The Magnetic and Velocity Fields of Solar Active Regions
ASP Conference Series, Vol. 46, 1993
Harold Zirin, Guoxiang Ai, and Haimin Wang (eds.)

ON THE INFERENCE OF MAGNETIC FIELD VECTORS FROM STOKES PROFILES: A GENERALIZATION OF THE WEAK-FIELD APPROXIMATION

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ABSTRACT A generalization of Jefferies et al's weak-field approximation for the inference of the strength and polar angle of magnetic field vectors from Stokes profiles is obtained while the inference of azimuthal angle follows a relationship derived by Landi Degl'Innocenti's perturbative solutions to the transfer equations in which the magneto-optical effect is taken account .It is found that the weak-field condition ($\Delta\lambda_H$ / $\Delta\lambda_D \leq 1$)is not necessary for the new method when the fitting is done in the line wings.

I. INTRODUCTION

Among the methods for inference of magnetic field vectors from Stokes I, Q, U, V profiles, the weak-field approximation(WFA) proposed by Jefferies et al. (1989, hereafter JLS,1991) is one of the most effective and the simplest.Under the assumption that the Zeeman splitting $\Delta\ \lambda_H$ is a fraction of the Doppler width $\Delta\lambda_D$, they represented Voigt function $H(a, v \pm v_b)$ (where $v_b = \Delta\lambda_H\ /\Delta\lambda_D$, $v = \Delta\lambda\ /\Delta\lambda_D$ and $\Delta\lambda$ is the distance from line center.) by the low-order terms of Taylor series as

$$H(a, v \pm v_b) = H(a, v) \pm v_b H^{'}(a, v) + \frac{v_b^2 H^{''}(a, v)}{2} \pm \frac{v_b^3 H^{'''}(a, v)}{6} + \cdots. \quad (1)$$

Approximate expressions for absorption coefficients η_I, η_Q, η_U and η_V were obtained by keeping only the former three terms of the series. Substituting these expressions into the transfer equations they found corresponding simplified equations.When considering the requirement for consistency between the simplified equations and making some assumptions,they obtained useful relationships to infer the magnetic field vectors from Stokes profiles (see JLS).

In the following,we use the Taylor series (1) for $H(a, v)$ up to the third order. An expression for v_b and the relationship between $tan2\chi$ and U/Q are obtained. All of these will be used to extract the magnetic field vectors from Stokes profiles.

II. THEORETICAL DERIVATION

We take the former four terms of the Taylor expansion for the Voigt function and insert it in the expressions for η_I, η_Q, η_U and η_V. This gives the same expressions as JLS' for η_I, η_Q and η_U while

$$\eta_V = -\eta_0 cos\gamma[v_b H'(a,v) + (\frac{v_b^3}{6})H'''(a,v)] + 0(v_b^5). \tag{2}$$

According to Landi Degl'Innocenti and Landi Degl'Innocenti (1973, hereafter LL), the magneto-optical effect is only of order $x^3(x = v_L/\Delta v_D, v_L$ is the Larmor frequency) in Q and U, but has no influence until the fourth order of x on I and V. v_b and γ are dominated by I and V, so the magneto-optical effect can be ignored. The transfer equations thus can be approximately written (the significance of symbols in this paper is the same as that of JLS) as

$$\begin{aligned}\mu\frac{dI}{d\tau} = {} & [1 + \eta_0[H(a,v) + (1 + cos^2\gamma)H''(a,v)(\frac{v_b}{2})^2]](I - B)\\ & -\eta_0 cos\gamma[v_b H'(a,v) + \frac{v_b^3}{6}H'''(a,v)]V\\ & -\eta_0(\frac{v_b sin\gamma}{2})^2 H''(a,v)[Qcos2\chi + Usin2\chi],\end{aligned} \tag{3a}$$

$$\mu\frac{dQ}{d\tau} = [1 + \eta_0 H(a,v)]Q - \eta_0(\frac{v_b sin\gamma}{2})^2 H''(a,v)(I - B)cos2\chi, \tag{3b}$$

$$\mu\frac{dU}{d\tau} = [1 + \eta_0 H(a,v)]U - \eta_0(\frac{v_b sin\gamma}{2})^2 H''(a,v)(I - B)sin2\chi, \tag{3c}$$

$$\mu\frac{dV}{d\tau} = [1 + \eta_0 H(a,v)]V + [\eta_0 v_b cos\gamma H'(a,v) + \eta_0(\frac{v_b^3}{6})cos\gamma H'''(a,v)](I - B). \tag{3d}$$

