

Measuring Closeness in Proportional Representation Systems

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Abstract

We provide closed-form solutions for measuring electoral closeness of candidates in proportional representation (PR) systems. In contrast to plurality systems, closeness in PR systems cannot be directly inferred from votes. Our measure captures electoral closeness for both open- and closed-list systems and for both main families of seat allocation mechanisms. This unified measure quantifies the vote surplus (shortfall) for elected (nonelected) candidates. It can serve as an assignment variable in regression discontinuity designs or as a measure of electoral competitiveness. For illustration, we estimate the incumbency advantage for the parliaments in Switzerland, Honduras, and Norway.

Keywords: proportional representation systems, regression discontinuity design, assignment variable, electoral competitiveness

1 Introduction

We develop a measure of electoral closeness for proportional representation (PR) systems. Measuring closeness is important for two broad strands of the literature. One focuses on close elections for identifying causal effects of winning elections using regression discontinuity (RD) designs both at the party and the candidate level (Caughey and Sekhon 2011; Eggers et al. 2015; Fourinaies and Hall 2014; Lee 2008). The other considers closeness as an important dimension of electoral competitiveness (Cox, Fiva, and Smith 2020; Grofman and Selb 2009). Both strands have primarily focused on plurality systems due to the challenges of measuring closeness in PR systems. However, PR is globally the most common electoral formula for legislative elections. According to the International Institute for Democracy and Electoral Assistance, 119 countries use a PR or mixed system, while 86 countries use a plurality system to elect their national parliament.¹

In plurality voting systems, an intuitive and widely used closeness measure is simply the gap in votes (or vote shares) to the marginally elected or nonelected candidate. In PR systems, however, measuring closeness is not straightforward as seats are assigned in two steps. First, seats are distributed to parties according to a transformation of party votes, which depends on the specific seat allocation mechanism. The two main families of seat allocation mechanisms are the highest average method and the largest remainder method. Second, these seats are assigned to individual candidates within a party based on the candidates' vote ranking (open-list systems) or their list position (closed-list systems). These characteristics of PR systems pose three challenges in defining a measure of closeness analogous to plurality systems. First, party votes are not directly informative about marginal winners and losers for different seats considered (Folke 2014; Freier and Odendahl 2015). Second, the measure of closeness is not simply the vote difference to these marginal winners or losers. Third, from the perspective of a candidate, there are potentially

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¹ www.idea.int/data-tools/; accessed November 25, 2019.

multiple margins to win (or lose) a seat: She can win (or lose) a seat for her party (in open- and closed-list systems) or pull ahead of (or fall behind) copartisan candidates (in open-list systems). In the following, we refer to these margins as the party and the candidate margin.

Let us illustrate the first two points using a simple example in a district with three seats and three parties P_1 , P_2 , and P_3 with 45, 35, and 20 votes. We assume that the D'Hondt method is employed for the seat distribution. With this method and three seats, we calculate D'Hondt ratios by dividing the votes by 1, 2, and 3 and then distribute the three seats by choosing the three highest D'Hondt ratios. In this example, P_1 receives two seats with D'Hondt ratios of 45 and 22.5. P_2 obtains one seat with a D'Hondt ratio of 35. To illustrate the first point from above, let us calculate P_2 's vote surplus for its seat. For this, we need to find out which party is next in line to win this seat. This cannot be inferred from raw votes but only from the other parties' D'Hondt ratios. The transformation from raw votes to D'Hondt ratios reveals that P_3 is closest to winning a seat with a D'Hondt ratio of 20. The vote surplus for P_2 's seat is the difference in the relevant D'Hondt ratios which, in this case, corresponds to the vote difference between P_2 and P_3 . However, we generally have to re-transform the D'Hondt ratio differences to calculate the vote shortfalls or vote surpluses. This is our second point mentioned above. We illustrate this point for the vote shortfall of P_2 's second seat. P_2 would have to win this seat at the expense of the second seat of P_1 . The difference in D'Hondt ratios to this marginal seat is the difference between the second-highest D'Hondt ratios of P_1 and P_2 , which is equal to 5. To express this difference again in terms of actual votes instead of D'Hondt ratios, we have to multiply the D'Hondt ratio difference by two as we consider the second seat of P_2 .

We propose the construction of a closeness measure for PR, which is applicable in open- as well as closed-list systems and we propose solutions for both main families of seat allocation mechanisms. The intuition of our approach is to compute the vote surplus of elected candidates to losing their seat as well as the vote shortfall of not elected candidates to winning a seat. We take into account that candidates of a party can potentially gain or lose a seat along two margins. First, they can win or lose by winning or losing seats for their party (open- and closed-list systems). Second, they can surpass or fall behind copartisan candidates (open-list systems). We explicitly calculate these margins and then define closeness as the minimal necessary movement along both margins to gain or lose a seat. Our approach can be extended to capture other features of electoral systems such as party alliances.

In the thought experiment underlying our approach, we assume that the votes of all other parties and candidates remain unchanged. Other thought experiments are possible. In particular, one could calculate the vote surplus and the vote shortfall by redistributing votes between parties, between candidates or both. Mathematically, the redistribution-based thought experiment necessitates considering twice as many inequality constraints as there are competing parties and an iterative procedure, hampering an analytical solution. Conceptually, it requires alternative assumptions about how likely it is that voters switch from one party to another and from one candidate to another. It is unclear whether one set of assumptions is generally better than the other. We favor our thought experiment because of its tractability and transparency.

