d \rightarrow e^- + u + \bar{\nu}_e$ can occur and that the corresponding energy loss rate is much greater than that of ordinary neutron matter (Iwamoto, 1980). Unfortunately, if ordinary neutron matter also contains a pion condensate, then the estimated cooling times are comparable to those of quark stars (Maxwell, 1979; Tsuruta, 1979; van Riper and Lamb, 1980).

An extension of the mass limit arguments developed for neutron stars has been applied to quark stars (Chapline and Nauenberg, 1977). They conclude that quark stars should have maximum masses less than 1.6 $M_\odot$.

The possibility of a stable quark matter mass peak distinct from the neutron star mass peak has also been discussed (Bowers et al., 1977a).

REFERENCES

Arnett, W.C. and Bowers, R.L.: 1977, Astrophys. J. Suppl. 33, p. 415.
Avni, Y.: 1977, 'Highlights of Astronomy' (Reidel, Netherlands), p. 137.
Baym, G. and Pethick, C.: 1975, Ann. Rev. Nucl. Sci. 25, p. 27.
Baym, G. and Pethick, C.: 1979, Ann. Rev. Astron. Astrophys. 17, p. 430.
Borner, G. and Cohen, J.M.: 1973, Astrophys. J. 185, p. 959.
Bowers, R.L.: 1977, Astrophys. J. Letters 216, p. L63.
Bowers, R.L., Gleeson, A.M., and Pedigo, R.D.: 1975, Phys. Rev. D12, p. 3043; p. 3056.

Bowers, R.L., Gleeson, A.M., and Pedigo, R.D.: 1977a, Astrophys. J. 213, p. 840.
Bowers, R.L., Gleeson, A.M., and Pedigo, R.D.: 1977b, Nuovo Cimento 41B, p. 441.
Bowers, R.L., Gleeson, A.M., and Pedigo, R.D.: 1978, 'High Spin Effects in Superdense Matter', Center for Particle Theory Preprint ORO 336, Univ. of Texas at Houston.
Brecher, K.: 1977, Astrophys. J. Letters 215, p. L17.
Canuto, V.: 1974, Ann. Rev. Astron. Astrophys. 12, p. 167.
Canuto, V.: 1975, Ann. Rev. Astron. Astrophys. 13, p. 335.
Canuto, V.: 1977, Ann. New York Acad. Sci. 302, p. 514.
Canuto, V., Datta, B., and Kalman, G.: 1978, Astrophys. J. 221, p. 274.
Chapline, G. and Nauenberg, M.: 1977, Ann. New York Acad. Sci. 302, p. 191.
Green, A.M. and Haapakoski, P.: 1974, Nucl. Phys. A221, p. 429.
Harrison, B.K., Thorne, K.S., Wakano, M., and Wheeler, J.A.: 1965, 'Gravitation Theory and Gravitational Collapse' (Chicago, Univ. of Chicago Press).
Hartle, J.: 1978, Phys. Rept. 46, p. 201.
Iwamoto, N.: 1980, Phys. Rev. Letters 44, p. 1637.
Joss, P.C. and Rappaport, S.A.: 1976, Nature 264, p. 219.
Lee, T.D. and Wick, G.C.: 1974, Phys. Rev. D9, p. 2297.
Leventhal, M.: 1977, private communication.
Leventhal, M., MacCallum, C.J., and Watts, A.C.: 1977, Astrophys. J. 216, p. 491.
Maxwell, O.V.: 1979, Astrophys. J. 231, p. 201.
Nauenberg, M. and Chapline, G.: 1973, Astrophys. J. 179, p. 277.
Oppenheimer, J.R. and Volkoff, G.M.: 1939, Phys. Rev. 55, p. 374.
Rhodes, C.E.: 1971, Unpublished Ph.D. Thesis, Princeton University.
van Riper, K.A. and Lamb, D.Q.: 1980, 'Neutron Star Evolution and Results from the Einstein X-ray Observatory', Univ. of Illinois preprint.
Ruderman, M.A.: 1972, Ann. Rev. Astron. Astrophys. 10, p. 427.
Sabbadini, A.G. and Hartle, J.B.: 1973, Astrophys. Space Sci. 25, p. 117.
Taylor, J.H. and McCulloch, P.M.: 1980, Ann. New York Acad. Sci. 336, p. 442.
Tsuruta, S.: 1979, Phys. Rept. 56, p. 237.
Walecka, J.D.: 1974, Ann. Phys. 83, p. 491.
Wheeler, J.W. and Gleeson, A.M.: 1980, private communication.