

ON THE ABSOLUTE ORIENTATION OF THE SELENODETIC REFERENCE FRAME

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I. INTRODUCTION

The selection of selenodetic reference coordinate system is an important problem in astronomy and selenodesy. For the purposes of reduction of observations, planning and executing space missions to the Moon, it is necessary, in any case, to know the orientation of the adopted selenodetic reference system in respect to the inertial coordinate system.

Let us introduce the following coordinate systems :

$C(\xi_c, \eta_c, \zeta_c)$, the Cassini system which is defined by the Cassini laws of the Moon rotation;

$D(\xi_d, \eta_d, \zeta_d)$, the dynamical coordinate system, whose axes coincide with the principal axes of inertia of the Moon;

$Q(\xi_q, \eta_q, \zeta_q)$, the quasi-dynamical coordinate system connected with the mean direction to the Earth, which is shifted by 254" West and 75" North from the longest axis of the dynamical system (Williams et al., 1973);

$S(\xi_s, \eta_s, \zeta_s)$, the selenodetic coordinate system, which is practically realized by the positions of the points on the Moon surface given in Catalogues;

$I(X, Y, Z)$, the space-fixed (inertial) coordinate system.

All the systems are selenocentric with the exception of $S(\xi_s, \eta_s, \zeta_s)$. On the whole, the origin of this system does not coincide with the center of mass of the Moon.

The figure 1 represents the interrelation between the selenocentric systems mentioned above. The following designations are used :

ϕ_i, ψ_i, θ_i ($i=0, 1, 2$) : Eulerian angles
 u_i, v_i, w_i ($i= 1, 2$) : small angles of rotation

The indexes 0, 1, 2 designate that the mentioned angles are calculated as follows : without taking into account the effect of physical libration; taking into account this effect with an accuracy of the second-degree; with the higher-than second-degree harmonics of the selenopotential.

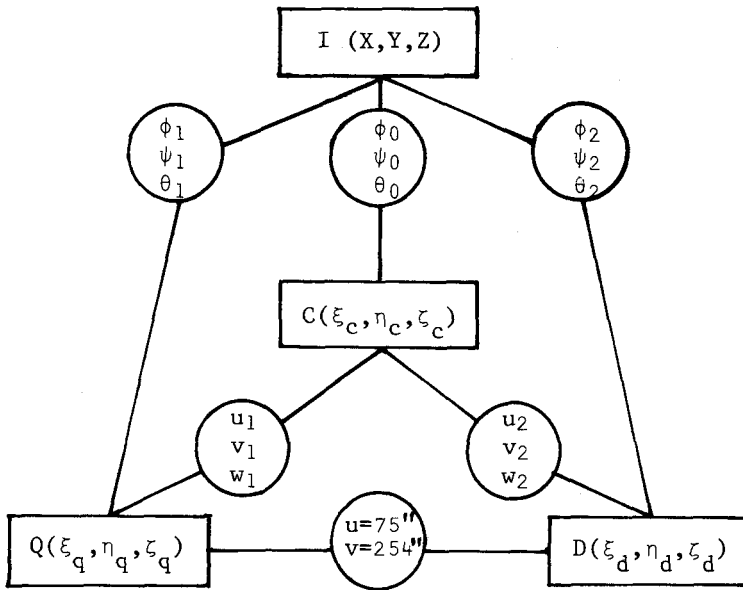


Figure 1. Scheme of interrelation of different selenocentric systems

It is well known that :

$$\begin{aligned}
 \phi_i &= 180^\circ + l_q - \Omega + \epsilon_i - \sigma_i \\
 \psi_i &= \Omega + \sigma_i \\
 \theta_i &= I + \rho_i
 \end{aligned}
 \tag{1}$$

and :

$$\begin{aligned}
 u_i &= -\rho_i \cos(l_q - \Omega) + I \sigma_i \sin(l_q - \Omega) \\
 v_i &= \tau_i \\
 w_i &= \rho_i \sin(l_q - \Omega) + I \sigma_i \cos(l_q - \Omega)
 \end{aligned}
 \tag{2}$$

where l_q is the mean longitude of the Moon, Ω is the longitude of the ascending node of the lunar orbit, I is the inclination of the Cassini equator to the ecliptic, τ , ρ , σ are the components of physical libration in longitude, inclination and node, respectively.