For illustration purposes, we implement our approach and estimate the individual incumbency advantage in three countries with electoral systems differing along two main dimensions of PR systems: the seat allocation mechanism (highest average versus largest remainder) and the list structure (open versus closed lists). We use data from 22 elections to the Swiss National Council in the years 1931–2015 (open lists; highest average), three elections to the Honduran National Congress in the years 2009–2017 (open lists; largest remainder), and eight elections to the Norwegian Storting in the years 1953–1981 (closed lists; highest average). For Switzerland and Norway, we identify substantial incumbency advantages. For Honduras, we find no incumbency advantage.

Previous research has proposed three different ways to measure closeness in PR systems. The first approach focuses only on the party margin for one extra seat, the second approach only considers the candidate margin, and the third approach uses simulation methods to measure overall closeness.

The first approach is based on identifying the marginal seat of the other parties and calculating the vote distance to this seat. Early variants of this idea include Blais and Lago (2009), Folke (2014), and Selb (2009).² We also build on calculating the party margin, but go well beyond existing implementations: First, we calculate the vote shortfall or vote surplus for all parties and for all seats a party can possibly achieve and not only for a one-seat deviation from the actual seat distribution. This allows researchers to address a wide range of issues, such as quantifying the vote shortfall of any candidate in closed-list PR systems to winning a mandate or a party's vote distance to winning a seat majority. Second, we present solutions for both the highest average and largest remainder methods (not only for the former). Third, we provide an overall measure of closeness at the candidate level for open-list PR systems, which requires considering and aggregating the party margin and the candidate margin.³

The second approach only considers the candidate margin and ignores the party margin (Cirone, Cox, and Fiva 2021; Fiva and Røhr 2018; Golden and Picci 2015; Hyytinen et al. 2018). Scholars who apply this approach basically calculate the distances of votes, vote shares or ranks to marginally elected or marginally not elected candidates. This approach suffers from the fact that the marginal candidate is not fixed. If the candidate of interest receives more votes, she not only catches up to other copartisan candidates, but she may also win an additional seat for her party and thereby change who is the marginal candidate. By ignoring the party margin, the approach makes many candidates appear to be less close than they are. This error is systematically related to the within-party vote distribution. In addition, this approach cannot measure closeness for candidates of parties winning no seat and for candidates of parties with all candidates elected. Section 3.4 illustrates these problems with data from our application for Switzerland. There we find that for many candidates the party margin, not the candidate margin, is binding. Our unifying approach overcomes these problems by simultaneously considering and aggregating the party margin from the first approach and the candidate margin from the second approach to an overall vote margin.

The third approach uses stochastic simulation methods to measure closeness in terms of election probabilities rather than vote distances. A major conceptual drawback of all methods based on simulated election probabilities is that the probabilities inherently depend on arbitrary choices of the simulation setup. For example, Kotakorpi, Poutvaara, and Terviö (2017) simulate elections by resampling votes from the actual vote distribution. They apply the relevant seat allocation mechanism, count the number of times a candidate is elected, and divide this number by the number of simulations to calculate the candidate's election probability. This probability essentially serves as a measure of closeness. However, this probability crucially depends on the size of the simulation samples. The probability approaches a candidate's election status with increasing sample size, it approaches the actual vote share of a candidate's party with decreasing sample size. In Section 3.4, we illustrate this problem by applying the approach of Kotakorpi et al. (2017) to our previous numerical example with the three parties P_1 , P_2 , and P_3 . This example demonstrates the arbitrariness of simulated election probabilities: For instance, depending on

- 2 Blais and Lago (2009) and Selb (2009) propose a vote margin for the marginal seat and the party closest to winning this marginal seat as a measure for district-level competitiveness. Folke (2014) calculates the vote margin for one additional (fewer) party seat to assess the importance of parties in an RD design.
- 3 Selb and Lutz (2015) simultaneously use the party margin and the candidate margin. They are interested in the relative importance of these margins for campaigning and therefore do not combine them into an overall closeness measure.

the number of resampled votes, the election probability of the first-ranked candidate of party P_2 ranges from 0.35 to 1.

In sum, we provide the first analytical measure of overall closeness at the candidate level in PR systems, applicable for both open- and closed-list systems and for both main families of seat allocation mechanisms.

2 Calculation and Aggregation of Vote Margins

Our closeness measure captures how many votes away a particular candidate is from winning or losing her seat. In PR systems, seats are distributed in two steps. First, seats are distributed to parties. Second, seats won by a party are allocated to individual candidates. Therefore, winning or losing votes affects a candidate’s election prospects in two ways: It affects her party’s total number of seats and, in open-list PR systems, her individual ranking within the party.⁴ The construction of our vote margin variable mirrors this two-step procedure. First, we construct a party margin variable capturing the vote change necessary so that the party obtains a particular number of seats (open- and closed-list PR). Second, we calculate the candidate margin capturing vote differences to other candidates from the party (open-list PR). Third, we aggregate the two margins to construct an overall measure of a candidate’s closeness. Throughout, negative numbers indicate vote shortfalls and positive numbers vote surpluses.

2.1 Party Margin

We construct the party margin for both families of seat allocation mechanisms, the highest average methods and largest remainder methods. Let us consider a district with J parties P_j indexed by $j \in \mathcal{P} \equiv \{1, 2, \dots, J\}$. The party index of all parties other than j is $-j \in \mathcal{P} \setminus \{j\}$. A total number of n seats are allocated. The index $i \in \{1, 2, \dots, n\}$ denotes the potential seats of party P_j . Similarly, \tilde{i} captures the potential seats of each of the other parties P_{-j} . We denote a party P_j ’s total number of votes by $votes_j$.