Comparing the magnitudes of the right-hand terms in the first equation, we can find that all the terms except the first one can be dropped, especially in the line wings where the magnetic field vectors are determined. Thus the first and last equations may be written

$$\mu\frac{dI^*}{d\tau} = (1 + \eta_0 H)I^* - \mu\frac{dB}{d\tau}, \tag{4a}$$

$$\mu\frac{dV}{d\tau} = (1 + \eta_0 H)V - \eta_0 v_b cos\gamma H'(1 + \frac{v_b^2}{6}\frac{H'''}{H'})I^* \tag{4b}$$

where

$$I^* = I - B. \tag{5}$$

https://doi.org/10.1017/S0252921100028943 Published online by Cambridge University Press

We use the same method as LL and JLS. Eqs.(4) require for consistency (set $\partial B/\partial v=0$)

$$V(v,\tau) = -v_b cos\gamma(1+\frac{v_b^2}{6}\frac{H'''}{H'})\frac{\partial I(a,v)}{\partial v} \tag{6}$$

provided

$$\mu\frac{d}{d\tau}[ln(v_b cos\gamma H' + \frac{v_b^3}{6}cos\gamma H''')] \ll 1+\eta_0 H(a,v). \tag{7}$$

The same expressions of Q and U of JLS and the corresponding assumption can be employed with good approximation for deriving $v_b sin\gamma$. We rewrite Eq.(6) and the expressions of Q and U of JLS in the following way:

$$v_b cos\gamma(1+\frac{v_b^2}{6}\frac{H'''}{H'}) = -\frac{V(v,\tau)}{\partial I/\partial v} \equiv SS1(v,\tau), \tag{8a}$$

$$(\frac{v_b sin\gamma}{2})^2 = |\frac{H'(a,v)}{H''(a,v)}\frac{(Q^2+U^2)^{1/2}}{\partial I/\partial v}| \equiv SS2(v,\tau). \tag{8b}$$

Neglecting the second order and the higher ones of H'''/H', we can obtain the following expressions through several algebraic steps from Eqs.(8)

$$v_b = [\frac{SS1^2+4SS2}{1+(SS1^2H'''/3H')}]^{1/2} \tag{9a}$$

$$sin\gamma = \frac{(4SS2)^{1/2}}{v_b}. \tag{9b}$$

For deriving the relationship between $tan2\chi$ and U/Q, we make use of LL's perturbative solutions, but some of their symbols are changed in order to keep agreement with JLS' .It is not difficult to find that LL and JLS are compatible with each other. In fact V, Q and U of JLS are equivalent to V_1, Q_2 and U_2 of LL respectively. if the varying with depth of the third orders of Q or U is neglected, i.e.

$$\frac{(dQ_3)}{d\tau} = \frac{(dU_3)}{d\tau} = 0, \tag{10}$$

one can obtain expressions for Q_3 and U_3 from LL ,with

$$Q_0 = U_0 = 0, \quad Q_1 = U_1 = 0, \tag{11}$$

and

$$\frac{U}{Q} = \frac{U_0+U_1+U_2+U_3+\cdots}{Q_0+Q_1+Q_2+Q_3+\cdots}, \tag{12}$$

We reach after several algebraic steps

$$tan2\chi = \frac{U/Q + v_b cos\gamma SS3(v,\tau)}{1 - (U/Q)v_b cos\gamma SS3(v,\tau)}, \tag{13}$$

where

$$SS3(v,\tau) \equiv \frac{2\eta_0}{(1+\eta_0 H)H''}[F'H'' - F''H'(1+\frac{v_b^2 H'''}{6H'})]. \tag{14}$$

It is obvious that when v_b is small enough or the magneto-optical effect is neglected or $\gamma = \pi/2$, Eq.(13) returns to the more familiar relationship

$$tan2\chi = \frac{U}{Q}. \tag{15}$$

Eqs.(9) and Eq.(13) are the basic formulae we need for inferring the magnetic field vectors from Stokes profiles. Finally a useful variable which represents the difference between the accurate and approximate expressions of the Voigt functions is defined as

$$D_{eff} = \frac{[H(a, v+v_b) - H(a, v-v_b)]}{2} - [v_b H'(a,v) + \frac{v_b^3}{6} H'''(a,v)], \tag{16}$$

where v_b is the value determined by Eqs.(9a),D_{eff} varies with wavelength within the line. We would expect that the less this variable is at one wavelength, the more reliable the fit parameters at the same wavelength are, i.e., a good fit has to be self-consistent.

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