II. ON THE EFFECTS OF INACCURACIES OF I AND f ON THE REALIZATION OF THE SELENODETIC REFERENCE SYSTEMS.

The discrepancies between the classical determinations of I and f by different authors are in the ranges of $1^\circ 29' - 1^\circ 34'$ and $0.50 - 0.90$

for I and f , respectively. The progress in specification of the values of these parameters has been achieved using new measuring techniques such as lunar laser ranging.

TABLE 1. Some determinations of the parameters I and f .

| Observations | I | f | references |
|-------------------------------|----------|-------|------------------|
| Heliometric | 1°32'04" | 0.633 | Koziel 1967 |
| Lunar Orbiter altimetry a) | 1 31 50 | 0.640 | Sagitov 1979 |
| Lunar Laser Ranging b) | 1 32 40 | 0.640 | King et al. 1976 |
| Lunar Laser Ranging + VLBI b) | 1 32 41 | 0.639 | King et al. 1976 |
| Adopted by IAU | 1 32 33 | 0.639 | Seidelmann 1977 |

a) The weighted averages of 16 series of harmonic coefficients C_{20} and C_{22} have been used.

b) The dynamical flattening β and γ have been used.

Table 1 shows that the estimates of parameter f agree well. But for I the case is different. This is due to the fact that the determination of the parameter I depends on the adopted model of internal structure of the Moon as well as on the assumptions made in the reduction of observational data.

According to the expressions (1) and figure 1, the inaccuracy of I limits the precise determination of the $C(\xi_C, \eta_C, \zeta_C)$ system. Besides it was shown that the error of $I = \pm 1'$ resulted the change of scale of ζ -direction in the $S(\xi_S, \eta_S, \zeta_S)$ system of the order ± 3 km (Kislyuk, 1972).

The inaccuracy of the parameter f limits the precision of calculating the lunar physical libration. Table 2 summarizes the main coefficients of physical libration in longitude, inclination and node for the different models of physical libration.

One can see from Table 2 that the Hayn model differs from those adopted by IAU. The first one should not be used for selenodetic reductions. Besides, the term of argument 2ω is different for the first two models which have been constructed by using the same values of I and f .

The 2ω -term is very sensitive to a change of the parameter f because of the nearness of this parameter to the resonant value $f_r = 0.662$ (see Figure 2). The partial derivative of the term $A_{2\omega}$ with respect to f is changed from $950''$ (to the left) to $2500''$ (to the right).

Table 3 gives the estimates of accuracy of components of the physical libration using an uncertainty of the parameter f of ± 0.01 .

TABLE 2. Coefficients of physical libration of the Moon according to different models.

| M O D E L S | | | | | |
|-------------|----------|----------|----------|---------|------|
| Argument | Eckhardt | Chikanov | Eckhardt | Hayn | |
| | 1976(a) | 1968 | 1970 | 1923 | |
| | I=5550" | I=5550" | I=5424" | I=5540" | |
| g g' ω ω' | f=0.639 | f=0.639 | f=0.633 | f=0.73 | |
| τ sin | 1 0 0 0 | -17" | -17" | -17" | -13" |
| | 0 1 0 0 | 90 | 90 | 92 | 65 |
| | 0 2-2 2 | 10 | 10 | 10 | 7 |
| | 0 0 2 0 | -19 | -34 | -15 | 10 |
| ρ cos | 1 0 0 0 | -99 | -102 | -98 | -106 |
| | 1 0 2 0 | 25 | 25 | 24 | 35 |
| | 2 0 2 0 | -11 | -11 | -11 | -11 |
| Iσ sin | 1 0 0 0 | -102 | -99 | -101 | -108 |
| | 1 0 2 0 | 25 | 25 | 24 | 35 |
| | 2 0 2 0 | -10 | -11 | -11 | -11 |

(a) See Doyle et al. (1977).

Note: The values g, g' are the mean anomaly of the Moon and the Sun; ω, ω' are the angular distance of the Moon and the Sun perigee from ascending node of the lunar orbit on the ecliptic.

TABLE 3. Accuracy of components of physical libration of the Moon (for $\delta f = \pm 0.01$).

| Argument | | g g' ω ω' | Errors of coefficients |
|-----------|---------|-----------|------------------------|
| | | | |
| δτ sin | 1 0 0 0 | | ± 0"4 |
| | 0 1 0 0 | | 2.9 |
| | 0 2-2 2 | | 0.3 |
| | 0 0 2 0 | | 10-25 |
| δρ cos | 1 0 0 0 | | ± 0"8 |
| | 1 0 2 0 | | 1.2 |
| δ(Iσ) sin | 1 0 0 0 | | ± 0"8 |
| | 1 0 2 0 | | 1.2 |