Highest Average Methods. We use the D’Hondt method to illustrate the calculation of the party margin for highest average methods.⁵ For each party, a total of n D’Hondt ratios, $votes_j/i$, are calculated. The first seat goes to the party with the biggest D’Hondt ratio, the second to the one with the second biggest D’Hondt ratio and so forth.

The *party margin* $_{i,j}$ captures the necessary change in the number of votes such that party P_j is at the margin of winning exactly i seats. This happens if the i th-highest D’Hondt ratio of party P_j equals the $(n - i + 1)$ th-highest D’Hondt ratio of all other $J - 1$ parties and, thus, if the i th seat for party P_j is decided by a coin flip. The $(n - i + 1)$ th-highest D’Hondt ratio is the one with the order statistic $(n(J - 1) - (n - i))$. Formally, the *party margin* $_{i,j}$ is defined as

$$party\ margin_{i,j} = votes_j - i \left(\frac{votes_{-j}}{\tilde{i}} \right)_{(n(J-1)-(n-i))}$$

In our example, the seats are allocated to three parties. Party P_1 has D’Hondt ratios of 45.0, 22.5, and 15.0, party P_2 ’s ratios are 35, 17.5, and 11.7, and party P_3 ’s ratios are 20, 10, and 6.7. The three highest D’Hondt ratios achieve a seat and thus in this case party P_1 receives two seats and party P_2 one seat.

4 In the following, we assume that every vote for a candidate counts toward her party’s vote total. This is the standard case for open-list PR systems, even though they can differ in many other dimensions. For example, voters might cast between one vote (e.g., Brazil and Finland) and a number of votes equalling the number of seats (e.g., Honduras and Switzerland).
 5 Other highest average methods, such as the Sainte-Laguë, modified Sainte-Laguë or Imperiali method, only differ by the denominator used to calculate the ratios.

Let us illustrate the calculation of the party margin for all three potential seats of party P_2 . The party's first seat is close if its highest D'Hondt ratio is equal to the third highest D'Hondt ratio of the other parties P_1 and P_3 which is 20 for the first seat of P_3 . The party could lose 15 votes to secure their first seat ($i = 1$) against party P_3 . Thus, its party margin is 15 for $i = 1$. If party P_2 wanted to win two seats, the second D'Hondt ratio of party P_2 would have to be equal to the second-highest D'Hondt ratio of all other parties which is 22.5. To achieve this, party P_2 would need 10 additional votes and thus the party margin for $i = 2$ is -10 . Similarly, if party P_2 wanted to achieve all three seats, the third-highest D'Hondt ratio of party P_2 would have to be equal to the highest D'Hondt ratio of all other parties which is 45. Thus, the party margin is -100 for $i = 3$.

Largest Remainder Methods. PR systems that apply the largest remainder method distribute seats by dividing the total votes of a party by a quota. For example, the Hare–Niemeyer method, which we employ for illustration, uses the division of the total number of party votes by a district's number of seats as a quota. The division of party votes by the quota results in a party-specific Hare–Niemeyer ratio comprising an integer and a fractional remainder. In a first round of the seat allocation, parties receive a total number of seats equalling their integer. In the second round of the seat allocation, parties obtain at most one of the remaining seats in decreasing order of their remainder. We denote the integer part of the Hare–Niemeyer ratio as $\text{floor}\left(\frac{\text{votes}_j}{\text{quota}}\right)$ and the remainder as $\text{mod}\left(\frac{\text{votes}_j}{\text{quota}}\right)$. We follow our notation for the highest average method and denote the total votes of our party of interest as votes_j , the votes of other parties as votes_{-j} , and the (Hare–Niemeyer) quota as $\frac{\text{votes}_j + \sum_{-j} \text{votes}_{-j}}{n}$.

In what follows, we quantify the vote shortfall for seat i of party P_j . In the first step, our procedure establishes the necessary votes such that party P_j receives $i - 1$ seats in the first round of the seat allocation and has the same remainder as another party P_{-j} . In a second step, we use these votes to determine the number of free seats to be distributed in the second round of the seat allocation and the ranking of the other parties' remainders. We repeat the first two steps for all parties P_{-j} and their possible seats. In the third step, we select the relevant solution from the first step. The relevant solution is the one for which the rank of party P_{-j} 's remainder equals the number of seats to be distributed in the second round of the seat allocation. Intuitively, the remainder that has the same rank as the number of seats to be distributed in the second round obtains the last seat. In the final step, we calculate the party margin as the difference between the actual number of votes and the necessary number of votes from the solution selected in the third step.

If the following condition holds, the parties P_j and P_{-j} achieve exactly $i - 1$ and $\tilde{i} - 1$ seats in the first round of the seat allocation and their remainders are identical:

$$\frac{\text{votes}_{i,\tilde{i},j,-j} \times n}{\sum_{-j} \text{votes}_{-j} + \text{votes}_{i,\tilde{i},j,-j}} - (i - 1) = \frac{\text{votes}_{-j} \times n}{\sum_{-j} \text{votes}_{-j} + \text{votes}_{i,\tilde{i},j,-j}} - (\tilde{i} - 1).$$

To get the necessary votes for party P_j for the first step outlined above, we can rewrite this condition as

$$\text{votes}_{i,\tilde{i},j,-j} = \frac{\text{votes}_{-j} \times n + (i - \tilde{i}) \sum_{-j} \text{votes}_{-j}}{n - (i - \tilde{i})}, \quad \text{where } 1 \leq \tilde{i} \leq n - i + 1.$$

For the second step, we substitute $\text{votes}_{i,\tilde{i},j,-j}$ for the actual votes of P_j and re-calculate the ratios of the $J - 1$ other parties. Because we repeat the first two steps for all parties indexed by $-j$, we need to introduce a new index $-\tilde{j}$ with the same values as $-j$ to express these ratios. The ratios are

$$\frac{votes_{-\tilde{j}} \times n}{\sum_{-\tilde{j}} votes_{-\tilde{j}} + votes_{i,\tilde{i},-\tilde{j}}}$$

Based on these ratios, we determine the number of seats to be distributed in the second round of the seat allocation

$$F_{i,\tilde{i},-\tilde{j}} = n - (i - 1) - \sum_{-\tilde{j}} \text{floor} \left(\frac{votes_{-\tilde{j}} \times n}{\sum_{-\tilde{j}} votes_{-\tilde{j}} + votes_{i,\tilde{i},-\tilde{j}}} \right)$$

and the ranking of all the remainders

$$R_{i,\tilde{i},-\tilde{j},-\tilde{j}} = \text{rank} \left(\text{mod} \left(\frac{votes_{-\tilde{j}} \times n}{\sum_{-\tilde{j}} votes_{-\tilde{j}} + votes_{i,\tilde{i},-\tilde{j}}} \right) \right)$$

As mentioned above, we repeat the first two steps for all $J - 1$ parties and all possible seat differences $i - \tilde{i}$. Therefore, we have $(J - 1) \times (2n + 1)$ possible solutions. In the third step, we select the relevant solution, $votes_{i,\tilde{i},-\tilde{j}}^*$, which satisfies the following conditions:

$$\begin{aligned} F_{i,\tilde{i},-\tilde{j}} &= R_{i,\tilde{i},-\tilde{j},-\tilde{j}}, \\ -\tilde{j} &= -j, \text{ and} \\ (i - 1) &= \text{floor} \left(\frac{votes_{i,\tilde{i},-\tilde{j}} \times n}{\sum_{-\tilde{j}} votes_{-\tilde{j}} + votes_{i,\tilde{i},-\tilde{j}}} \right). \end{aligned}$$

In the fourth step, we define the party margin as the difference between the actual votes and the necessary votes selected in the third step:

$$\text{party margin}_{i,j} = votes_{i,j} - votes_{i,\tilde{i},-\tilde{j}}^*$$

In our example with three seats and 45, 35, and 20 votes for the parties P_1 , P_2 , and P_3 , the Hare–Niemeyer quota is 33.33. The party P_1 has a ratio of 1.35, the party P_2 of 1.05, and the party P_3 of 0.6. Thus, the parties P_1 and P_2 receive one seat each in the first round of the seat allocation and the remaining seat is distributed to party P_3 in the second round based on its largest remainder. Let us demonstrate our procedure to find the party margin for the second seat ($i = 2$) of party P_2 . In the first step, we calculate the necessary number of votes such that party P_2 gets one seat ($i - 1$) in the first round, the comparison party, P_1 or P_3 , gets zero or one seat(s) ($\tilde{i} - 1$, where $1 \leq \tilde{i} \leq 3 - 2 + 1$) in the first round, and party P_2 and the comparison party have the same remainder. For example, for the comparison party P_3 and $\tilde{i} = 1$, the solution is 62.5. If party P_2 had 62.5 votes, the new Hare–Niemeyer quota would be 42.5, and the ratios of the three parties would be 1.06, 1.47, and 0.47. In the second step, we determine that one seat would remain to be allocated in the second round of the seat distribution and that the remainder of the relevant comparison party P_3 would rank first. In the third step, we select 62.5 as the relevant solution because the number of seats to be distributed in the second round and the rank of comparison party P_3 are both equal to one. Finally, we calculate the party margin for the second seat of P_2 as $35 - 62.5 = -27.5$.

2.2 Candidate Margin

We now construct the candidate margin that measures how far an individual candidate is from reaching the same position as another copartisan candidate. This variable is only relevant for open-list PR systems. We extend the setting and introduce candidates C_h of party P_j . They are

indexed by $h \in \mathcal{H} \equiv \{1, \dots, n_j\}$ where n_j is the number of candidates in party P_j . Similarly, all other candidates of party P_j are indexed by $-h \in \mathcal{H} \setminus \{h\}$.

The vote change required to be on par with the i th highest copartisan candidate, that is, the one with the order statistic $(n_j - i + 1)$, is simply the difference between the two candidates' number of votes. The candidate margin for candidate h in party j to position i is defined⁶:

$$\text{candidate margin}_{h,i,j} = \begin{cases} \text{votes}_{h,j} - (\text{votes}_{-h,j})_{(n_j-i+1)}, & \text{if } i < n_j, \\ \text{votes}_{h,j}, & \text{if } i = n_j. \end{cases}$$

We continue our example and calculate the candidate margin for candidate C_2 of party P_2 . We assume the following individual vote distribution: 22 votes for C_1 , 8 votes for C_2 , and 5 votes for C_3 . Candidate C_2 needs 14 votes to be on par with the highest-ranked copartisan candidate, she can lose three votes to be on par with the second-ranked candidate among all other copartisans, and she can lose all her eight votes and still be the lowest-ranked candidate. The candidate margin for candidate C_2 of party P_2 is -14 for $i = 1$, 3 for $i = 2$, and 8 for $i = 3$.