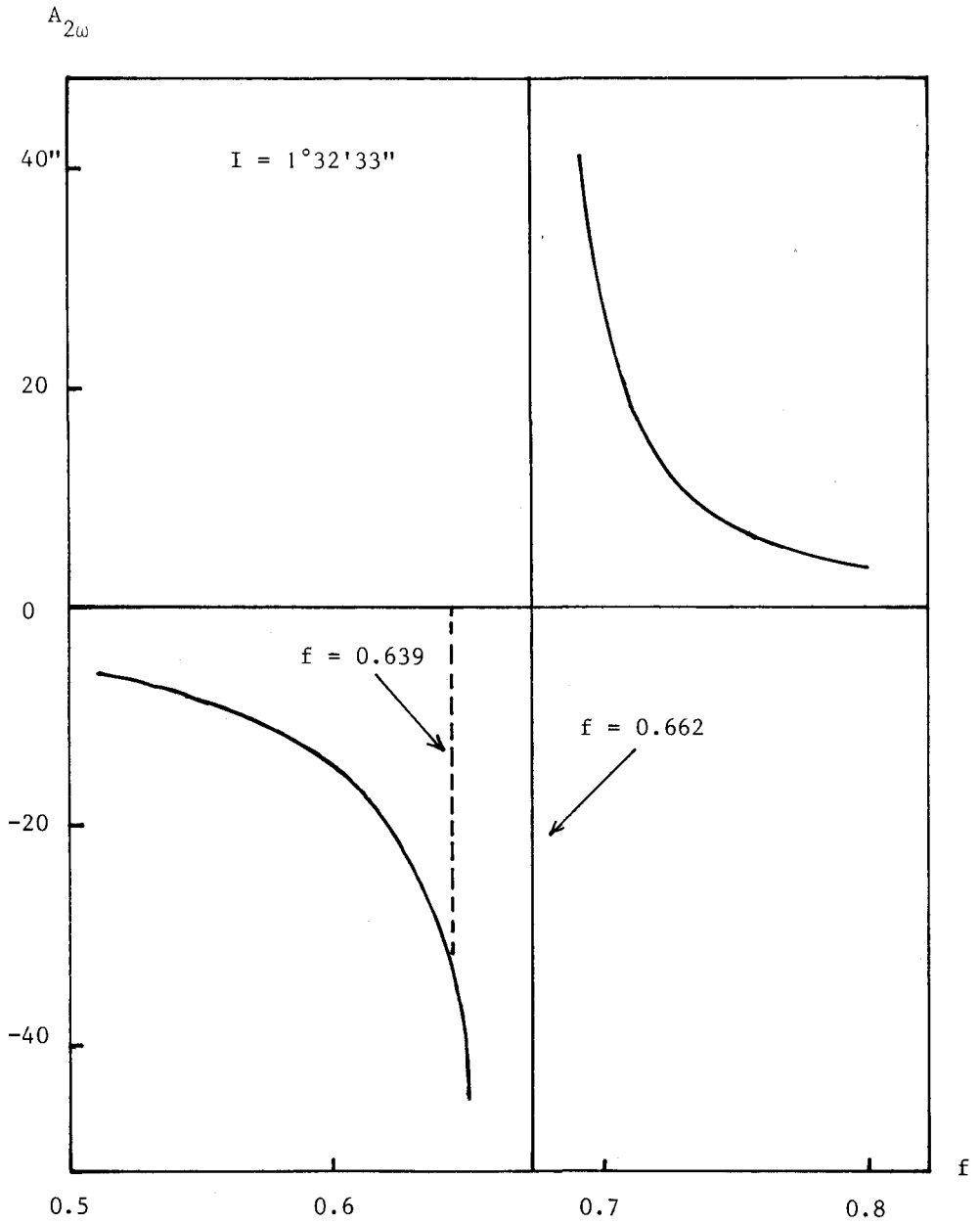


Fig.2. Dependence of 2ω -term on f according to the Chikanov tables (1968)

III. CONCLUSION

1/ For calculating the physical libration with an accuracy of $\pm 1''$, it is necessary to know the value of f with an error less than ± 0.001 .

2/ The accurate absolute orientation of the reference coordinate system $Q (\xi_q, \eta_q, \zeta_q)$ is limited mainly by the inaccuracy of I .

3/ The orientation of the system $S (\xi_s, \eta_s, \zeta_s)$ in respect to the coordinate system $Q (\xi_q, \eta_q, \zeta_q)$ may be defined by using the Earth-ground photographic observations of the Moon on a stellar background.

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