2.3 Aggregation

In the final step, we aggregate the party and candidate margin to our overall measure of closeness, that is, the vote margin. For each candidate C_h there are n possibilities such that her election outcome is decided by a coin flip: The party gains one seat and candidate C_h is on par with the best-ranked copartisan candidate, the party receives two seats and C_h is on par with the second-best-ranked copartisan candidate, and so on. A final possibility is that C_h is the lowest-ranked candidate and the party is at the margin of winning n seats.

We can think of our aggregation as a two-step process. First, we identify the binding margin such that party P_j obtains exactly i seats and candidate C_h is on par with the candidate on position i . The binding margin is the minimum of the party margin and the candidate margin. Second, we identify the combination with the narrowest binding margin for each h . Among all binding margins, we choose the smallest absolute vote change that would alter the election status of candidate C_h . This is the maximum among all binding margins identified in the first step. Our overall vote margin of candidate C_h in party P_j is defined as

$$\text{vote margin}_{h,i,j} = \max_{i \in \{1, \dots, n\}} \{ \min \{ \text{party margin}_{i,j}, \text{candidate margin}_{h,i,j} \} \}.$$

In our D'Hondt example, the candidate C_2 of party P_2 has three possibilities to win a seat by a coin flip. First, party P_2 receives one seat ($i = 1$) and candidate C_2 makes up the vote difference to the highest-ranked copartisan candidate. The respective party and candidate margins are 15 and -14 . In other words, the candidate C_2 could lose 15 votes in order for party P_2 to still get one seat but she would require 14 additional votes to have a chance to win this seat. The binding margin for this first possibility is -14 . We underline this binding margin in the first row of Table 1. Second, party P_2 obtains two seats and candidate C_2 ties with the second-ranked copartisan candidate. The respective party margin and candidate margins are -10 and 3 . The binding margin for this possibility is the party margin of -10 . Third, party P_2 receives three seats and candidate C_2 is the lowest-ranked candidate. For this possibility, the party margin is -100 , the candidate margin is 8 and thus the binding margin is -100 . Candidate C_2 is not elected and her easiest possibility to get elected is the second possibility. It is the one with the highest of the binding margins which is -10 , overlined in Table 1. Thus, her overall vote margin is -10 .

6 To be the lowest-ranked candidate ($i = n_j$), candidate C_h could lose all her votes.

Table 1. Aggregation of party and candidate margins.

Party	Candidate	Seat	Margin	
			Party	Candidate
P_2	C_2	1	15	-14
P_2	C_2	2	-10	3
P_2	C_2	3	-100	8

2.4 Extension

The electoral systems of many jurisdictions are characterized by additional institutional features. Our approach can accommodate such features. As an illustration, we introduce party alliances. Party alliances are common around the world and relevant for our empirical application for Switzerland, where parties can form alliances that influence the seat allocation but have no further implications beyond this. In Section A of the Supplementary Material, we show how to adapt the construction of the party margin to account for alliances.

3 Empirical Illustration

We assess the individual incumbency advantage for three countries with PR to illustrate the use of our closeness measure as an assignment variable in RD designs. Our main focus is on open-list systems, which require calculating and aggregating party and candidate margins. As examples, we assess the prospects of election winners in subsequent elections for the Swiss National Council with an electoral formula from the highest average family and for the Honduran National Congress with one from the largest remainder family. For completeness, we also estimate the incumbency advantage for a closed-list system, which only involves measuring the party margin. Here, we use the case of Norway previously analyzed by Fiva and Smith (2018).

3.1 Highest Average Methods: Switzerland, 1931–2015

Institutional Background. The National Council is the larger chamber of the Swiss parliament. Since 1963, it consists of 200 members, for the earlier years in our sample, its size varied between 187 and 196. The electoral districts are the 26 cantons (25 before 1979) with between 1 and 35 seats. The D’Hondt method and open-list ballots are key elements of the electoral system. Parties can form alliances and suballiances (Section 2.4). We use data from the years 1931–2015. In these years, elections take place every 4 years and National Council members face no term limit.

Data. We collect data on all candidates’ votes, election outcome, name, party, age, and gender and on all parties’ votes and alliances from the Federal Gazette for the early years and from the Swiss Federal Statistical Office for later years. We link observations from the same candidate and canton across election years to construct a panel at the candidate level.⁷ Starting with 41,555 observations (excluding vote totals of unnamed individual candidates in single-member districts), we delete 92 observations from tacit elections. We remain with 41,463 observations from 26,629 candidates.

To account for the variation in district magnitude, we divide the vote margin by the number of eligible voters (Cox et al. 2020). The data on voters are from the Swiss Federal Statistical Office. The variable *relative vote margin* is the assignment variable in our RD analysis:

$$relative\ vote\ margin = \frac{vote\ margin}{eligible\ voters}$$

⁷ We complement Stata’s record linkage procedure *relink* with several rounds of manual checks by research assistants.

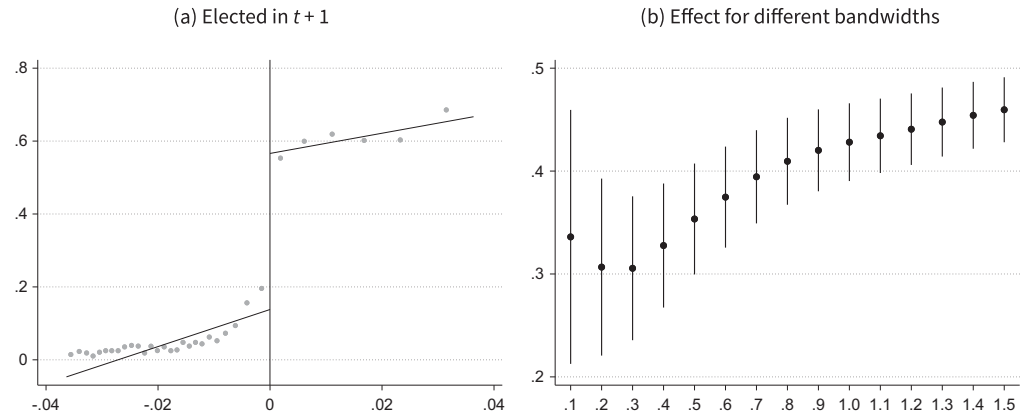


Figure 1. Incumbency advantage estimates for Switzerland. Panel (a) depicts the relationship between the assignment variable in t (x -axis) and the probability of being elected in $t + 1$ (y -axis), whereby t indexes elections, with the average values in quantile-spaced bins and linear regressions on both sides of the threshold. On both sides, we use the same optimal bandwidth for the election probability in $t + 1$ (Calonico et al. 2017) and a triangular kernel. Panel (b) depicts point estimates and 95% confidence intervals (y -axis) for bandwidths ranging from 10% to 150% of the optimal bandwidth (x -axis). Confidence intervals account for candidate-level clustering (Calonico et al. 2017).

Figure B.1 in the Supplementary Material depicts the smooth distribution of the assignment variable around the threshold. In contrast to other settings, the assignment variable's density is not informative about its manipulation. Election manipulation moving one candidate above the threshold necessarily moves another candidate below the threshold as the number of seats is fixed. This symmetry prevents bunching.

Results. Figure 1a depicts the relationship between a candidate's relative vote margin in the election in t and her election probability in the subsequent election in $t + 1$. We focus on the unconditional probability of being elected and not on the probability conditional on running. For both elected and nonelected candidates, this probability increases with the relative vote margin. However, there is a clear discontinuity between the two groups of candidates at the relative vote margin of zero. Figure 1b presents the estimates for varying bandwidths scaled relative to the optimal bandwidth. It demonstrates that the effect robustly lies between 30 and 45 percentage points. Given the average election probability of 11% for the years 1931–2011 this amounts to a substantial incumbency advantage. In Table B.1 in the Supplementary Material, we show that the estimated size of the incumbency advantage is robust to using second-order polynomial regressions.

For the validity of our RD design, it is necessary that only the probability of being elected changes discontinuously at the relative vote margin of zero. We examine this assumption by looking at discontinuities of candidate characteristics. Panels (B) and (C) of Table B.1 in the Supplementary Material show that characteristics are generally well balanced.

3.2 Largest Remainder Methods: Honduras, 2009–2017

Institutional Background. The National Congress is the unicameral parliament of Honduras with 128 members from 18 departments that constitute the electoral districts. There are between 1 and 23 representatives per department. Honduras employs the Hare–Niemeyer method and open-list ballots for the parliamentary elections. Honduran voters can vote for candidates from different parties by marking the respective candidates on a ballot that contains all the candidates from the department. This system is only in place since 2005 (Muñoz-Portillo 2013). However, for 2005, no candidate information is available. Therefore, we use the three elections 2009, 2013, and 2017.

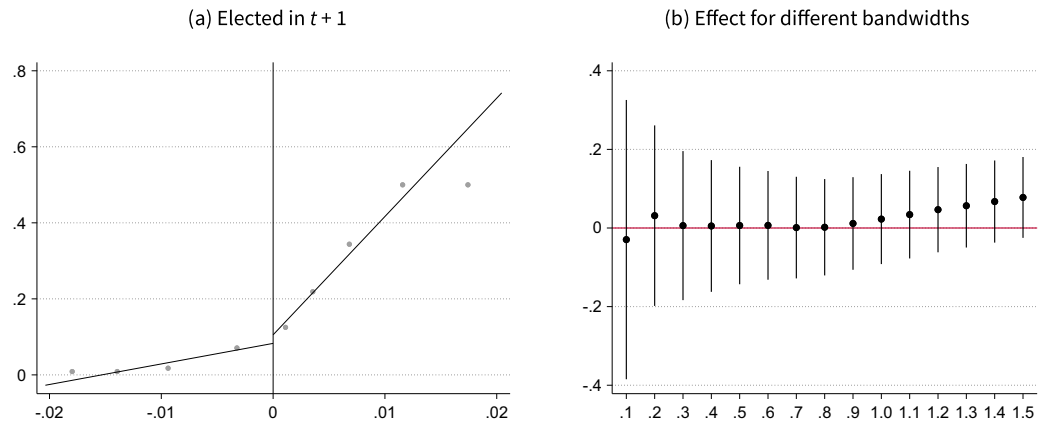


Figure 2. Incumbency advantage estimates for Honduras. See Figure 1.

Data. We collect information on candidates' votes, election outcome, name, and party from the Honduran Tribunal Supremo Electoral. We code candidates' gender from the pictures on the ballots and their first names. For the elected candidates in 2009 and all candidates in the elections of 2013 and 2017, the data from the Honduran Tribunal Supremo Electoral contain unique candidate identifiers. For the remaining candidates, we use a similar procedure as in Switzerland to construct the candidate-level panel.⁸ We have 3,025 observations from 2,644 candidates.

Again, we divide the vote margin by the number of eligible voters to accommodate differences in district magnitude. Figure B.2 in the Supplementary Material depicts the density of the assignment variable.

Results. According to Figure 2a, a close election victory in the election in t does not increase the election probability in the subsequent election in $t + 1$. This result is confirmed with different bandwidths (Figure 2b) and in second-order polynomial regressions (Table B.2 in the Supplementary Material). The validity tests raise no concerns. All candidate characteristics are balanced between elected and nonelected candidates (Panels (B) and (C) of Table B.2 in the Supplementary Material).

3.3 Closed-List Highest Average Methods: Norway, 1953–1985

In this last application, we briefly illustrate how our measure of closeness can be applied in the context of closed-list electoral systems, where only the calculation of the party margin is required. We use the data of Fiva and Smith (2018) for Norwegian parliamentary elections. The Norwegian Storting is a unicameral parliament with between 150 and 157 members in the relevant period. Elections are based on closed lists and a highest average method (modified Sainte-Laguë).

Our approach allows us to calculate the vote margin for all candidates and, thereby, to go beyond the focus on candidates next to win or lose a seat (hereafter “marginal candidates”). The focus on marginal candidates is an *a priori* restriction of the estimation sample. Non-marginal candidates can locate as close to the threshold as marginal ones. In this application, closer to the threshold marginal candidates dominate (Figure B.3 in the Supplementary Material). However, in other applications, this pattern might be less pronounced. More importantly, using all, marginal as well as non-marginal, candidates permits a data-driven sample selection.

Figure 3 depicts the incumbency advantage for marginal candidates in panel (a) and for all candidates in panel (b). In this application, the *a priori* restriction of the sample to marginal candidates makes a substantial difference. With the inclusion of all candidates, the effect is larger.

⁸ We have five observations of the 2009 elections with missing candidate information. We cannot know if these candidates run in later years.

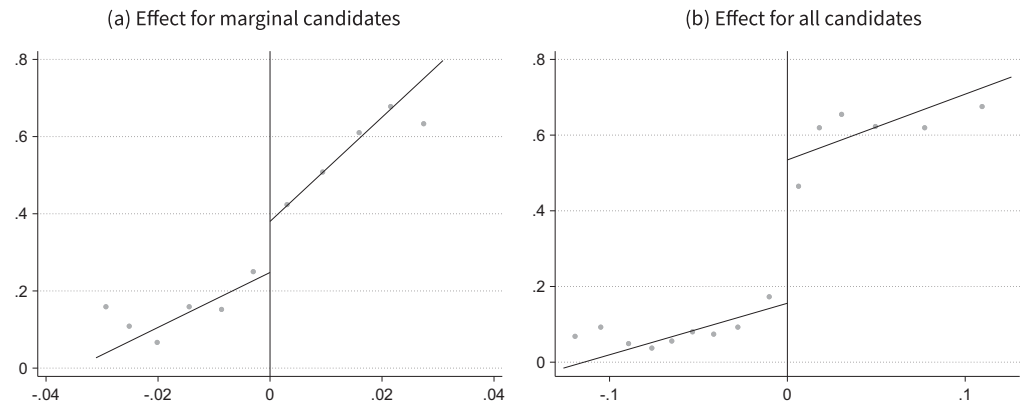


Figure 3. Incumbency advantage estimates for Norway. This figure depicts the relationship between the assignment variable in t (x -axis) and the probability of being elected in $t + 1$ (y -axis), whereby t indexes elections, with the average values in quantile-spaced bins and linear regressions on both sides of the threshold. On both sides, we use the same optimal bandwidth for the election probability in $t + 1$ (Calonico et al. 2017) and a triangular kernel. Panel (a) restricts the sample to the marginal candidates, Panel (b) includes all candidates.

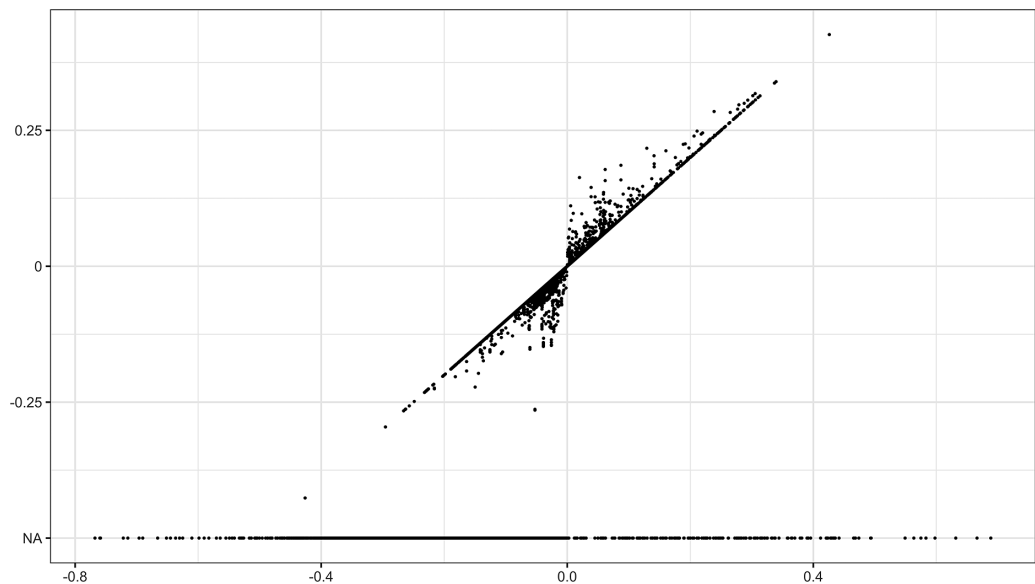


Figure 4. Comparing our overall margin with the candidate margin. This figure documents the relationship between our overall vote margin (x -axis) and the candidate margin (y -axis). Both variables are rescaled by the number of eligible voters in a canton.

3.4 Our Measure in Comparison

We briefly discuss how our approach relates to the three approaches summarized in the introduction. Our approach is closest to the first one, which is based on vote distances. In contrast to this first approach, ours allows calculating the vote margin for both families of seat allocation methods, closed-list and open-list systems, and non-marginal candidates. Our application for Norway demonstrates that the inclusion of non-marginal candidates matters. It increases the sample size, allows for a data-driven bandwidth selection, and—in our application—increases the estimated incumbency advantage substantially.

The second approach focuses exclusively on the candidate margin, which introduces systematic measurement error and makes it impossible to calculate the vote margin for all candidates. We exemplify these points with data from our application for Switzerland in Figure 4. For 44% of the candidates, the candidate margin is not defined as their party won no seat or as all their copartisan

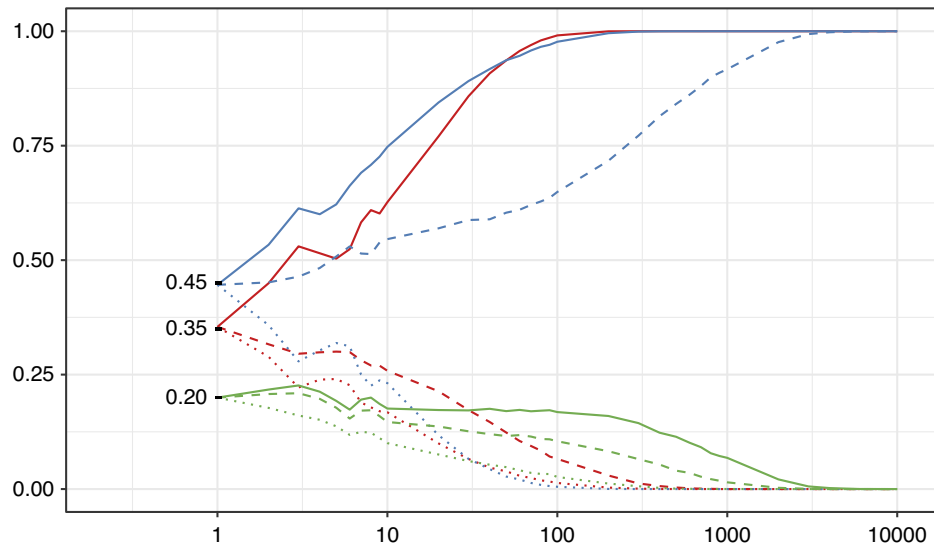


Figure 5. Arbitrariness of simulated election probabilities. The figure depicts simulated election probabilities (y -axis) for different sizes of simulation samples on a logarithmic scale (x -axis). The election probabilities are simulated for our numerical example with vote shares of 0.45 for party P_1 (blue lines), 0.35 for P_2 (red lines), and 0.2 for P_3 (green lines). For each party, the election probabilities are simulated for the first-ranked (solid line), second-ranked (dashed line), and third-ranked (dotted line) candidate. The vote shares of the individual candidates are 0.25, 0.15, and 0.05 for P_1 , 0.22, 0.08, and 0.05 for P_2 , and 0.08, 0.07, and 0.05 for P_3 . The simulation is based on the approach of Kotakorpi et al. (2017).

candidates were elected. These candidates are depicted at the bottom of the figure (“NA”). For 3% of the candidates, the party margin, not the candidate, is binding. These are the candidates off the 45-degree line.

The third approach suffers from the conceptual problem that simulated election probabilities heavily depend on arbitrary simulation settings. Figure 5 illustrates this for our example with three parties P_1 , P_2 , and P_3 and three seats. With one resampled vote, the simulated election probabilities of all candidates coincide with their party’s vote share of 0.45 for P_1 , 0.35 for P_2 , and 0.2 for P_3 . Already for around 5,000 resampled votes, the simulated election probabilities of all candidates converge to 0 for nonelected candidates or to 1 for elected candidates. Thus, for example, for the first ranked candidate of P_2 , the simulated election probability ranges from 0.35 to 1. In addition, the third approach fails for technical reasons: With our data for Switzerland, for many election districts and years, the simulation method fails to meet the necessary convergence criterion (see footnote 3, Kotakorpi et al. 2017), even after over 2,000,000 resamplings.

4 Conclusion

PR systems are the dominant electoral formula around the world. Hence, our measure of electoral closeness in PR systems opens up rich research opportunities. It allows researchers to investigate the determinants and consequences of electoral competitiveness down to the candidate level. Examples are the relationship between competitiveness and turnout, candidate characteristics, and electoral and campaigning strategies of parties and candidates. Such research could combine our closeness measure with candidate surveys such as the Comparative Candidate Survey (similar to Selb and Lutz 2015). We propose a unified measure for different PR systems and provide a consistent assignment variable in candidate-level RD designs, thus making this credible research design applicable to PR systems. In addition, our closed-form solution is flexible. As we demonstrate in the illustration for Switzerland, it can accommodate particularities of an electoral system such as party alliances. This flexibility is important because hardly any two PR systems look the same.

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Competing Interest

The authors declare no competing interest exists.

Supplementary Material

For supplementary material accompanying this paper, please visit <https://doi.org/10.1017/pan.2023.22>.

Data Availability Statement

Replication data and code can be found in Harvard Dataverse at <https://doi.org/10.7910/DVN/FBZMWN> (Luechinger, Schelker, and Schmid 2023).

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